Report: Physics Constrained Stochastic Statistical Models for Extended Range Environmental Prediction

Andrew J. Majda
New York University
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, NY 10012
Phone: (212) 998-3323 Fax: (212) 995-4121 Email: jonjon@cims.nyu.edu

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The timing of the receipt of the MURI funds in August 2012 hampered the assembly of the full MURI team in this initial reporting period since typically post docs in applied math only became available for hiring after June, July, August, 2013. Despite this difficulty there are exciting results on eight major topics in the MURI during the past year by MURI-PI, Andrew Majda, Co-PI's , Sam Stechmann (U. Wisconsin), John Harlim (Penn State), Duane Waliser (JPL-UCLA), and Dimitri Giannakis (Courant Institute) with their post docs and Ph.D. students:

A. Reemergence Mechanisms for North Pacific Sea Ice Revealed through Nonlinear Laplacian Spectral Analysis
B. Symmetric and Antisymmetric Madden-Julian oscillation signals in tropical deep convective systems
C. Limits of Predictability in the North Pacific Sector of a Comprehensive Climate Model
D. Stochastic Skeleton Model for the MJO
E. Observations and the Stochastic Skeleton Model
F. Physics Constrained Nonlinear Regression Models for Time Series
G. Statistically Accurate Low Order Models for Uncertainty Quantification in Turbulent Dynamical Systems
H. Mathematical Techniques for Quantifying Uncertainty in Complex Systems with Model Error with Prototype Applications

I. Findings: Scientific Results

A. Reemergence Mechanisms for North Pacific Sea Ice Revealed through Nonlinear Laplacian Spectral Analysis

Andrew Majda, Dimitris Giannakis, and CAOS PhD student Mitch Bushuk have submitted a paper (MURI-6) to the Journal of Climate on the coupled variability of sea ice and sea surface temperature (SST). The paper studies spatiotemporal modes of variability of sea ice concentration and SST in the North Pacific sector of the Community Climate System Model version 3 (CCSM3). These modes were obtained via nonlinear Laplacian spectral analysis (NLSA), a recently developed data analysis technique for high-dimensional nonlinear datasets, which is also used in other MURI objectives such as item B) ahead. The existing NLSA algorithm was modified to allow for a scale-invariant coupled analysis of multiple variables in different physical units. The coupled NLSA modes were utilized to investigate North Pacific sea ice reemergence: a process in which sea ice anomalies originating in the melt season (spring) are positively correlated with anomalies in the growth season (fall) despite a loss of correlation in the intervening summer months. It was found that low-dimensional families of NLSA modes are able to reproduce the lagged correlations observed in sea ice data from the Bering and Okhotsk seas, the two regions of high sea ice variability in the North Pacific (see Figure 1). Each mode family is closely related with a low-frequency pattern of North Pacific SST variability: the Bering Sea family with the North Pacific Gyre Oscillation (NPGO), and the Okhotsk Sea family with the Pacific Decadal Oscillation (PDO). Moreover, these mode families provide a mechanism for sea ice reemergence, in which summer SST anomalies store the memory of spring sea ice anomalies, allowing for sea ice anomalies of the same sign to appear in the fall season. Lagged correlations are significantly strengthened by conditioning on certain low-frequency modes being active, in either positive or negative phase.

Ongoing and future developments in this area include incorporation of sea level pressure in the analysis in order to elucidate the role of the atmosphere in sea ice reemergence. Moreover, the experience gained in the previous year's MURI publication by Giannakis and Majda on limits of predictability in the North Pacific will be employed here to formulate regression models for the coupled low-frequency variability of sea ice, ocean, and atmosphere. A further avenue of research will be to perform this analysis in the Barents sea, which is another region where sea ice reemergence is known to occur.

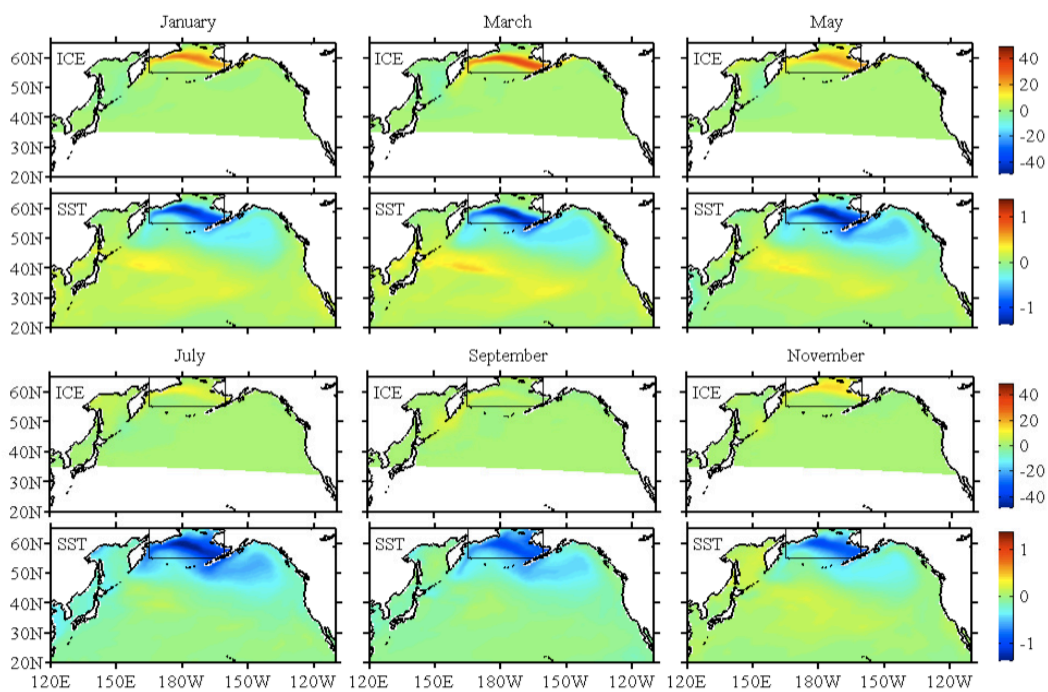

Figure 1: Sea Ice and SST patterns extracted using NLSA. The Bering Sea (boxed) exhibits a spring-fall sea ice reemergence. Positive spring sea ice anomalies imprint negative SST anomalies as they move northward during the melt season. The SST anomalies persist through the summer months, and when the ice returns in the growth season, the positive sea ice anomaly is reproduced.

B. Symmetric and Antisymmetric Madden-Julian oscillation signals in tropical deep convective systems

Andrew Majda, Dimitris Giannakis, and MURI collaborator Wen-wen Tung of Purdue University, have submitted a two-paper series to the Journal of Atmospheric Science (MURI 5) on the significance of asymmetry in systems of organized convection associated with the Madden-Julian oscillation (MJO). Satellite infrared brightness temperature data for the period 1983--2006 were decomposed into subsets symmetric and antisymmetric about the equator. Using NLSA, modes of variability were extracted representing symmetric and antisymmetric features of MJO convective systems, along with a plethora of other modes of tropical convective variability. It was found that the boreal winter MJO emerges as a single pair of modes in both symmetric and antisymmetric convection signals (see Figure 2). Phase composites of the corresponding kinematic and thermodynamic anomalous fields were constructed using reanalysis data. It was found that the predominantly symmetric convective systems are potentially short-lived due to equatorial dry air intrusion eradicating equatorial convection when the MJO crosses the Maritime Continent. The predominantly antisymmetric convective systems, however, are less affected by dry intrusion; the strength of the MJO convective systems as well as anomalous circulations can be maintained and enhanced in the West Pacific. It was also found that the off-equatorial convective systems enhanced during the MJO are mostly deep convection and stratiform anvils, unlike the typical complex of shallow-congestus-deep convection to stratiform anvils on the equator. The multiscale interactions between the diurnal, MJO, and ENSO modes of convection were studied. It was found that the symmetric component of MJO convection is out of phase with the symmetric component of the diurnal cycle, while the antisymmetric component of MJO convection is in phase with the antisymmetric diurnal cycle. The former relationship breaks down during strong El Nino events, and both relationships break down during prolonged La Ninas.

Ongoing and future work in this area will be to apply the information-theoretic techniquedeveloped by Giannakis and Majda , J. Climate, 25(6), 2012, p. 1793 to quantify the predictability of the MJO signals represented by the NLSA temporal modes. In particular, precursors for MJO initiation and termination will be sought. This effort is currently being pursued in collaboration with MURI postdoc Eniko Szekely at Courant. Moreover, the analysis of meridionally-averaged brightness temperature fields in the recently submitted JAS papers will be extended to 2D datasets. Numerical work for this study is being curried out at the dedicated computing cluster at Courant acquired through the MURI.

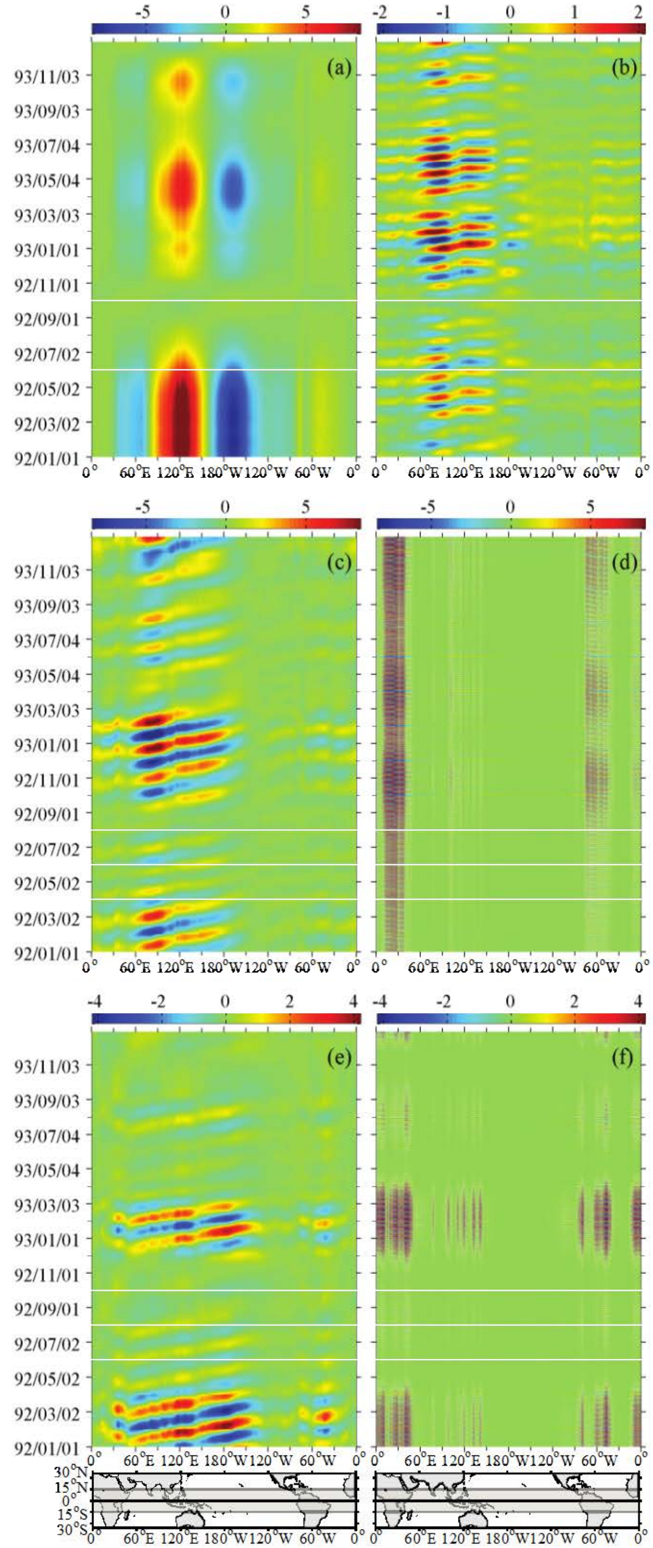

Figure 2: Multiscale spatiotemporal modes of the averaged temperature brightness field for 1992—1993 extracted using NLSA. (a) Symmetric interannual (ENSO) mode; (b) symmetric Maritime Continent mode; (c) symmetric MJO pair; (d) symmetric diurnal pair; (e) antisymmetric MJO pair; (f) antisymmetric diurnal pair.

C. Limits of Predictability in the North Pacific Sector of a Comprehensive Climate Model

Majda and Courant Institute Assistant Professor and MURI collaborator, Dimitri Giannakis, have published a paper in GRL (MUR 11). We study limits of interannual to decadal predictability of sea surface temperature (SST) in the North Pacific sector of the Community Climate System Model version 3 (CCSM3). Using a set of low-frequency and intermittent spatiotemporal SST modes acquired through nonlinear Laplacian spectral analysis (a nonlinear data manifold generalization of singular spectrum analysis), we build a hierarchy of regression models with external factors to determine which modes govern the dynamic evolution and predictability of prominent large-scale patterns, namely the Pacific Decadal Oscillation (PDO) and North Pacific Gyre Oscillation (NPGO). Retaining key triple correlations between prognostic variables and external factors, as well as the seasonality of the data, we find that the PDO and NPGO modes of CCSM3 can be described with remarkably high fidelity as an outcome of forcing by the intermittent modes (with phase demodulation by the seasonal cycle) and cubic interactions between the low-frequency modes. Our results differ from the classical picture of ENSO-driven autoregressive models for North Pacific SST variability, providing evidence that intermittent processes, such as variability of the Kuroshio current, limit long-range predictability in this climate model.

This use Nonlinear Laplacian Spectral Analysis (NLSA) for a comprehensive climate model opens the way for further applications of these techniques to reveal significant intermittent behavior which limits predictability.

D. Stochastic Skeleton Model for the MJO

Majda, co-PI Stechmann, and Courant Institute MURI Postdoc Sulian Thual have just submitted a paper to JAS which is already accepted with minor revisions (MURI 4). We design and study a stochastic MJO skeleton model that accounts for (i) the intermittent generation of MJO events and (ii) the organization of MJO events into wave trains with growth and demise, as seen in nature. These features are in addition to the most fundamental MJO features that were recovered by the original MJO skeleton model of Majda and Stechmann: (iii) a slow eastward speed of roughly 5 ms-1, (iv) a peculiar dispersion relation with dw/dk=0, and (v) a horizontal quadrupole vortex structure. The additional intermittent aspects (i) and (ii) are achieved by developing a simple stochastic parameterization for the unresolved details of synoptic-scale activity, that is coupled to otherwise deterministic processes in the skeleton model. In particular, the intermittent initiation, propagation and shut down of MJO wave trains in the skeleton model occur through these stochastic effects. This includes examples with a background warm-pool where some initial MJO-like disturbances propagate through the western region but stall at the peak of background convection/heating corresponding to the maritime continent in nature.

This stochastic MJO skeleton model provides a basis for investigating a low-dimensional model for predicting the MJO.

E. Observations and the Stochastic Skeleton Model

Dr. Justin Stachnik completed his Ph.D. at Texas A&M University in early 2013 and began as Postdoctoral Scholar with UCLA JIFRESSE on April 8, 2013. His previous research includes an observational analysis of the heating and moisture characteristics of low-latitude clouds and precipitating systems associated with the large-scale tropical circulation. Dr. Stachnik also has experience evaluating atmospheric reanalyses and other numerical datasets (e.g., global climate model [GCMs]). He is currently working in residence at the NASA Jet Propulsion Laboratory (JPL) under the guidance of co-PI Waliser.

WORK COMPLETED AND EARLY RESULTS

Given the unique relationship between: 1) the time tendency of the sub-planetary scale wave activity (i.e., convective heating) and 2) planetary scale, lower-tropospheric water vapor anomalies implemented in the skeleton model, an initial attempt was made to further quantify the relationship between MJO heating and moisture using a mixture of satellite and radiosonde data following the approach in Stachnik et al. (J. Climate 2013). Although the technique was initially developed to account for the heating effects of weakly precipitating systems (e.g., anvil regions and shallow convection) and redistribution of heating by non-precipitating clouds that remain undetected in current satellite retrievals, the composite profiles at best only showed weak low-level heating in advance of the main MJO deep convective anomalies.

Although the new results for the corresponding calculations of the apparent moisture sink did correctly identify the maximum values in the lower troposphere (i.e., below 850 hPa), the canonical westward tilt with height was likewise entirely absent from the MJO phase composites in both the Indian and west Pacific Oceans. Moreover, the low-level moistening anomalies were out of phase with the deep convective heating (i.e., wave activity). Additional calculations using a restricted training dataset from only local oceanic domains (e.g., radiosonde observations from the Mirai Indian Ocean Cruise for the Study of the MJO-Convection Onset [MISMO]) did reveal a better westward tilt with height for the apparent moisture sink, though the overall representativeness remains uncertain given the lower sampling number and likely aliasing of mid-level clouds (e.g., altocumulus) as a predominantly low-level regime in the satellite data throughout this domain.

The current approach relies upon a k-means clustering algorithm with a semi-objective method for selecting the appropriate number of clusters and cloud regimes. Considering the discrepancies in the composite cloud properties over land and ocean for the cumulus regime, we speculate additional clusters/regimes are needed in order to properly quantify the effects of shallow clouds. Future work would benefit from using an entirely objective definition for the number of clusters following the objective entropy criteria outlined in Giannakis et al. (2012). Furthermore, retrievals of the MJO vertical structure would also likely improve by incorporating radiosonde observations from the Dynamics of the Madden-Julian Oscillation (DYNAMO) campaign when calculating the average profiles for each cloud regime. Therefore, we have temporarily tabled this work.

These results may eventually support closely related MURI projects by providing potential validation and model input for work intended by co-PI Stechmann to include multi-cloud stochastic effects into the MJO skeleton model. Recent efforts involving PI Majda have indicated that a stochastic multi-cloud model can improve conventional parameterizations (Khouider et al. 2010; Frenkel et al. 2012).

CURRENT AND FUTURE WORK

A recent advance incorporated into the stochastic skeleton model is the ability to reproduce the intermittent generation of MJO events, in addition to organizing MJOs into wave trains that experience growth and demise as often seen in nature. These wave trains are poorly captured by most GCMs and Co-PI Waliser and his postdoc (Justin Stachnik) are beginning to investigate the simulated MJO heating and moisture variability in the stochastic skeletal model compared to satellite-derived observations and reanalyses. In particular, they intend to generate vertical and spatial composites of the thermodynamic fields related to the initiation of primary and successive/wave train MJO events, as well as those precursor conditions leading to quiescent periods of the MJO. The work is currently underway and abstract has been submitted for reporting preliminary results at the AGU 2013 Fall Meeting. These results may also support other MURI projects by providing guidance on the specification of parameters in the low-dimension dynamic model.

F. Physics Constrained Nonlinear Regression Models for Time Series

Majda and co-PI John Harlim have published a paper in Nonlinearity on this important topic (MURI 10). The objective of the work in this article was to solve the following problem, which often arises in various scientific disciplines that involve predictability issues:

Given partial observations of complex dynamical systems, develop low-dimensional, data driven nonlinear dynamical models for statistical prediction!

Purely data driven linear regression models, fitting variance and lagged autocorrelation of time series under stationary assumptions, have been developed and applied with some skill for El Nino Southern Oscillation prediction. In various applications, however, there is an inherent need to incorporate nonlinearity to model low frequency patterns with non-Gaussian distributions. Recent approach such as the purely data driven multi-level quadratic regression models, unfortunately, can have finite-time blow up of statistical solutions and/or pathological behavior of their invariant measure. This clearly indicates a severe limitation of the skill of these methods for long range forecasting. Nevertheless, the use of multi-level nonlinear regression can potentially incorporate important memory effects of unresolved degrees of freedom in a low order model.

To address this problem, we developed a new class of physics constrained multi-level quadratic regression models. These models were designed to incorporate the memory effects in time as well as an additional nonlinear noise from energy conserving nonlinear interaction to avoid the pathological finite-time blow up. Theoretically, we applied the conditions for geometric ergodicity to provide mathematical guidelines for the non-degenerate (non blow-up) solutions of these physics constrained multi-level regression models. Subsequently, we use the available data set (or partial observations) to estimate the parameters in these models. Here, we implemented an existing scheme which combines extended Kalman filter (EKF) with the Belanger's noise estimation scheme to estimate the parameters associated with the deterministic part of the dynamical system and the stochastic noise amplitudes. In our most complex application, we numerically checked the fidelity of the statistical prediction skill of the proposed physics constrained multi-level regression models given only noisy observations of the first Fourier mode of the TBH (truncated Burgers-Hopf) model without any direct knowledge that the time series are solutions of this model.

PI Majda, co-PI Harlim, and A. Mahdi have submitted a new paper to J. Computational Physics on this topic (MURI 3). In there, they introduced a novel, improved parameter estimation algorithm to avoid numerical blow-up that sometimes can occur with the existing EKF based algorithm when observations are sparse even when the physics constrained nonlinear model has non blow-up solutions.

Several stringent tests and applications of the method are developed here. In the most complex application, the perfect model has 57 degrees of freedom involving a zonal (east–west) jet, two topographic Rossby waves, and 54 nonlinearly interacting Rossby waves; the perfect model has significant non-Gaussian statistics in the zonal jet with blocked and unblocked regimes and a non-Gaussian skewed distribution due to interaction with the other 56 modes. We only observe the zonal jet contaminated by noise and apply the ensemble filter algorithm for estimation. Numerically, we find that a three dimensional nonlinear stochastic model with one level of memory mimics the statistical effect of the other 56 modes on the zonal jet in an accurate fashion, including the skew non–Gaussian distribution and autocorrelation decay. On the other hand, a similar stochastic model with zero memory levels fails to capture the crucial non-Gaussian behavior of the zonal jet from the perfect 57-mode model.

As more practical applications of this technique are developed by the MURI team, we plan to keep developing the theory as needed.

G. Statistically Accurate Low Order Models for Uncertainty Quantification in Turbulent Dynamical Systems

Majda and Sapsis (MIT) have achieved a potential major breakthrough with a new class of methods for UQ.

Turbulent dynamical systems are characterized by both a large dimensional phase space and a large dimension of instabilities i.e. a large number of positive Lyapunov exponents on the attractor. Turbulent dynamical systems are ubiquitous in many complex systems with fluid flow such as for example, the atmosphere, ocean, and coupled climate system, confined plasmas, and engineering turbulence at high Reynolds numbers. In turbulent dynamical systems, these linear instabilities are mitigated by energy conserving nonlinear interactions which transfer energy to the linearly stable modes where it is dissipated resulting in a statistical steady state. Uncertainty quantification (UQ) in turbulent dynamical systems is a grand challenge where the goal is to obtain statistical estimates such as the change in mean and variance for key physical quantities in the nonlinear response to changes in external forcing parameters or uncertain initial data. These key physical quantities are often characterized by the degrees of freedom which carry the largest energy or variance and an even more ambitious grand challenge is to develop truncated low order models for UQ for a reduced set of important variables with the largest variance. This is the topic of the research by Majda in the MURI [2, 9].

In the work of Sapsis and Majda [2, 9] a systematic strategy is developed for building statistically accurate low order models for UQ in turbulent dynamical systems. First, exact dynamical equations for the mean and the covariance are developed; the possibly intermittent effects of the third order statistics on these low-order statistics are present in the exact equations.

Secondly, an approximate nonlinear dynamical system for the evolution of the mean and covariance constrained by covariance forcing from minimal damping and random forcing on the unperturbed attractor is formulated; it is required that this dynamical system has the unperturbed mean and covariance as a stable fixed point. In the third calibration step, the effect of the third moments on the mean and the covariance in the approximate dynamical system for the statistics are calibrated efficiently at the unperturbed steady state using only the measured first and second moments. The result at this stage is a very recent algorithm for UQ called Modified Quasilinear Gaussian (MQG) closure [9] which applies on the entire phase space of variables. In the fourth step, the MQG algorithm is projected on suitable leading EOF patterns with further efficient calibration of the effect of the unresolved modes at the unperturbed statistical steady state. This final step defines the reduced order MQG (ROMQG) method for UQ in turbulent dynamical systems. The research of Sapsis and Majda includes two highly nontrivial applications of the ROMQG method to UQ. The first application involves the Lorenz 96 (L-96) model which is a non-trivial forty dimensional turbulent dynamical system which mimics mid-latitude atmospheric turbulence and is a popular model for testing methods for statistical prediction, data assimilation or filtering, FDT, and UQ. The advantage of the forty mode L-96 with many features of turbulent dynamical systems is that very large ensemble Monte-Carlo simulations can be utilized for validation in transient regimes. Here the ROMQG algorithm has remarkably robust skill for UQ in the transient response to general random external forcing for truncations as low as one, two or three leading Fourier (EOF) modes. The second application involves a prototype example of two-layer ocean baroclinic turbulence. Here the turbulent system has over 125,000 degrees of freedom so validation through transient Monte-Carlo simulations is impossible and only the nonlinear statistical steady state response to the change in shear can tested for various perturbed shear strengths. Here the ROMQG algorithms for UQ utilizing 252 EOF modes (less than 0.2% of the total modes) are able to capture the nonlinear response of both the one-dimensional energy spectrum and heat flux spectrum at each wavenumber with remarkable skill for a wide range of shear variations. Majda plans to work with MURI funded post doc D. Comeau and CAOS Ph.D. student DiQi, to apply this method to prototype midlatude atmospheric dynamics in the next stage.

H. Mathematical Techniques for Quantifying Uncertainty in Complex Systems with Model Error with Prototype Applications

Development of new uses of information theory to quantify uncertainty, irreducible impression, sensitivity, and long range forecasting skill. This work includes expansion of the concept of information barriers developed by Majda and co-workers to filtering or data assimilation with model error (MURI7) and a new information theoretic analysis of the skill of multi-model ensembles of imperfect models in intermediate and long range forecasting (MURI 1). Understanding of the phenomena of catastrophic filter divergence in data assimilation has been developed (MURI 8).

Majda plans to pursue further developments for information barriers in Lagrangian tracers with model error with MURI post doc, Xin Tong, and CAOS Ph.D. student, Nan Chen. Majda and MURI post doc, Yoonsang Lee, are working on multi-scale data assimilation.

II. Contributions to Discipline

In A), B), Giannakis and Majda have developed a novel physics constrained mathematical technique for large data sets, NLSA, to capture both intermittency and low frequency variability; the North Pacific sea ice and Claus data sets provide important new applications of the method.

In F) Harlim and Majda have developed new physics constrained algorithms to use data to build low order nonlinear regression models for time series from physical systems. In D) Thual, S. Stechmann, and Majda have developed a novel stochastic parameterization of synoptic scale activity to build a stochastic skeleton model for the MJO; this is the first low order model of the MJO which reproduces its key features including initiation, propagation and demise of MJO wave trains as seen in nature. The new approach to UQ for turbulent systems developed by Sapsis and Majda in G) involves novel mathematical developments to physically constrain stochastic statistical models.

III. Contributions to Outside Disciplines

The application of NLSA by Tung, Giannakis, and Majda to Claus data provides a new observational perspective on various types of MJO's seen in nature. Stachnik and Waliser have taken the first steps in linking the dynamics of the low order stochastic skeleton model to nature. An important agenda problem in the next year for the MURI team is to identify observational surrogates for the synoptic scale activity in the stochastic MJO skeleton model. The application of NLSA to ice and SST on the North Pacific sector of the NCAR climate model by Bushuk, Giannakis, and Majda reveals a concise low order description of these coupled variables involving both intermittency and low frequency variability. The low order predictive model for SST developed by Giannakis and Majda for SST in the North Pacific sector of NCAR's climate model reveals the important role of intermittent modes associated with the Kuroshio current in controlling the predictive skill of the low frequency modes. The inclusion of the role of the atmosphere through sea level pressure in both of these low order representations is a high priority in the short term.

PERSONNEL

The full MURI team is now assembled with ten post docs and four CAOS – CIMS Ph.D. students (Chen, DiQi, Yang, Brenowitz). Here is the list and their starting dates for the post docs

MURI Post docs

I. Courant NYU (Majda and Giannakis)
   - S. Thual (January 2013)
   - Y. Lee (June 2013)
   - E. Szekely (July 2013)
   - J. Zhao (September 2013)
   - D. Comeau (September 2013)
   - X. Tong (September 2013)

II. Penn State (J. Harlim)
   - T. Berry (August 2013)

III. U. Wisconsin (S. Stechmann)
   - H.R. Ogrosky (July 2013)
   - S. Chen (September 2013)

IV. Cal Tech - UCLA (D. Waliser)
   - J.Stachnik (April 2013)

We held a preliminary MURI workshop at Cal Tech JPL during January 2013 hosted by Duane Waliser with active participation of his tropical observation/prediction group. An extensive one week workshop with the Full MURI Team is planned for January 2014.

PUBLICATIONS

1. M. Branicki and A. Majda “An information-theoretic framework for improving imperfect predictions via Multi Model Ensemble forecasts” submitted, J. Nonlinear Science, July 2013
2. T. Sapsis and A. Majda “Statistically Accurate Low Order Models for Uncertainty Quantification in Turbulent Dynamical Systems” published, PNAS, July 2013
3. J. Harlim, A. Mahdi and A. Majda “An Ensemble Kalman Filter for Statistical Estimation of Physics Constrained Nonlinear Regression Models” submitted, J. Comp. Phys., June 2013
4. S. Thual, A. Majda, S. Stechmann “A Stochastic Skeleton Model for MJO” submitted, J. Atmos. Sci., June 2013
5. W. Tung, D. Giannakis and A. Majda “Symmetric and antisymmetric Madden-Julian oscillation signals in tropical deep convective systems” submitted, J. Atmos. Sci., April 2013
6. M. Bushuk, D. Giannakis and A. Majda “Reemergence Mechanisms for North Pacific Sea Ice Revealed Through Nonlinear Laplacian Spectral Analysis” submitted, J. Climate, April 2013
7. M. Branicki and A. Majda “Quantifying Filter Performance for Turbulent Dynamical Systems through Information Theory” submitted, Comm. Math. Sci., February 2013
8. G. Gottwald and A. Majda “A mechanism for catastrophic filter divergence in data assimilation for sparse observation networks” submitted, Nonlinear Processes in Geophysics, December 2012
9. T. Sapsis and A. Majda “Blending Modified Gaussian Closure and Non-Gaussian Reduced Subspace Methods for Turbulent Dynamical Systems” submitted, J. Nonlinear Science, November 2012
10. A. Majda and J. Harlim "Physics Constrained Nonlinear Regression Models for Time Series", Nonlinearity, 2013 Volume 26 #1, 201–217
11. D. Giannakis and A. Majda "Limits of predictability in the North Pacific sector of a comprehensive climate model", Geophysical Research Letters, December 18, 2012, Volume 39, Issue 24