Report: Low Frequency Predictive Skill Despite Structural Instability and Model Error

Andrew J. Majda
New York University
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, NY 10012
Phone: (212) 998-3323 Fax: (212) 995-4121 Email: jonjon@cims.nyu.edu

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I. Findings: Scientific Results

The PI, Andrew Majda, and the postdoctoral associates, Themis Sapsis and Michal Branicki (funded by the DRI) have submitted the following papers all with the PI as author (and other collaborators listed). Sapsis and Branicki have recently accepted tenure track positions at major universities MIT and U. Edinburgh respectively.

A) Statistically Accurate Low Order Models for Uncertainty Quantification in Turbulent Dynamical Systems

Majda and his DRI post doc Sapsis have achieved a potential major breakthrough with a new class of methods for UQ.

Turbulent dynamical systems are characterized by both a large dimensional phase space and a large dimension of instabilities i.e. a large number of positive Lyapunov exponents on the attractor. Turbulent dynamical systems are ubiquitous in many complex systems with fluid flow such as for example, the atmosphere, ocean, and coupled climate system, confined plasmas, and engineering turbulence at high Reynolds numbers. In turbulent dynamical systems, these linear instabilities are mitigated by energy conserving nonlinear interactions which transfer energy to the linearly stable modes where it is dissipated resulting in a statistical steady state. Uncertainty quantification (UQ) in turbulent dynamical systems is a grand challenge where the goal is to obtain statistical estimates such as the change in mean and variance for key physical quantities in the nonlinear response to changes in external forcing parameters or uncertain initial data. These key physical quantities are often characterized by the degrees of freedom which carry the largest energy or variance and an even more ambitious grand challenge is to develop truncated low order models for UQ for a reduced set of important variables with the largest variance. This is the topic of the research by Sapsis and Majda in the DRI [2, 5, 6, 7].

In the work of Sapsis and Majda [2, 5, 6, 7] a systematic strategy is developed for building statistically accurate low order models for UQ in turbulent dynamical systems. First, exact dynamical equations for the mean and the covariance are developed; the possibly intermittent effects of the third order statistics on these low-order statistics are present in the exact equations.

Secondly, an approximate nonlinear dynamical system for the evolution of the mean and covariance constrained by covariance forcing from minimal damping and random forcing on the unperturbed attractor is formulated; it is required that this dynamical system has the unperturbed mean and covariance as a stable fixed point. In the third calibration step, the effect of the third moments on the mean and the covariance in the approximate dynamical system for the statistics are calibrated efficiently at the unperturbed steady state using only the measured first and second moments. The result at this stage is a very recent algorithm for UQ called Modified Quasilinear Gaussian (MQG) closure [7] which applies on the entire phase space of variables. In the fourth step, the MQG algorithm is projected on suitable leading EOF patterns with further efficient calibration of the effect of the unresolved modes at the unperturbed statistical steady state. This final step defines the reduced order MQG (ROMQG) method for UQ in turbulent dynamical systems. The research of Sapsis and Majda includes two highly nontrivial applications of the ROMQG method to UQ. The first application involves the Lorenz 96 (L-96) model which is a non-trivial forty dimensional turbulent dynamical system which mimics mid-latitude atmospheric turbulence and is a popular model for testing methods for statistical prediction, data assimilation or filtering, FDT, and UQ. The advantage of the forty mode L-96 with many features of turbulent dynamical systems is that very large ensemble Monte-Carlo simulations can be utilized for validation in transient regimes. Here the ROMQG algorithm has remarkably robust skill for UQ in the transient response to general random external forcing for truncations as low as one, two or three leading Fourier (EOF) modes. The second application involves a prototype example of two-layer ocean baroclinic turbulence. Here the turbulent system has over 125,000 degrees of freedom so validation through transientMonte-Carlo simulations is impossible and only the nonlinear statistical steady state response to the change in shear can tested for various perturbed shear strengths. Here the ROMQG algorithms for UQ utilizing 252 EOF modes (less than 0.2% of the total modes) are able to capture the nonlinear response of both the one-dimensional energy spectrum and heat flux spectrum at each wavenumber with remarkable skill for a wide range of shear variations.

B) Mathematical Techniques for Quantifying Uncertainty in Complex Systems with Model Error with Prototype Applications

Development of new uses of information theory to quantify uncertainty, irreducible impression, sensitivity, and long range forecasting skill. This work includes expansion of the concept of information barriers developed in this DRI to filtering or data assimilation with model error (3) and a new information theoretic analysis of the skill of multi-model ensembles of imperfect models in intermediate and long range forecasting (1). Understanding of the phenomena of catastrophic filter divergence in data assimilation has been developed (4).

II. Contributions to Discipline

The new approach to UQ for turbulent systems developed by Sapsis and Majda involves novel mathematical developments in stochastic statistical modeling.

III. Contributions to Outside Disciplines

The work of Sapsis and Majda has the potential to do UQ for long range forecasting for the atmosphere and ocean. The work of Branicki and Majda on multi-model ensembles with model error has the potential to improve strategies for ensemble prediction in NMME Project. Branicki reported this approach at the recent San Diego meeting.

PUBLICATIONS

1. (M. Branicki & A. Majda) "An information-theoretic framework for improving imperfect predictions via Multi Model Ensemble forecasts" submitted, J. Nonlinear Science, July 2013 (PDF)
2. (T. Sapsis & A. Majda) "Statistically Accurate Low Order Models for Uncertainty Quantification in Turbulent Dynamical Systems" accepted, PNAS, July 2013 (PDF)
3. (M. Branicki & A. Majda) "Quantifying Filter Performance for Turbulent Dynamical Systems through Information Theory" submitted, Comm. Math. Sci., February 2013 (PDF)
4. (G. Gottwald & A. Majda) "A mechanism for catastrophic filter divergence in data assimilation for sparse observation networks" submitted, Nonlinear Processes in Geophysics, December 2012 (PDF)
5. (T. Sapsis & A. Majda) "Blending Modified Gaussian Closure and Non-Gaussian Reduced Subspace Methods for Turbulent Dynamical Systems" submitted, J. Nonlinear Science, November 2012 (PDF)
6. (T. Sapsis & A. Majda) "Blended reduced subspace algorithms for uncertainty quantification of quadratic systems with a stable mean state" submitted, Physica D, October 2012 (PDF)
7. (T. Sapsis & A. Majda) "A statistically accurate modified quasilinear Gaussian closure for uncertainty quantification in turbulent dynamical systems", Physica D, February 26, 2013, doi:10.1016/j.physd.2013.02.009 (PDF)