

## **An Integrative Wave model for the Marginal Ice Zone based on a Rheological Parameterization**

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### **LONG-TERM GOALS**

To enhance wave forecasting models such as WAVEWATCH III (WW3) so that they can predict the marginal ice zone wave climate in the present and future Arctic seas.

### **OBJECTIVES**

1. To build a comprehensive wave-ice interaction mathematical framework for a wide range of ice conditions observable in the Marginal Ice Zone (MIZ);
2. To identify a minimum set of rheological parameters that can reproduce all existing wave-ice interaction types;
3. To test the sensitivity of various parameters in the new wave-ice interaction model;
4. To relate physically detectable ice cover parameters from remote sensing to its rheological properties;
5. To establish a strategy for WW3 to implement the wave-ice interaction mechanisms;
6. To test the model performance and validate it using WW3.

### **APPROACH**

**For objective 1:** Complete the viscoelastic theory. Key individuals are the PI and a PhD student.

*Task 1:* Use an analytical method to determine the propagation of waves through a floating viscoelastic mat for a wide range of effective viscosity and elasticity parameters. This range should include all possible ice cover types in the MIZ. The outcome of this task is a direct relation between viscoelastic properties and the wave dispersion, including the group velocity and the attenuation.

*Task 2:* Obtain the energy flux between different regions of ice covers via analytical solutions, i.e. the transmission and reflection energy between different viscoelastic mats.

**For objective 2:** Justify the rheological parameters. Key individuals are the PI and a PhD student, in collaboration with Vernon Squire and Ben Holt.

*Task 3:* Assemble all existing laboratory and field data of wave propagation in ice covers.

*Task 4:* Determine if all existing evidence of wave property changes across ice covers is reflected in the theoretical model, i.e. verify that the proposed viscoelastic theory is capable of producing qualitatively similar range of the observations.

**For objective 3:** Test the sensitivity of parameters. Key individuals are the PI and a PhD student.

*Task 5:* Apply statistical analysis to examine the sensitivity of model dependence on the rheological parameters. It is likely that over certain ranges of parameters the resulting wave property change may either be highly sensitive or insensitive to changes of the parameter values. This part of the study will help to simplify the wave-ice interaction model.

**For objective 4:** Relate accessible ice cover data to its rheological properties. Key individuals are the PI, a postdoc, and a graduate student, in collaboration with Vernon Squire. Additional information will be collected from all other PIs in this DRI.

*Task 6:* Perform inverse analysis to map accessible wave dispersion and reflection/transmission data to the viscoelastic parameters. The accessible data will include all old and new data from field, laboratory, and remote sensing studies, as well as those derived from the scattering theory.

**For objective 5:** Develop tools to implement the rheological model into WW3. Key individuals are the PI, a postdoc, and a PhD student.

*Task 7:* Determine the best strategy to implement the rheological theory to WW3. The mathematical theory contains many parameters. Each set of parameters produces different wave group velocity, attenuation coefficient, and transmission and reflection properties. Results from the theory are input that needs to feed into WW3. These results are both time and space dependent. Implementation method will be established keeping in mind the resolution, accuracy, and computational efficiency.

**For objective 6:** Test the updated WW3. Key individuals are the PI, a postdoc, and Erick Rogers.

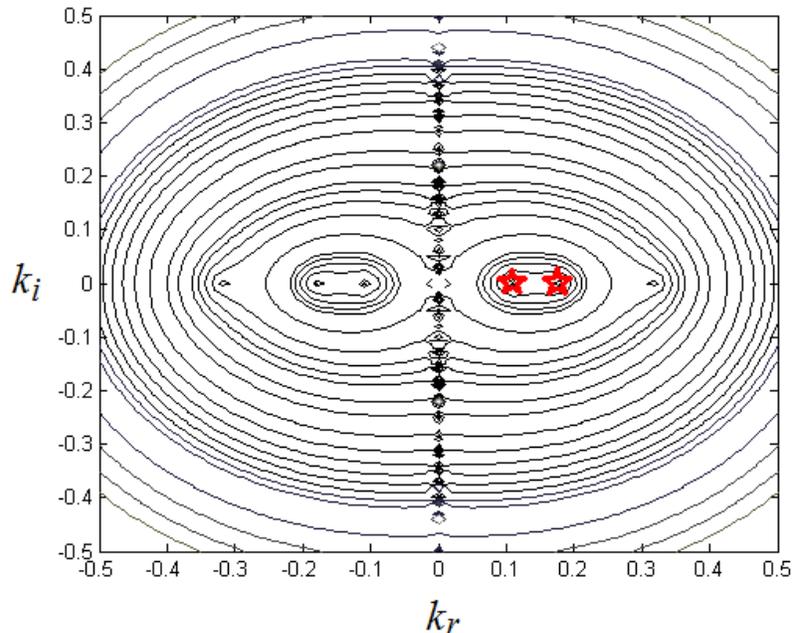
*Task 8:* The modified WW3 will be tested using both hind- and fore-casts.

## **WORK COMPLETED**

Tasks 1 and 7 have made progress. The theoretical dispersion relation for a simple linear viscoelastic rheology was obtained prior to the start of this project (Wang and Shen, 2010). An example of the relevant information from this theory is shown in Fig. 1, where the contours of the residuals of the dispersion relation are plotted on the complex wavenumber plane. The roots of the dispersion relation are in the centers of the contours. The contours are for a specific set of physical parameters as shown in the figure caption. Different physical parameter sets produce different contours and roots. Although there are multiple solutions, two of them are the most important. Their amplitudes dominate the rest. For the parameters shown, these two solutions are marked by stars on the complex plane. Each solution has a real and an imaginary component,  $k_r$  and  $k_i$ , corresponding to the wave number and the attenuation coefficient, respectively. Depending on the viscoelastic parameters, one of these two

modes contains most of the wave energy. For practical applications, this mode is the one used to complete the theoretical framework. A set of FORTRAN subroutines has been developed to compute the wave number/wave speed and the attenuation rate for any given set of parameters. Erick Rogers has implemented these subroutines into WW3. There have been no difficulty encountered yet. The run time is about 10 times longer than the former case which only considered ice concentration, and used it to linearly scaled down the wave energy flux.

Task 2 has also made progress. Theoretical solutions for wave transmission and reflection have been obtained. This work is published recently (Zhao and Shen, 2013). These solutions are obtained using an approximation method. Further study is underway to check if the approximation method is sufficiently accurate. Our approach is to begin with a model that represents the physical processes as closely as possible, then simplify it while still preserve its fundamental behaviour.



**Fig. 1. Roots of the dispersion relation for ice cover thickness 0.5m, water depth 100m, wave period 6s, viscous parameter  $0.05\text{m}^2/\text{s}$ , and elastic parameter  $10^4\text{Pa}$ . Focusing on the right side of the domain, there are three roots shown in the complex plane. The two marked by red stars are the most important ones since they share nearly 100% of the wave energy.**

## RESULTS

Visual representation of how the wave number  $k_r$  and the attenuation coefficient  $k_i$  vary with different ice property and wave frequency is helpful. It can provide a physical intuition of how significant each parameter affects the wave propagation. However, as we began tabulating the results of wave speed and attenuation for different values of frequency, water depth, and the effective elasticity and viscosity of the ice cover, we realized that tabulation was inefficient for implementing into WW3. Instead, directly computing the wave speed and attenuation using the nonlinear dispersion relation provided in the above reference was faster. The solution procedure relies on the initial guess of the true solution. By choosing initial guess from the open water case we can obtain the wave number and attenuation for long waves. These long wave solutions are used incrementally to provide the initial guess for successively shorter waves. With this method, fast convergence is achieved. A set of subroutines in

FORTRAN was created to interface with WW3. So far it runs well. The computing time appears to increase by an order of magnitude from the previous simple algorithm discussed in Tolman (2003). Graphical representation of the dispersion relation over a wide range of parameter space is still desirable to gain comprehensive view of the sensitivity of wave dispersion under different viscoelastic covers. This part will be done in the coming year.

The transmission and reflection of waves from different viscoelastic covers depend on the physical parameters of the adjacent ice covers. An example of the solution is shown in Fig. 2. In this figure we show two cases of the transmission and reflection between different ice covers. The first case (Fig. 2a) is between open water and an ice cover. The second case (Fig. 2b) is between two ice covers. In the case of open water connecting with an ice cover, there are two transmitted waves with transmission coefficients labelled  $T(1)$  and  $T(2)$  respectively. There is one reflection coefficient  $R$ . Between two different ice covers, again there are two significant transmitted modes labelled as  $T(1)$ ,  $T(2)$ . The reflected side also has two significant modes labelled as coefficients  $R(1)$  and  $R(2)$ . The results for a specific set of parameters are given in Fig. 2c. In this specific case, the ice region on the right is a pure elastic cover. The region on the left is either open water or a pure viscous layer over water. These results show that aside from the existence of  $R(2)$  which is absent in the open water case, all other coefficients are the same whether it is an open water joined with an elastic sheet or a viscous layer joined with an elastic sheet. This result means that the scattering theory developed by Squire's group (e.g. Fox and Squire, 1994, Meylan and Squire, 1996, Bennetts and Squire, 2009) is also applicable to the case of ice floes imbedded in a frazil slurry.

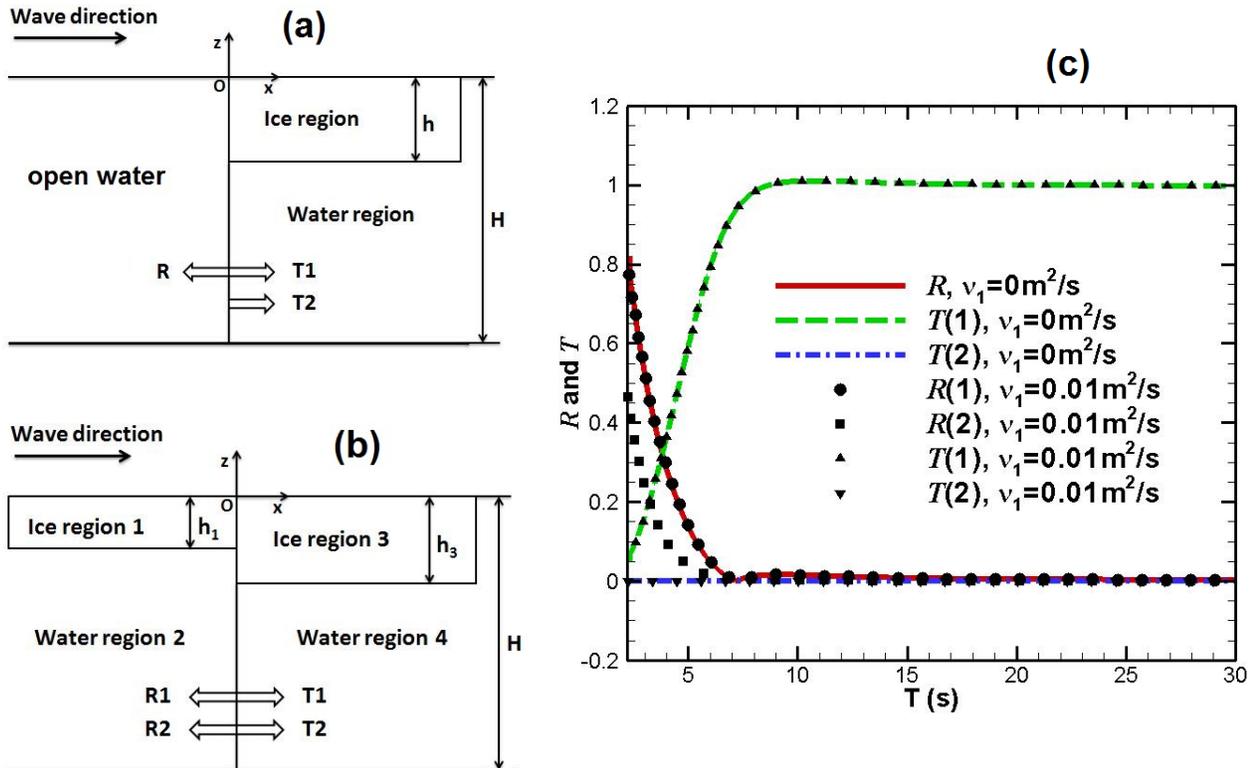


Fig. 2. Reflection and transmission coefficients with respect to wave period. (a) From open water to an ice cover; (b) between two different ice covers; (c) results for the case of wave propagation into a pure elastic ice cover; curves are for case (a) and symbols are for case (b). The elastic parameter for the ice region on the right is 0.05Gpa in both cases. In case (b), ice thickness on both sides are 1m and the viscous parameter of ice region on the left is  $0.01\text{m}^2/\text{s}$ .

## IMPACT/APPLICATIONS

More accurate wave models are necessary tools for many naval operations and environmental protection purposes: such as navigation route planning, offshore structure design in the Arctic, and coastal erosion prevention. They may also be coupled with the ocean circulation models to study the effects of a more dynamic upper surface on the water body underneath.

## RELATED PROJECTS

A related proposal submitted to the Singapore Ministry of Education Academic Research Fund (AcRF) Tier 2 in March, 2013 has been funded. The proposal entitled: “Wave drift and attenuation of viscoelastic floating substances” was submitted by Prof. Adrian Wing-Keung Law ([http://research.ntu.edu.sg/expertise/academicprofile/pages/StaffProfile.aspx?ST\\_EMAILID=CWKLAW&CategoryDescription=watersustainability](http://research.ntu.edu.sg/expertise/academicprofile/pages/StaffProfile.aspx?ST_EMAILID=CWKLAW&CategoryDescription=watersustainability)). The PI is the international collaborator in the project which will run from Oct. 2013-Sept. 2016. This project emphasizes laboratory experiment. A small wave flume (30cm wide with a length to be determined) will be built. PDMS (a viscoelastic material) with adjustable and precisely measured viscoelastic properties will be used as floating materials (Nase and Linder, 2008). The drift of such materials and their effects on wave transmission/reflection will be measured. This project will serve as a necessary check for our wave-ice interaction project. The data obtained will be useful to first validate the theory and then used as a true “continuum” analog of the fragmented ice covers over large scale.

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