Mathematical modeling of space-time variations in acoustic transmission and scattering from schools of swim bladder fish (FY13 Annual Report)

Christopher Feuillade
Pontificia Universidad Católica de Chile
Facultad de Física
Av. Vicuña Mackenna 4860
Santiago, Chile
phone: +56 2 354 4800 fax: +56 2 354 4491 email: chris.feuillade@gmail.com

Award Number: N00014-111-0161

LONG-TERM GOALS

The goal is the development of a time-domain acoustical method for investigating the spatial and temporal stochastic variations in fish density within fish schools, and thereby to study the statistical fluctuations in the scattering of sound from these objects.

OBJECTIVES

The objective of this research is to develop a complete time-domain theory of acoustic scattering from, and propagation through, schools of swim bladder fish at and near the swim bladder resonance frequency, including multiple scattering and coherent interaction effects between the individual fish. This should then lead to a prescriptive capability for modeling the evolution of sound pulses as they are scattered from, and pass through, fish schools. Subsequently, it should be possible to identify discrete segments of the signal with different sections of the fish school, and to develop an enhanced understanding of signal scattering, and extinction, by the school, and the fluctuations in these properties.

APPROACH

The personnel participating in this work at the present time are: Principal Investigator: Christopher Feuillade - Ph. D. (Physics), Manchester, UK, 1977. (Visiting Professor, Pontificia Universidad Católica de Chile); Assistant: Maria Paz Raveau Morales - Civil Engineer in Sound and Acoustics - INACAP, Chile, 2009. (Doctoral student, Pontificia Universidad Católica de Chile), Simon Alfaro Jimenez (Undergraduate student, INACAP), Camilo Pichuante Alderete (Undergraduate student, INACAP).

(1) Steady-state solution for a school: In 1996, a low-frequency scattering model for fish schools was introduced (see Ref. 1) using a formalism based upon the harmonic solution of sets of coupled differential equations. It incorporates a verified swim bladder scattering kernel for the individual fish, includes multiple scattering interactions between the fish, and calculates the aggregate scattering field by coherent summation. In this work, the Love swim bladder model is used as the kernel (Ref. 2).

When an external pressure field is incident upon an ensemble of N fish, the scattering from each fish is determined not just by its response to the incident field, but also by its response to the aggregate scattered field reaching it from all of the other fish. The behavior of the whole ensemble can be represented by the following set of coupled second-order differential equations:

$$\begin{aligned} m_1\ddot{v}_1 + b_1\dot{v}_1 + \kappa_1 v_1 &= -P_1 e^{i(\omega t+\phi_1)} - \sum_{j\neq 1}^{N} \frac{\rho e^{-ikr_{j1}}}{4\pi r_{j1}}\ddot{v}_j \quad ; \\ &\cdots \\ m_n\ddot{v}_n + b_n\dot{v}_n + \kappa_n v_n &= -P_n e^{i(\omega t+\phi_n)} - \sum_{j\neq n}^{N} \frac{\rho e^{-ikr_{jn}}}{4\pi r_{jn}}\ddot{v}_j \quad ; \\ &\cdots \\ m_N\ddot{v}_N + b_N\dot{v}_N + \kappa_N v_N &= -P_N e^{i(\omega t+\phi_N)} - \sum_{j=1}^{N-1} \frac{\rho e^{-ikr_{jN}}}{4\pi r_{jN}}\ddot{v}_j \quad . \end{aligned} \tag{1}$$

Here, P_n and ϕ_n are the amplitude and phase of the external field incident on the n-th bubble, and r_{jn} is the distance between the centers of the j-th and n-th bubble, etc. The variable quantities m_n , b_n , κ_n , etc., appearing in these equations, allows for different values for the swim bladder radii, damping, etc., to be incorporated into the model, in order to represent a diverse range of individual fish properties within the school. Note, on the RHS of every equation in (1), the each bubble is driven by an aggregation of the external field and the scattered fields from the other bubbles.

Looking for harmonic steady state solutions, by substituting $v_1 = \bar{v}_1 e^{i\omega t}, v_2 = \bar{v}_2 e^{i\omega t}, \ldots$ etc., in (1), a matrix equation is obtained which may be written $\mathbf{Mv} = \mathbf{p}$, where $\mathbf{v} = \{\bar{v}_1, \ldots, \bar{v}_n, \ldots, \bar{v}_N\}$ and $\mathbf{p} = \{-p_1 e^{i\phi_1}, \ldots, -p_n e^{i\phi_n}, \ldots, -p_N e^{i\phi_N}\}$ are column vectors containing the steady-state volume oscillation amplitudes and external fields, respectively, for the individual bladders, and $\mathbf{M}$ is an $N \times N$ matrix with elements:

$$M_{nn} = \kappa_n - \omega^2 m_n + i\omega b_n \quad ; \quad M_{nj} = \frac{-\omega^2 \rho e^{-ikr_{jn}}}{4\pi r_{jn}} \quad (n \neq j) \quad . \tag{2}$$

Each diagonal term [i.e., $M_{jj}\,(j = 1, \ldots, N)$] describes the resonance behavior of an individual bladder, as if it were uncoupled from all the others. Variations in size, damping, depth etc., may be incorporated into these diagonal elements. Every off-diagonal element [i.e., $M_{nj}\,(n, j = 1, \ldots, N;\ n \neq j)$] describes the radiative coupling between two of the bladders due to their scattered acoustic fields. It is clear therefore, from (2), that the matrix falls into two distinct parts. The diagonal elements depend only on the individual characteristics of the bladders and not on their spacing or distribution. The off-diagonal elements depend only on the spacing and distribution of the bladders but not on their individual characteristics. The solution of the matrix equation (i.e., $\mathbf{v} = \mathbf{M}^{-1}\mathbf{p}$) enables the description of steady-state scattering from the whole ensemble of swim bladders as a function of the external field amplitude and frequency. Once the solutions $\bar{v}_n$ are found, the total scattered pressure field for the whole school may be readily obtained using coherent summation. This can be done for a receiver placed at any arbitrary orientation with respect to the fish school, and for any bistatic angle with respect to the acoustic source, including both back and forward scattering geometries.

(2) Time-domain solution for a school: The aim of this research is to obtain explicit analytic time-domainsolutions of a more generalized form of equations (1) for arbitrary time-dependent external input fields, i.e., $[P_1(t), \cdots, P_n(t), \cdots, P_N(t)]$, which are not necessarily harmonic.

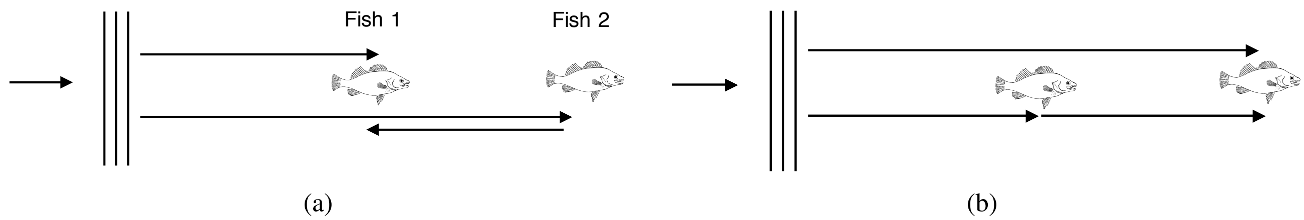


Figure 1: Ensonification of two identical fish lying along an axis between the source and receiver:(a) field contributions on the fish nearest from the source;(b) field contributions on the fish furthest to the source. Let r denote the distance between the two fish.

Three main approaches are currently being investigated for solving the time-domain equations: (a) reformulating the *N* second-order differential equations for *N* fish as an equivalent set of 2*N* first-order differential equations, which are then treated as state equations which can be analytically solved using eigen- decomposition techniques; (b) using numerical techniques to march out a solution of the coupled equations computationally; (c) Calculating the steady-state solution described above for a comb of equally spaced frequencies extending through and above the resonance frequency region (which provides accurate complex amplitudes for the scattered fields at these frequencies) and then performing an inverse FFT to this set of frequency solutions, to obtain the corresponding impulse response of the fish school. The primary intention of this last approach is to provide a benchmark against which the accuracy of the first two techniques [i.e., (a) and (b)] can be assessed.

The purpose is to then to use these solutions to describe the evolution of sound pulses as they are scattered from, and pass through, fish schools, and to identify discrete segments of the acoustic signal with different sections of the fish school, leading to an improved understanding of signal scattering, signal extinction, and their fluctuations.

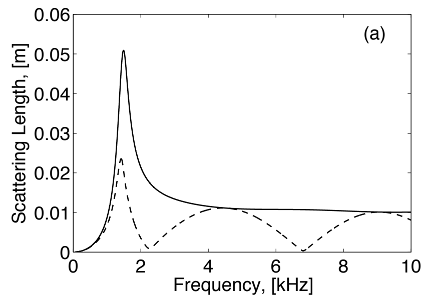
WORK COMPLETED

During FY13 we have completed an extensive analysis of the comparative results obtained by investigating the forward scattered field of a fish school, rather than the back scattered field, and thereby the sound extinction due to the school, using the current steady-state model (Ref. 1) to study the acoustic field. An analysis of transmission data obtained by Diachok (Ref. 3), using this forward scattering analysis, predicts significantly larger estimates of fish numbers than corresponding back scattering studies. Presentations have been given at the 162nd Meeting of the Acoustical Society of America in San Diego, California 31 October - 4 November 2011, and the 164th Meeting of the Acoustical Society of America in Kansas City, Missouri, 22-26 October 2012 (see Refs. 4 and 5). A paper has been written and submitted to the Journal of the Acoustical Society (Ref. 6).

RESULTS

(a) Comparison of the back scattered and forward scattered field

Consider the case of two fish, and assume that the source, the fish, and the receiver, are all located along a common axis. Figure 1 depicts, in schematic form, the ensonification of the two fish in this configuration. Figure 2 shows the variations in the aggregate scattering length for the two fish, for back



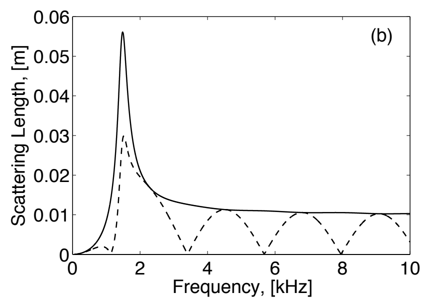


Figure 2: Scattering length variations for two identical fish lying along an axis between the source and receiver. (a) The separation between the fish is $r = L$. (b) The separation between the fish is $r = 2L$. In both cases, the solid line indicates the forward scattering response, and the dashed line indicates the back scattering response.

scattering (dashed line), and forward scattering (solid line), for a separation between the fish of $r = L$ (Figure 2a) and $r = 2L$ (Figure 2b), where L is the fish length. When the two back scattered pressure fields are detected at the receiver, they sum coherently together and, as a result of the difference between the source-fish distance, they interfere either constructively or destructively depending on the frequency, leading to a "comb filter" effect with periodicity equal to $c/2r$ Hz, where c is the water sound speed. This effect is notably absent in the forward scattering response.

Figure 3 shows the frequency variation of the scattering length averaged over 10 different random schools of 500 fish ($L = 0.165$ m, swim bladder radius $a_0 = 7.9$ mm), at 60 m depth, with an average spacing $r = L$, where radiative interactions between the fish have been incorporated using the school scattering model. Figure 3a shows the back scattering case, and figure 3b shows the forward scattering case. The vertical dashed line, in both figures, indicates the resonance frequency f_0 of an individual fish at 60 m depth. In back scattering, interference effects degrade and modify the resonance response in a way that can frequently make the identification of the actual school resonance peak problematic. However, in the forward direction, the scattered field is free of interference effects. The resonance peak of the school is identified as the lower of the two peak features indicated in the figure. Note also, from the different scales for the scattering length on the vertical axes of figure 3a and figure 3b, that acoustical scattering in the forward direction is generally much stronger than back scattering, thus indicating that there is a strong azimuthal variation of scattering from the school.

(b) Analysis of the forward scattered pressure for ensembles of fish

In fig. 1b the fish furthest from the source is ensonified by the external pressure field, and also by the

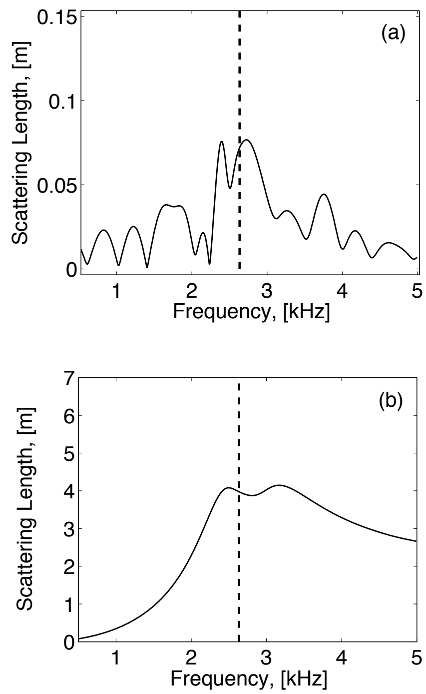


Figure 3: Scattering Length of a school of 500 fish, at 60 m depth (a) Back scattering (b) Forward scattering. Note the different scales for scattering length on the vertical axes.

field reradiated by the other fish, in such a way that the distance travelled by the waves is the same in both cases. In contrast, for the fish closest to the source, the distance traveled by the incident field directly from the source to the fish is not the same as the distance traveled to the other fish plus the distance traveled by the reradiated field back to it. There is a difference in the travel paths of $2r$ (see fig. 1a). The effect of the extra propagation distance for the reradiated field leads to the incorporation of a factor e^{-2ikr}, which depends upon the frequency. This phase factor causes a marked difference in the scattered field amplitude from the two fish, depending on their positions with respect to each other. The scattering from the fish nearer the receiver is suppressed.

This behavior is depicted here for a larger ensemble. Figure 4(b) shows the spatial distribution of a school with 500 fish. The 250 fish with the lowest scattered energies are indicated by triangles. It is seen that these 250 fish lie predominantly on the side of school nearer to the receiver (i.e., on the RHS of the figure, since the external field is incident from the LHS). Figure 4(a) contains two curves showing the frequency variation of the scattering length, evaluated in the forward direction, for two of the 500 fish: that with the highest scattered energy (dashed line); and that with the lowest scattered energy (solid line). It can be seen that resonance scattering from the fish nearer the receiver is suppressed.

Figure 5 shows the frequency variation of the average scattering length evaluated in the forward direction (obtained by dividing the scattering length for the whole school by the number of fish) for schools with 5, 10, 50, 100 and 500 fish. It may be seen that, as the number of fish is increased, the peak frequency f_0' of the overall school resonance response decreases, and that a hole in the region of the original resonance frequency (indicated by the vertical dashed line) progressively emerges. As this hole

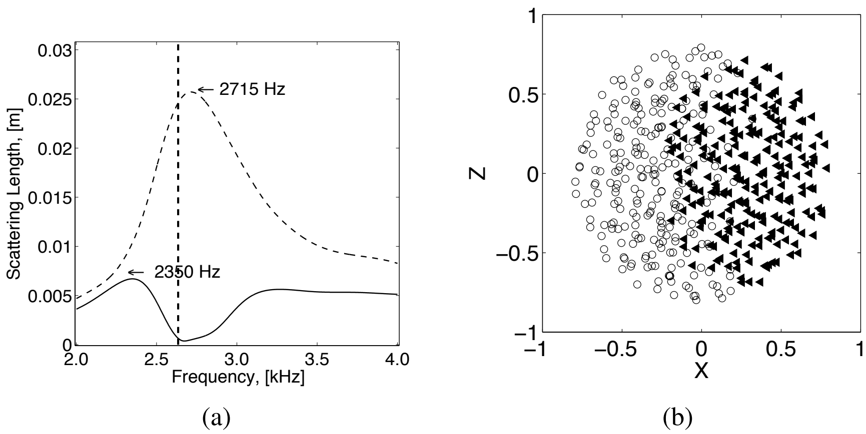


Figure 4: (a) Frequency variation of the scattering length evaluated in the forward direction of two fish from a school of 500 fish, with their corresponding peak resonance frequencies indicated. Dashed line: the fish with the highest scattered energy. Solid line: the fish with the lowest scattered energy. Forward scattering from a school of 500 fish at 60 m depth. (b) Spatial configuration of the fish school. All of the fish lie in the XZ plane. The source and receiver lie on the x axis. The source is located on the LHS of the fish (with negative x) and the receiver lies on the RHS (with positive x). The triangles indicate the 250 lowest-energy radiating fish.

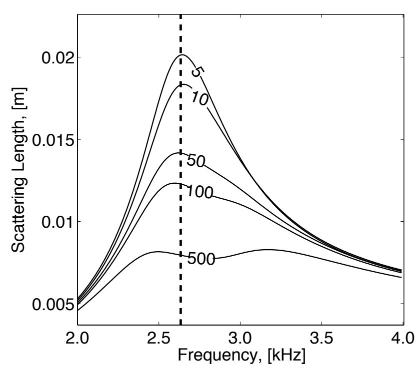


Figure 5: Effect of school size on scattering length. Average scattering length for a single fish within schools containing 5, 10, 50, 100 and 500 fish. In each case the average scattering length is obtained by dividing the scattering length for the whole school by the number of fish in the school.

develops, this leads to the appearance, on the high frequency edge of the hole, of a feature that imitates a second higher frequency peak. These phenomena are explained by the interference effects affecting the resonance behavior, as previously discussed.

(c) Data analysis

In the 1999 article by Diachok (Ref. 3), an analysis of acoustic transmission data obtained during the Modal Lion experiment performed in the Gulf of Lion in September 1995 was presented that produces estimates of the number of fish (specifically, sardines) in a school, based upon assumptions about the scattering properties of fish schools derived from models of acoustic back scattering. By analyzing the same data, and adopting essentially the same biological assumptions and analytical approach as Diachok, but matching the data against a forward scattering model, rather than a back scattering model, significantly different estimates of the numbers of fish contained in the schools were obtained.

Using the school scattering model described previously, simulations were performed to determine the variation of the extinction cross-section for schools of sardines of individual length L = 16.5 cm, with an effective swim bladder radius, at the surface, of 0.79 cm. The extinction cross-section was computed using an oblate spheroidal form for the schools of sardines, assuming, in every case, an aspect ratio 1/3, but with variable dimensions and numbers of fish contained. Each school was also assumed to be ensonified along a direction which is perpendicular to the axis of rotational symmetry of the oblate spheroid.

This reanalysis of the data obtained during the Modal Lion experiment indicates that the present forward scattering analysis leads to much higher estimates of the number of fish N than those of Diachok, for the same values of f_0/f_0 and r, and for schools of sardines both in daytime (65 m depth) and at night (25 m and 60 m depth) [See Table 1].

Comparison of the two analyses						
Measured f_0	Depth	f_0/f_0	r	Diachok	Present	Alternatives
1.5 ± 0.1 kHz	60 m	0.57	$0.6L$	$N = 3,000$	$N = 5,000$	$N = 7,000, r = 0.8L$ $N = 3,000, r = 0.5L$
1.7 ± 0.1 kHz	65 m	0.6	$0.8L$	$N = 5,000$	$N = 7,000$	$N = 10,000, r = 0.89L$ $N = 5,000, r = 0.67L$
0.9 ± 0.1 kHz	25 m	0.69	$2L$	$N = 10,000$	$N = 300,000$	$N = 10,000, r = 1.5L, f_0/f_0 = 0.61$
2.0 ± 0.2 kHz	65 m	0.72	$1.5L$	$N = 10,000$	$N = 30,000$	

Table 1: A direct comparison is made between the data interpretations described by Diachok (Ref. 3) and the present analysis. The right-most column indicates several other combinations of** N **and** r **which could be used to fit the data.

REFERENCES

(1) C. Feuillade, R.W. Nero and R. H. Love, " A low frequency acoustic scattering model for small schools of fish," J. Acoust. Soc. Am., **99**, 196-208 (1996).
(2) R. H. Love, " Resonant acoustic scattering by swim bladder-bearing fish," J. Acoust. Soc. Am. **64**, 571-580 (1978).
(3) O. Diachok, " Effects of absorptivity due to fish on transmission loss in shallow water," J. Acoust.

Soc. Am., **105**(4), 2107-2128 (1999).
(4) M.P. Raveau and C. Feuillade, “ Forward scattering at low acoustical frequencies from schools of swim bladder fish,” J. Acoust. Soc. Am. 130, 2337 (2011).
(5) M.P. Raveau and C. Feuillade, “ Low frequency acoustical scattering properties of large schools of swim bladder fish,” J. Acoust. Soc. Am. 132, 1882 (2012).
(6) M. Raveau and C. Feuillade, “ Sound extinction by fish schools: Forward scattering theory and data analysis,” J. Acoust. Soc. Am. (submitted, and in review).

IMPACT/APPLICATIONS

This work indicates the estimation of transmission loss due to extinction by fish schools in the water column should be based upon forward scattering models of scattering from these schools. The use of a back scattering paradigm can introduce significant errors.

This work also shows that levels of back scattering from a fish school can vary strongly with frequency depending on the specific spatial structure of the school, since the ensemble back scattering is strongly affected by interference effects from the scattered fields of the individual fish. This does not occur in forward scattering, leading to stable estimates of transmission loss which are not contingent on the exact configuration of the fish in the school.

TRANSITIONS

None at the present time.

RELATED PROJECTS

Experimental and other work by other participants in the ONR BRC, Fish Acoustics program.