The Energy Flux Method for Reverberation: Modeling and Inversion

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LONG-TERM GOALS

The long-term goals of this work are: (1) to develop a reverberation model for predicting the reverberation level and the echo-to-reverberation ratio in shallow water (SW), and (2) to characterize the seabed scattering in a frequency band of 100-3000 Hz from long-range broadband reverberation measurements.

OBJECTIVES

The scientific objectives of this work include: (1) to integrate the energy flux method for SW reverberation with physics-based seabed scattering models for reverberation prediction and inversion, and (2) to analyze the mechanisms of low-frequency (LF) seabed scattering, obtained from reverberation data and the Biot seabed geo-acoustic model.

APPROACH

(1) The energy flux (angular power spectrum) method for reverberation, based on the W.K.B approximation to the normal-mode solution [1], is integrated with physics-based seabed scattering models [2] to get SW reverberation expressions; (2) The effective Biot acoustic model for sandy and sand-silt mixture bottoms, derived from broadband LF field measurements at many locations [3], is used to model two-way sound propagation in the reverberation; (3) The broadband reverberation data are used to characterize the seabed scattering; (4) The LF seabed scattering cross-sections, derived from the long-range reverberation data, will be used to estimate the geo-physical parameters of the seabed roughness and the sediment inhomogeneity, and to analyze LF seabed scattering mechanisms.

RESULTS

(1) The energy flux method for SW reverberation is integrated with the rough bottom scattering (RBS) model and the sediment volume scattering (SVS) model. This integration directly and intuitively results in general expressions for SW reverberation in the angular domain and in the modal domain. The resultant reverberation expressions can conveniently be used for predicting SW reverberation for the Pekeris waveguide or non-Pekeris waveguide, respectively. The reverberation intensity in shallow water, derived from the energy flux method, can be expressed by a summation over normal modes:

$$R(r,z;z_0)=\frac{c_w\tau_0}{\pi r}e^{-2\alpha_w r}\sum_m\sum_n\frac{k_m(k_m-k_{m+1})^2\exp(-2\beta_m r)}{\left[k_w^2(z_0)D(z_0)+k_w^2(z_0)-k_m^2\right]^{1/2}\left[k_w^2(h)D(h)+k_w^2(h)-k_m^2\right]^{1/2}}$$
$$\times\sigma(\theta_m,\theta_n)\times\frac{k_n(k_n-k_{n+1})^2\exp(-2\beta_n r)}{\left[k_w^2(h)D(h)+k_w^2(h)-k_n^2\right]^{1/2}\left[k_w^2(z)D(z)+k_w^2(z)-k_n^2\right]^{1/2}} \qquad (1)$$

where k_n is the longitudinal wave number of the nth mode, and β_n is the modal attenuation coefficient. r is distance, and h is the water depth; z_0 is the source depth and z is the receiver depth. α_w is the water absorption coefficient. $\sigma[\theta_m,\theta_n]$ is the bottom scattering cross section per unit area; $\theta_{\mathrm{m}}(h)$ is the incident grazing angle of mode *m* at the bottom; and $\theta_n(h)$ is the scattering angle of mode *n*. $k_w(z)=2\pi f/c_w(z)$ is the wave number in the water. τ_0 is the signal pulse duration. $D(z)$ is a minor modification factor for overcoming a problem when a receiver is near a turning point,

$$D(z)=0.875\left|\frac{1}{\pi f}\frac{dc_w(z)}{dz}\right|^{2/3} \qquad (2)$$

For arbitrary velocity profiles, Eq. (1) can be used to calculate SW long-range reverberation intensity by using the classic cross sections of seabottom scattering and any available normal-mode code. This only requires two parameters: real and imaginary parts of the modal eigenvalue k_m, and β_m.

(2) The integration of the energy flux method with seabed scattering models also results in a simple relationship between the classic boundary scattering cross sections and the modal scattering matrix in SW waveguides:

$$MSM(m,n)=\frac{\Phi_m^2(h)}{\left|1+V_{ww}(\theta_m)\right|^2}\times\sigma(\theta_m,\theta_n)\times\frac{\Phi_n^2(h)}{\left|1+V_{ww}(\theta_n)\right|^2} \qquad (3)$$

Here $MSM(m,n)$ is defined as the modal scattering matrix in the SW waveguide between an incident mode m and a scattering mode n. $\Phi_m(h)$ is the amplitude of m^{th} mode at the water-bottom interface where the modal angle $\theta_m=\cos^{-1}[k_m/k_w(h)]$. $\sigma(\theta_m,\theta_n)$ can be either the RBS cross section or the SVS cross section. V_{ww} is the sound reflection coefficient with the scripts ww added to indicate that the incident and reflected fields are both in the water.

(3) The data-model comparisons show that the reverberation can almost be equally well predicted by using the RBS model and SVS model. Thus, the scattering parameters for the RBS model and the SVS model have been inverted from reverberation data. However, the reverberation level as a function of time, $RL(t)$, at one single frequency cannot uniquely determine the seabed scattering parameters. As an example, Figure 1 shows the internal parameter coupling for the RBS model between the spectral exponent γ_2 and the spectral strength ω_2: three different pairs of γ_2 and ω_2 would result in an identical $RL(t)$. An increase (decrease) in γ_2 can be compensated for by an increase (decrease) in ω_2.

(4) The wideband reverberation data as a function of frequency, $RL(f)$, at given reverberation times can uniquely determine a set of the bottom roughness spectra or the sediment volume scattering cross section. As an example, Figure 2 shows that the wideband reverberation data are used to uniquely invert a set of seabed scattering parameters for the RBS and SVS models. The bottom roughness spectrum and the sediment volume scattering cross section, inverted from the broadband long-range reverberation data at the ASIAEX site, exhibit different characteristics from those commonly used in HF. For the HF, the roughness spectrum exponent is restricted to the range of $2 \le \gamma_2 \le 4$ with a mean value of 3.0; the LF roughness spectrum exponent from long-range reverberation data has smaller values with a mean value around $\gamma_2 \approx 1.3$. The HF sediment volume scattering cross section σ_V is generally assumed to have linear frequency dependence. However, Figure 3 shows that the σ_V, inverted from long-range reverberation data, exhibits much stronger frequency dependence in the 150-2500 Hz range: $\alpha_V = (0.00238 \pm 0.00004) f^{(3.805 \pm 0.021)}$, here f is frequency in a unit of kHz.

(5) All the LF seabed scattering parameters for the RBS model and the SVS model, inverted from the long-range (3 to 12 s) reverberation data, are listed in Table I. The standard deviations between the measured reverberation data and the theoretical predictions for all the frequencies are also listed in Table I. These seabed scattering parameters for the RBS model and the SVS model may be used for future analyzing which one (or both) is dominating LF scattering mechanism at low grazing angles. In Figure 2 and Table I, α_p (in a unit of *dB/m*) is the sound attenuation in the sea bottoms that obeys the Biot model [3].

IMPACT/APPLICATIONS

The resultant reverberation expressions with the physics-based seabed scattering models can conveniently be used to predict the SW reverberation and to characterize the LF seabed scattering.

RELATED PROJECTS

NONE

REFERENCES

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PUBLICATIONS

1. J.X. Zhou and X. Z. Zhang, "Shear wave velocity and attenuation in the upper layer of ocean bottom from long-range acoustic field measurements," J. Acoust. Soc. Am., **132**(6), 3698-3705, Dec. 2012. [published, refereed].
2. J.X. Zhou and X.Z. Zhang, "Integrating the energy flux method for reverberation with physics-based seabed scattering models: Modeling and inversion," J. Acoust. Soc. Am., **134** (1), 55-66, July 2013. [published, refereed].

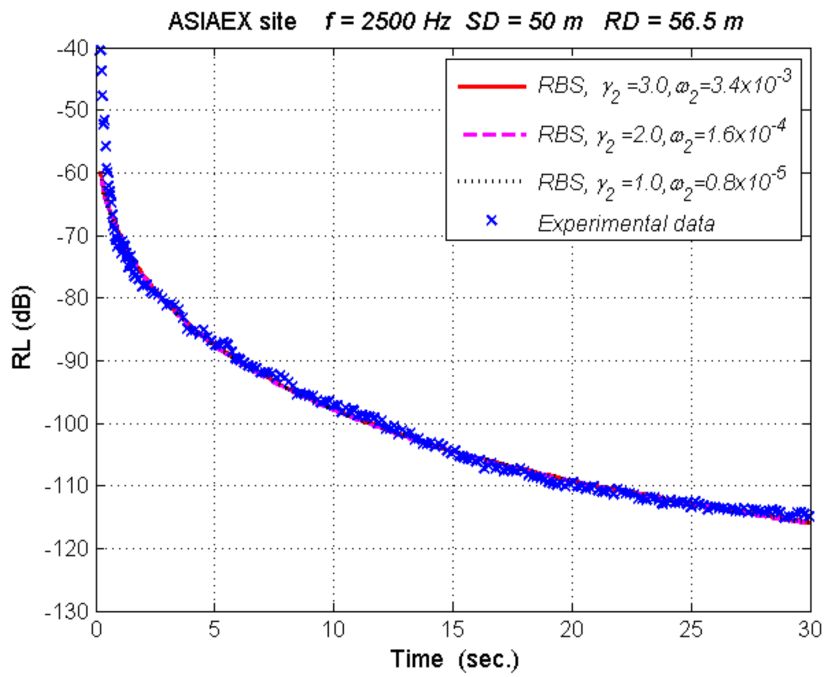

Fig. 1. Internal parameter coupling between the spectral exponent γ_2 and the spectral strength ω_2 for the RBS model. Three different pairs of γ_2 and ω_2 result in a identical $RL(t)$. An increase in γ_2 can be compensated for by an increase in ω_2.

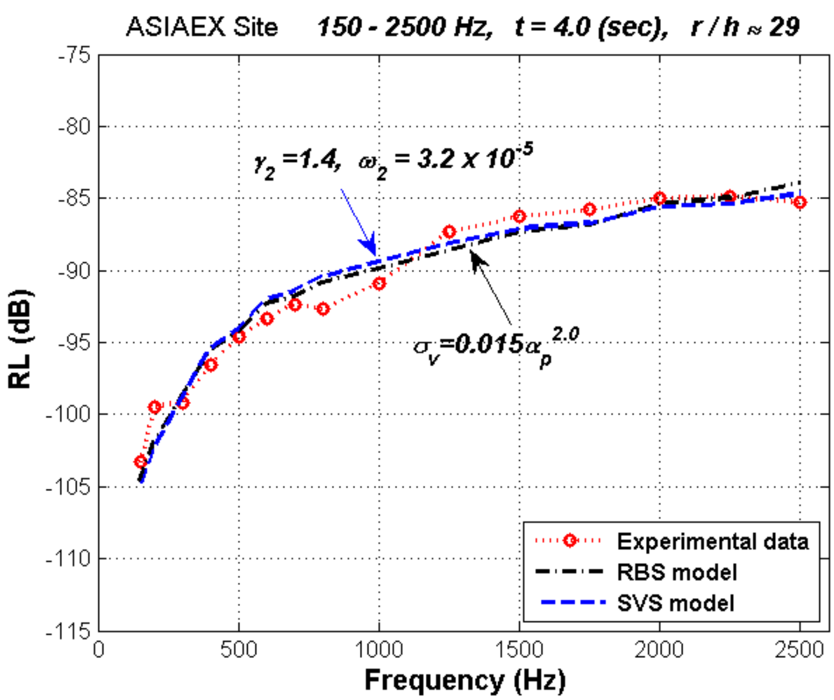

Fig. 2. Wideband reverberation data at the ASIAEX site are used to invert a set of seabed scattering parameters for the RBS and SVS models.

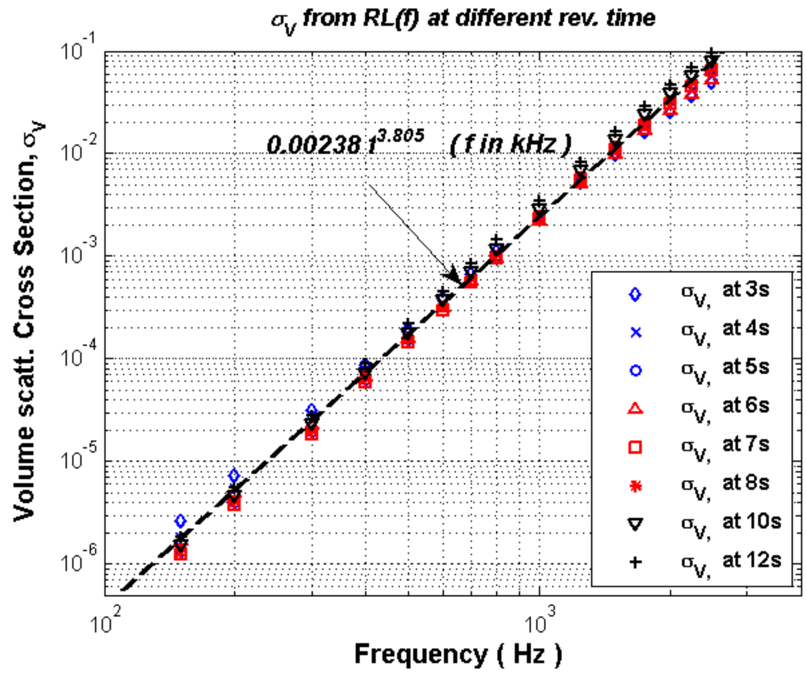

Fig. 3 The sediment volume scattering cross section, inverted from long-range reverberation data at the ASIAEX site, exhibits strong frequency dependence: $\alpha_V = 0.00238 f^{3.805} \quad (f \ \ in \ \ kHz)$

Table I. The LF seabed scattering parameters for the RBS model and the SVS model, inverted from long-range (3 to 12 s) reverberation data at the ASIAEX site

Models		RBS : Bottom roughness spectrum $W(\Delta K)=\omega_2/(\Delta K)^{\gamma_2}$			SVS: Volume scattering cross section: $\sigma_V=\sigma_2^*\alpha_p^{n_v}$		
Rev. time and rang-to-depth ratio		γ_2	$\omega_2 \times 10^{-5}$	$std_{RBS}(dB)$	σ_2^*	n_v	$std_{SVS}\ (dB)$
$t_R=3.0$	$r/h=$ 22	1.6	4.9	1.419	0.014	1.9	1.603
4.0	29	1.4	3.2	1.192	0.015	2.0	1.348
5.0	37	1.3	2.6	1.719	0.017	2.1	1.874
6.0	44	1.3	2.5	1.473	0.014	2.0	1.668
7.0	52	1.2	2.0	1.547	0.016	2.1	1.725
8.0	59	1.2	2.1	1.672	0.017	2.1	1.855
10.0	73	1.2	2.4	1.191	0.020	2.1	1.243
12.0	88	1.2	2.9	1.513	0.024	2.1	1.578