Spatio-Temporal Characterization of Bio-acoustic Scatterers in Complex Media

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LONG TERM GOALS

To develop a numerically efficient methodology for modeling the acoustic response of large aggregate of biological scatterers to parameterize acoustic models for long-range SONAR measurements

OBJECTIVE

Characterization of biologically-induced ocean reverberation features is key to effectively parameterize acoustic models and thus ultimately improve the detection performance of long-range SONAR systems. In particular, scattering from fish schools can significantly contribute to volume reverberation in the open ocean measured by mid-frequencies tactical SONAR (1kHz-10kHz), especially if the resonance frequencies of the fish' air-filled swim bladder is excited. Furthermore, multiple scattering effects from the incident acoustic wave and the collective arrangement of fish lead to complicated frequency response functions. The bio-acoustics properties of the fish body and geometry can also contribute to the scattering response and can be incorporated into an accurate scattering model.

The objective of this research is to characterize the relevant *spatial and temporal scales* of bio-acoustic scatterers generating ocean reverberation to effectively parameterize acoustic models and improve the detection performance of long-range SONAR systems. To do so, we developed an efficient modeling technique to predict the scattered fields from large fish schools (which can cause especially high false-alarm rate for mid-frequency SONAR systems.), which readily account for the fish acoustic properties, school's spatial configuration and multiple scattering effects.

Fig. 1 School of Fuseliers fishes (Papua New Guinea), photo by Randy Harwood.

WORK COMPLETED

Two key observations can be made about large fish schools. First, in large schools, fishes typically swim in a *periodic* arrangement where fishes are regularly spaced by approximately one-body length in all three dimensions (see Fig. 1). Second large fish schools (e.g. several km wide) can have dimensions exceeding several thousand of wavelengths of the SONAR's frequency. Hence, based on these two observations, we will treat large fish schools as an *infinitely* large system generated by tessellating in 3D a unit volume cell containing a single fish (e.g. see Fig. 2). This infinite system can be modeled as a periodic phononic crystal which is a special class of well-studied periodic materials that support acoustic or elastic waves. In our case, the infinite phononic fish crystal (FC) contains a host medium (homogenous, quiescent water) and periodically spaced inclusions (fish bodies).

Incident acoustic energy onto the FC causes an infinite number of multiple scattering, Bragg scattering, and inclusion resonance phenomena. These scattering events establish a dispersion relationship which governs harmonic wave propagation in the FC. Notably, there are frequency ranges, termed band gaps, where acoustic waves are *prohibited* from propagating in the crystal [1]. Analyzing the FC dispersion relationship yields an infinite set of mode shapes, with corresponding harmonic frequencies and wavenumbers, analogous to a finite vibrational system, that describe the pressure field. The Bloch theorem [2], which accounts for *all orders* of multiple scattering, is employed to calculate the dispersion relationship and modes of the infinite fish crystal. Use of this theorem allows one to study a *single unit cell* and simultaneously gain knowledge of the entire domain. For simplicity, we will focus on two dimensional geometry only, where now the FC is an infinite array of periodically spaced fish bodies in a water background. Fig. 3a shows a representative 2D FC with a unit cell (Fig. 3b) containing a circle of radius R corresponding to the air filled swimbladder $\rho_2 = 1.22\,\text{kg/m}^3$, $c_2 = 343\,\text{m/s}$ in a water background with $\rho_1 = 1000\,\text{kg/m}^3$, $c_1 = 1500$ m/s. Here the dimensions of the unit cell are $a_1 = a_2 = 40$ cm and R = 2 cm which represent a single air filled fish swimbladder [3].

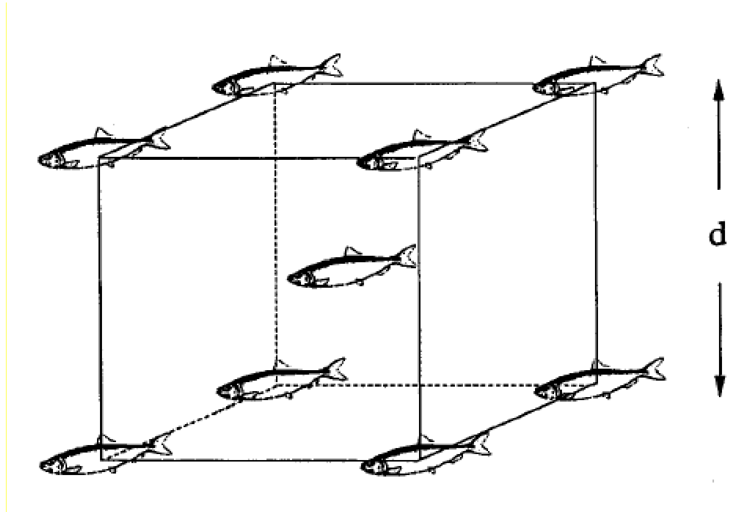


Fig. 2 A basic school unit. The schools are constructed from cubic cellular units (or unit cell). The packing density is determined by the distance d between closest neighbors (Reproduced from Feuillade et al.) [3].

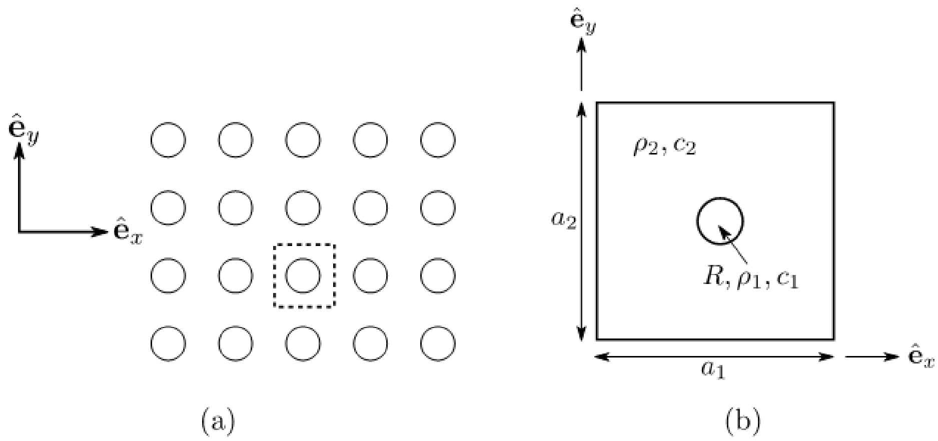


Fig. 3 (a) A representative two dimensional periodic fish crystal and (b) the unit cell studied where the circular inclusion represents the fish' air filled swimbladder.

In a two dimensional periodic lattice the pressure $p(\mathbf{x},t)$ at any point $\mathbf{x}$ in the FC can be represented using the Bloch theorem (assuming harmonic time dependence of frequency ω) by

$$p(\mathbf{x},t) = \tilde{p}(\mathbf{x})\exp\left[i(\mathbf{k}\cdot\mathbf{x}-\omega t)\right] \tag{1}$$

where $\tilde{p}(\mathbf{x})$ is a function periodic on the unit cell, hereby termed a wave mode, and the vector $\mathbf{k}$ is the Bloch wavevector and is related to ω by a dispersion relationship. Every field quantity in the FC obeys the relation of Eq. (1) – a periodic function modulated by a harmonic plane wave. Periodicity of the FC imposes that unique Bloch waves, given by Eq. (1), are found with the real part of the Bloch wavevector components within the range $-\pi / a_j \leq k_j \leq \pi / a_j$. However, the imaginary components of the wavevector are unbounded [4].

To investigate the acoustic reflection from a large fish school, we suppose an incident homogenous water medium adjacent to a semi-infinite FC half space, see Fig 4. At a given frequency and incident wavevector $\mathbf{k}_i$ we can find a finite number of propagating Bloch waves, given by the constraints of the FC periodicity, and an infinite number of evanescent waves which account for the near field refraction behavior at the FC boundary [4]. The propagating and evanescent Bloch waves constitute a complete basis [5] and justify a Bloch wave expansion for the wave field within the FC. We will utilize the Bloch wave expansion, first outlined in [6], along with a plane wave expansion for the reflected field to compute the acoustic reflection from the semi-infinite fish school.

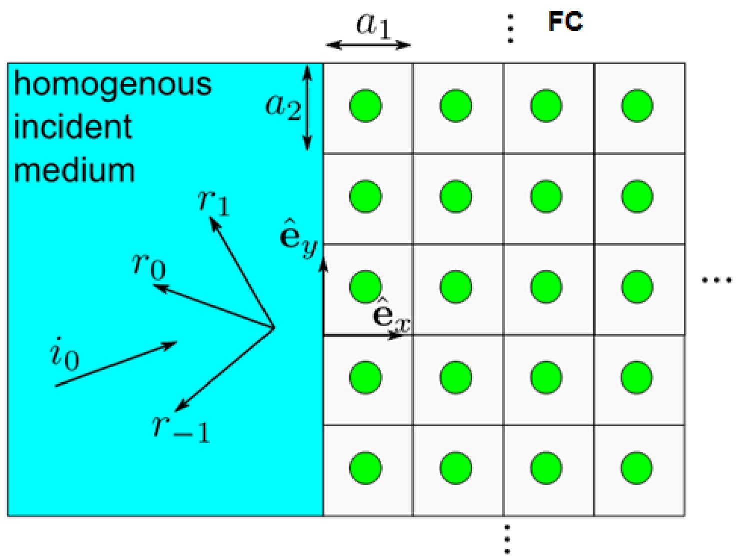


Fig. 4 A half space consisting of a homogenous water medium and a PC. Three reflected wave orders and the incident plane wave are included for an example.

Identification of the Bloch waves that constitutes the expansion set requires studying the Helmholtz equation with spatially varying density ρ and speed c

$$\frac{1}{\rho(\mathbf{x})c(\mathbf{x})^2}\frac{\partial^2 p(\mathbf{x},t)}{\partial t^2}-\nabla\cdot\left[\frac{1}{\rho(\mathbf{x})}\nabla p(\mathbf{x},t)\right]=0. \tag{2}$$

Insertion of the Bloch theorem from Eq. (1) and simplification results in a homogenous equation that is next discretized on the unit cell using the finite element method (FEM). Enforcement of matching parallel wavelengths between the incident plane wave and the Bloch waves and application of periodic boundary conditions on the finite element mesh of the unit cell results in a quadratic eigenvalue problem

$$\left[\mathbf{D}(\omega)+k\mathbf{A}+k^2\mathbf{B}\right]\tilde{\mathbf{p}}=\mathbf{0} \tag{3}$$

where here $k=k_x$ is the unknown wavenumber of the periodic Bloch wave mode $\tilde{\mathbf{p}}$ (discretized version of $\tilde{p}$) [7]. The FEM matrices in Eq. (3) contain the unit cell's geometry and swimbladder's biological information. Solution of Eq. (3) yields propagating and/or evanescent Bloch waves depending on the frequency; propagating waves with a group velocity vector pointing away from the FC medium are discarded from the expansion as these waves violate energy conservation for this particular problem.

At this point we have N Bloch waves (N eigenvalues k and eigvenfunctions $\tilde{p}_n$), and subsequently use N reflected wave orders. The periodicity of the interface creates a diffraction grating and constrains the parallel component of the reflected wavevectors $\mathbf{k}_m$ [6]. We write an expansion for the transmitted (into FC) and reflected wave fields as

$$p_t(\mathbf{x},t)=\sum_{n=1}^{N} t_n \tilde{p}_n(\mathbf{x})\exp\left[i\left(\mathbf{k}_n\cdot\mathbf{x}-\omega t\right)\right] \tag{4}$$

$$p_r(x,\mathrm{t})=\sum_{m=-N/2}^{N/2} r_m \exp\left[i\left(\mathbf{k}_m\cdot\mathbf{x}-\omega t\right)\right] \tag{5}$$

where now t_n, r_m are unknown modal coefficients to be solved for. Enforcing continuous pressure and normal particle velocity boundary conditions between the homogenous medium and FC results a system of linear equations for the modal coefficients. The entire pressure field within the FC and reflected wave field can now be calculated. We can define an average power reflection coefficient R_W from the resulting reflected and incident time-averaged intensity fields $\mathbf{I}_r$, $\mathbf{I}_i$

$$R_W=\frac{\int_0^{a_2}\mathbf{I}_r(0,y)\cdot-\hat{\mathbf{e}}_x dy}{\int_0^{a_2}\mathbf{I}_t(0,y)\cdot\hat{\mathbf{e}}_x dy} \tag{6}$$

Note this coefficient depends on frequency ω and the angle of the incident plane wave θ ($\theta=0$ for normal incidence), as well as the spatial and biologic properties of the FC.

To verify the Bloch wave expansion and associated reflected wave field, we have compared our methodology with independent finite element software. The finite element model contains over 4000 unit cells, chosen to be approximately infinite in extent, and specialized radiation boundary conditions. Excellent agreement is seen between the real and imaginary parts of the pressure field between the Bloch wave expansion and the finite element model in Fig. 5.

It was earlier stipulated that wave propagation is forbidden for frequencies within a band gap of the FC's dispersion relationship. Here, the power reflection is unity, as all acoustic energy is reflected away and a decaying field remains in the FC domain. Analysis of the dispersion relationship not only yields band gaps, but regions of incident wave angles whereby perfect acoustic reflection also occurs. Following the analogy of acoustic reflection from homogenous medium, we term these angles 'critical'. The strong acoustic reflection from the FC could lead to the possibility of predicting strong SONAR returns for given incident wave parameters. In Fig 6a, at $\omega=39$ krad/s , we graphically display the critical angle data – an incident wavevector in the shaded region will yield perfect acoustic reflection from the FC. Fig. 6b compares the power reflection predicted from the critical angles to the power reflection resulting from direct numerical evaluation.

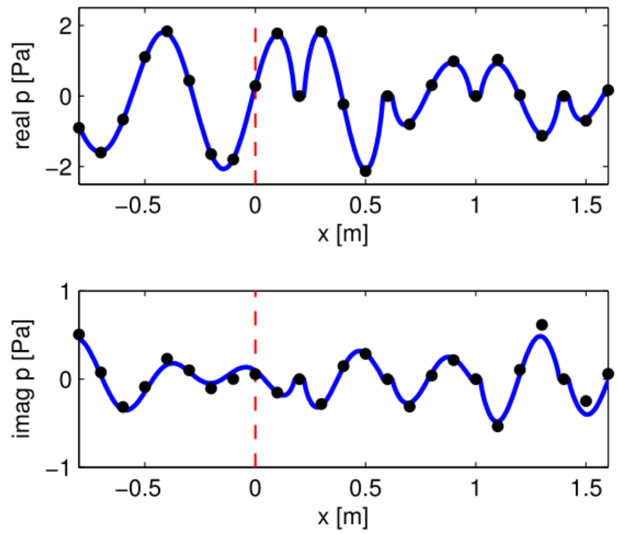
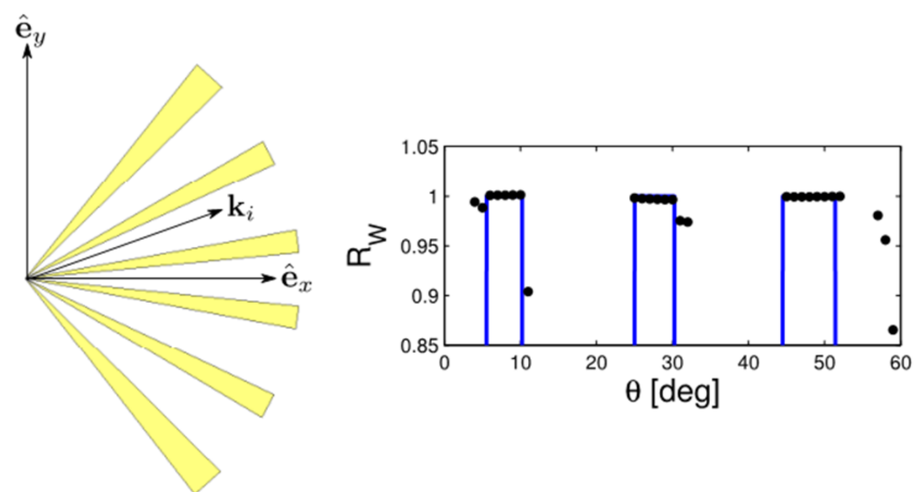


Fig. 5 A comparison of the real (top) and imaginary (bottom) components of the pressure field from the Bloch expansion (line) and the numerical model (dots) field, along a horizontal line centered at y = 8.2 m. Here ω = 24000 rad/s and θ = 45 degrees and the interface is demarcated by a vertical dashed line.



(a) (b)

Fig. 6 (a) A graphical display of the critical angles where shaded regions indicate ranges of critical angles. (b) Validation of the critical angle observation versus direct numerical calculation (dots). Solid lines are from wedge regions from (a).

CONCLUSIONS

The work here enforces the approximation of a large number of scattering bodies as an infinite periodic system amenable to Bloch analysis. Modeling the fish school reflection using this approach has a few benefits: (1) computational advantage of studying a single swimbladder which can have an *arbitrary* shape and biological properties via the FEM (2) there are no frequency limitations to the model, (3) all orders of multiple scattering are easily included and (4) the possibility of predicting frequencies and angles that yield strong SONAR returns. Recent work has even extended this modeling methodology to three dimensions. Future work will compare the infinite FC model to very large, yet finite sized, fish schools and the effect of random perturbations of fish within the school.

IMPACT

It is conjectured that the results of this study could potentially be used to reduce the uncertainty of SONAR prediction modeling tools due to marine biota, thus ultimately improving the remote detection, classification and localization of marine biota using long-range mid-frequency SONAR systems.

PUBLICATIONS

J. A. Kulpe, M. J. Leamy, K. G. Sabra, "Modeling the acoustic scattering from large fish schools using the Bloch-Floquet theorem, "*Proceedings of Meetings on Acoustics*. Vol. 19. International Commission on Acoustics, June 6th, Montreal QC (2013)

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