

## **Transport Theory for Propagation and Reverberation**

Eric I. Thorsos

Applied Physics Laboratory, University of Washington  
Seattle, Washington 98105

phone: (206) 543-1369 fax: (206) 543-6785 email: [eit@apl.washington.edu](mailto:eit@apl.washington.edu)

Award Number: N00014-13-1-0216

### **LONG-TERM GOALS**

Development of computationally efficient modeling methods for shallow water propagation and reverberation that can account for the effects of multiple forward scattering from waveguide boundary roughness and volume heterogeneity such as internal waves.

### **OBJECTIVES**

Previously, our shallow water propagation model based on transport theory was extended to include reverberation, and it was found that sea surface forward scattering could have very important effects on reverberation level at mid frequencies, e.g., at 3 kHz. One objective in FY13 was to obtain reverberation measurements during TREX13 that can be used for a definitive verification of these important effects. An additional objective in FY13 was to use transport theory results to support the development of an effective surface reflection loss model that can approximately account for effects of surface forward scattering in ray-based or mode-based propagation and reverberation codes.

### **APPROACH**

Accurate propagation and reverberation modeling is important for many prediction methods that are important for Navy applications and for underwater acoustics systems development. While acoustic propagation and reverberation modeling has been extensively developed for many years, significant limitations still exist on current capability, particularly in the area of computation speed. In addition, the modeling problem increases in complexity as the frequency is raised from the low frequency region (< 1 kHz) to the mid frequency region (1–10 kHz). At mid frequencies (and higher) the effect of forward scattering from the sea surface and bottom has a greater effect on propagation and reverberation than in the low frequency region, especially in shallow water environments.

The available options for modeling forward scattering in propagation are very limited, and are largely confined to computationally intensive methods that can yield benchmark solutions for certain simplified problems. When PE is used for practical propagation modeling, only large-scale bathymetry variations are included with small-scale boundary roughness ignored, and internal waves are also generally ignored. Even the simple expedient of using a loss at the boundary to approximately account for boundary roughness is not conveniently included in PE propagation simulations. Similarly, normal mode methods generally ignore mode coupling due to boundary roughness in forward propagation, and in reverberation simulations only a single scattering (the backscattering) is included. In order to include the stochastic effects of boundary forward scattering and internal wave forward scattering in

propagation simulations, investigators have typically applied a full-wave method, such as PE, and performed propagation simulations using many realizations of the fluctuating environment in a “Monte Carlo” approach. Averaging the results over the set of realizations can then give accurate results for averages (or moments) of the field, and by using a sufficient number of realizations even pdfs of field amplitudes or intensities can be obtained. In the case of boundary roughness scattering, simulations using the finite element method have also been used. The computational demands for full-wave Monte Carlo simulations for propagation and particularly for reverberation are severe. Instead of doing time consuming Monte Carlo simulations, much faster solutions for field moments can be obtained if equations governing the evolution of the moments themselves can be obtained and solved. Any method that works with evolution equations for the moments of the propagating quantities can be described as a “transport theory,” though not always referred to as such.

Therefore, the need exists for much faster computational approaches for obtaining moments of the field for propagation and reverberation at mid frequencies that can account for boundary and internal wave scattering. Our approach is based on expanding the acoustic field in modes, and therefore would most readily apply at mid-frequencies and below, and in relatively shallow water environments such as on the continental shelf.

We have focused on the case where forward scattering is due to scattering from sea surface roughness. Evolution equations are obtained for the first and second moments of the mode amplitudes, accounting for mode coupling due to scattering from a rough sea surface using first-order perturbation theory [1]. Comparisons with rough surface PE simulations [2] have been used to verify the accuracy of the transport theory method for one-way propagation. It should be kept in mind that transport theory is much faster than full wave approaches that use a Monte Carlo method with many rough surface realizations. Also, any number of forward scattering interactions can be accounted for as the field propagates along the waveguide.

While rough surface PE simulations has shown the accuracy of transport theory predictions for average mode amplitude decays in one-way propagation, the effects of sea surface forward scattering on reverberation level has been found to be even more significant than on one-way propagation. Thus, it is important to verify reverberation transport predictions as well. TREX13, a propagation and reverberation experiment carried out near Panama City, Florida in the spring of 2013, was planned to obtain suitable reverberation results to give a definitive test of transport theory predictions, since the environment was characterized in sufficient detail to highly constrain reverberation modeling. DJ Tang and Todd Hefner from APL-UW were the co-Chief Scientists for TREX13.

Because transports theory has shown the importance of accounting for sea surface forward scattering in accurately modeling shallow water reverberation at mid frequencies, it becomes imperative to develop as approximate way to include these effects into traditional ray-based or mode-based reverberation codes. A separate project supported by PMW-120 (M. Speckhahn) has been ongoing with this particular goal in mind. The effect of surface forward scattering is treated with an effective surface reflection loss model for the total field (referred to as TOTLOS), where the total field is the combination of the coherent (or reflected) component, and the incoherent (or scattered) component. The original approach in developing TOTLOS was to base it on the results of Monte Carlo rough surface PE results, but as transport theory became available it became clear that results from it were much more suitable to support TOTLOS development. As a result TOTLOS development has become an important secondary goal of the present project.

The approach being used in the development of TOTLOS will be summarized briefly. Because our transport theory is mode-based, it readily provides mode amplitudes as a function of range for any particular shallow water environment of interest. Each mode amplitude can be associated with a particular grazing angle at the sea surface. The decay of each mode amplitude over a cycle distance (the distance between surface interactions assuming reflected rays) is first determined, and the contribution of loss at the bottom is removed. What remains is identified as a loss in a single surface interaction, and in many cases that loss is negative, which means that there is a gain. In such a case more energy is being forward scattered into a particular mode than is being lost into the bottom in one cycle distance. With this information determined as a function of range for each mode, it is possible to form an effective reflection loss (the TOTLOS model) that will replicate the transport theory results for propagation when surface forward scattering occurs. The model can then be tested in reverberation geometries using TOTLOS in a ray-based code such as CASS-GRAB and making comparisons with transport theory reverberation results.

The TOTLOS model depends not only on the sea surface roughness and frequency, but on range and on the water column and bottom properties, i.e., the TOTLOS model is scenario dependent. To avoid the need to tune the model to each scenario with appropriate transport runs, the approach is to develop an algorithm using quasi-analytic expressions for the model parameters based on a selection of transport runs, and then use that algorithm to define the parameters for the model in general.

The key individuals assisting with the transport theory work are Frank Henyey, Jie Yang, and Tim Elam, all at APL-UW. Todd Hefner, also at APL-UW, has been assisting with TOTLOS model development.

## **WORK COMPLETED**

During the spring of 2013, reverberation data sets at mid frequency were successfully obtained during TREX13 that will be suitable for rigorous testing the effect of sea surface forward scattering on reverberation level as predicted by transport theory.

When extending the TOTLOS model to account for a range of sediment sound speeds, it was found that it was not possible to obtain satisfactory accuracy with a straightforward extension of the previous algorithm. Therefore, a much more detailed fit to the transport theory mode decays was implemented to permit as accurate extension of TOTLOS to a range of sediment sound speeds. In this initial TOTLOS version, the sound speed profile is assume isovelocity, and a simple surface roughness model has been assumed (an isotropic Pierson-Moskowitz spectrum). The initial version can account for variations in wind speed, frequency ( $\leq 3$  kHz), and water depth. Future versions will account for more realistic roughness spectra and general sound speed profiles.

At a more fundamental level, work has been completed on resolving a longstanding puzzle about why our transport theory method is as accurate as rough surface PE simulations show it to be. This has been a puzzle because analysis suggests that for the conditions being used in our simulations the perturbation approximation employed to account for mode coupling in forward scattering from the rough sea surface should be accurate for only the lowest modes and not accurate for higher modes. The small parameter for the perturbation expansion is the product of the mode vertical wave number at the surface and the rms surface height. As the mode number becomes high at a frequency of 3 kHz, for example, this parameter routinely becomes too large to expect accuracy, yet our transport theory comparisons with PE simulations for the average intensity show quite good agreement nevertheless.

This has been one final issue (in addition to experimental verification mentioned above) that needed resolution before our transport theory could be considered finally completed.

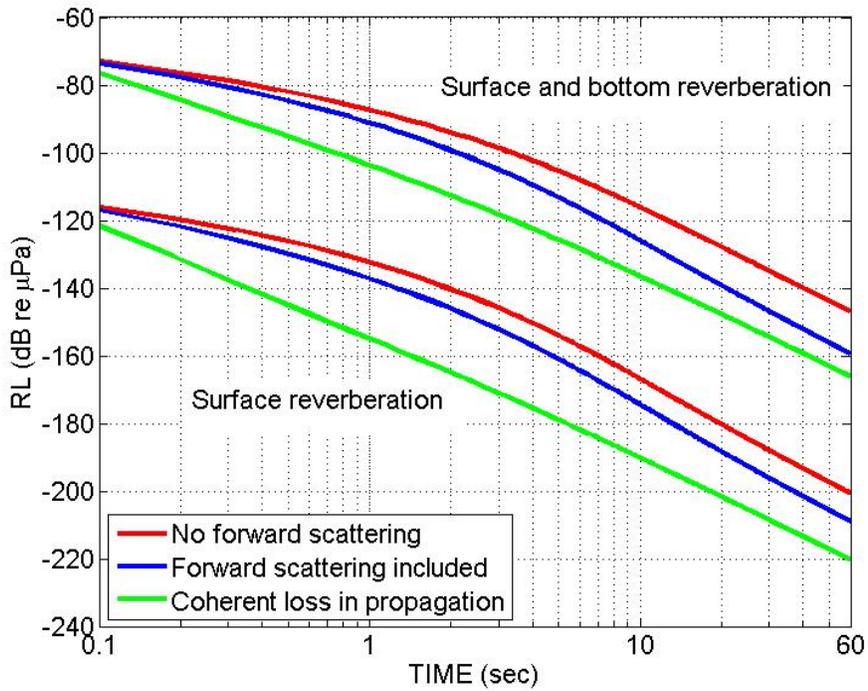
To resolve this apparent puzzle, a more sophisticated mode coupling method has been developed that is uniformly accurate for all modes, but too complicated to readily incorporate into transport theory. Nevertheless, it has allowed us to understand that even though our initial mode coupling method indeed shows error when applied to individual realizations for rough sea surfaces, the results when averaged over an ensemble of realizations are accurate, and that is the result that transport theory is attempting to model, resolving the puzzle.

## RESULTS

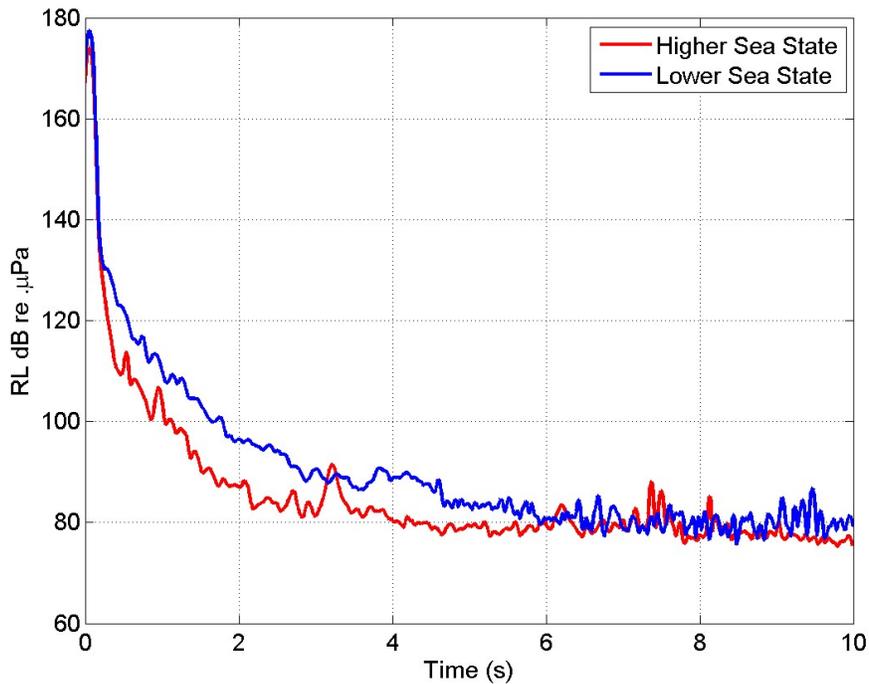
TREX13 was successfully completed during April and May 2013, with many aspects covered in separate reports. As mentioned above, one important goal of TREX13 was to provide reverberation measurements that can be used to verify the accuracy of transport theory predictions. One important prediction is that at mid frequencies the reverberation level will decrease as the sea state rises, even when the bottom reverberation dominates. This is illustrated in Figure 1, reproduced from the FY12 report. For this example the frequency is 3 kHz, the rough sea surface is modeled with an isotropic Pierson-Moskowitz roughness spectrum for a wind speed of 7.7 m/s giving an rms wave height of 0.31 m, the sound speed is taken as isovelocity at 1500 m/s over a water depth of 50 m, and the bottom roughness is described by the Reverberation Modeling Workshop “typical roughness” model [3]. For the top set of curves, bottom reverberation dominates, the red curve is a prediction corresponding to a very low sea state, while the blue curve is the transport theory result that fully accounts for the effects of surface forward scattering. The reverberation levels can be considered relative levels, since a typical source level has not been included.

Figure 2 shows a corresponding result obtained during TREX13, where bottom reverberation is known to dominate, and the water depth is about 19 m. The measured reverberation has qualitatively similar trends to the transport theory predictions shown in Figure 1, until the reverberation level merges into the noise at a level of about 80 dB re 1  $\mu$ Pa. As the TREX13 environmental measurements are fully processed, detailed data/model comparisons will be made for this and other cases to give definitive tests of transport theory predictions for propagation and reverberation.

Turning now to the new work on the accuracy of transport theory, an example will be presented to illustrate the findings. For the example the water depth is 50 m with an isovelocity sound speed of 1500 m/s, and the surface waves are realizations consistent with a Pierson-Moskowitz spectrum for a wind speed of 7.7 m/s, the same as for Figure 1. For this case the frequency is 2 kHz, and for simplicity, a Dirichlet bottom boundary condition is used, and the source is taken to be a single mode given by mode 8. Two treatments of mode coupling based on first-order perturbation theory are considered: For the first treatment the coupling term is linear in the surface height and is the same form employed for mode coupling in our transport theory; results from it are denoted by “LI” in the figures that follow. Analysis would appear to show that for the first treatment to be accurate that not only the rms surface height  $h$  needs to be small in the sense that  $k_{zN}h$  is small, where  $k_{zN}$  is the vertical wave number for mode  $N$ , but that  $Nk_{zN}h$  needs to be small. The second treatment is based on a uniform approximation in the number of modes, and therefore should be more accurate, though too complicated to incorporate into a transport theory formalism; results from the second treatment are denoted “DI” in

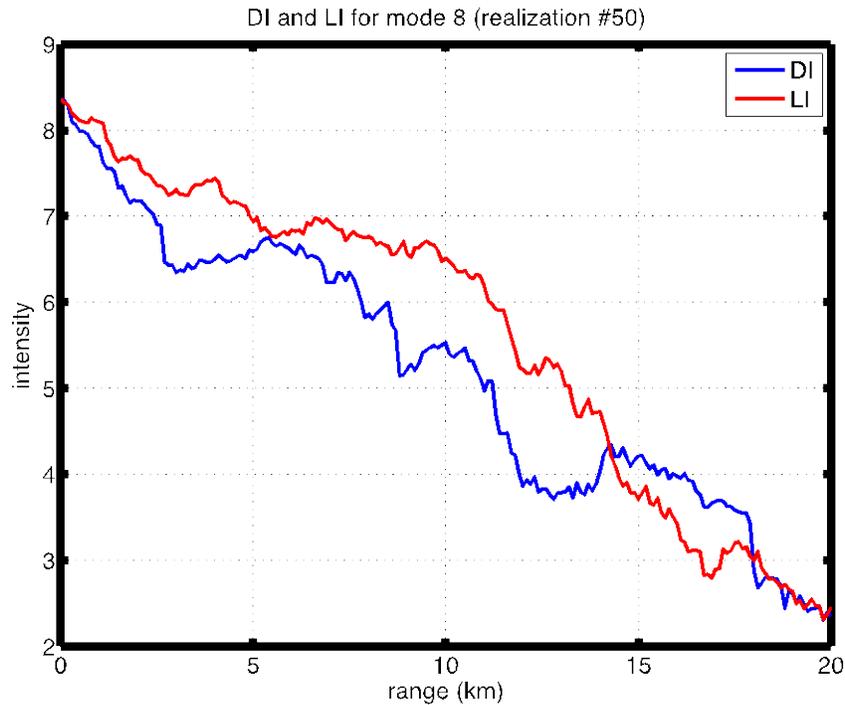


*Figure 1. Reverberation predictions at 3 kHz obtained with transport theory. The red curves ignore all effects of boundary roughness during propagation. The blue curves account for surface forward scattering. The green curves approximate the effect of surface forward scattering in terms of a coherent loss.*



*Figure 2. TREX13 reverberation data at 3.5 kHz.*

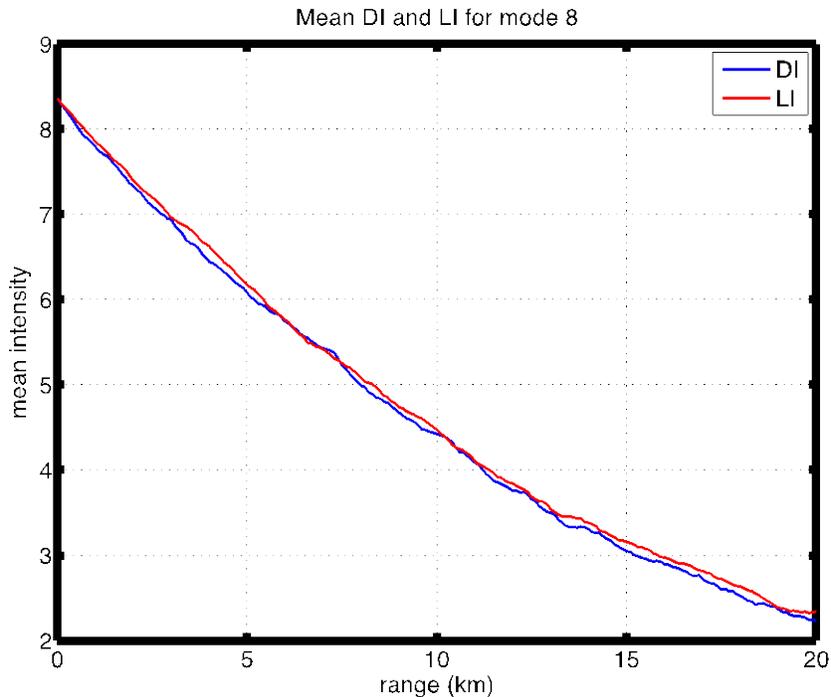
the figures that follow. A comparison for the intensity of mode 8 (in arbitrary units) as a function of range for the same rough surface realization is shown in Figure 3. The intensity of mode 8 decreases with range as surface forward scattering leads to energy transfer to other modes. Indeed, the first treatment leads to significant differences from the more accurate second treatment, which might cause doubt on the accuracy of transport theory results.



**Figure 3. Mode 8 intensity as a function of range. The blue curve is the result for the more accurate treatment of mode coupling, and the red curve is the result for the less accurate treatment used with transport theory.**

However, transport theory deals with ensemble averages, not results for single realizations as shown in Figure 3. When the results of 50 realizations are used to obtain the average intensity as a function range for mode 8, the agreement between the two methods is quite good, as shown in Figure 4. Since the goal of transport theory is to give the correct average mode intensity as a function of range, we find that our approach can indeed accomplish that goal, with a very fortunate cancellation of errors that arise from a mode coupling treatment that is not uniformly accurate in the number of modes.

This interesting finding helps explain the quite good agreement found in previous comparisons between ensemble averaged rough surface PE simulations and transport theory results. It also paves the way for our formal publication of this body of work that has been developed over a number of years. A paper on the development of the mode coupling treatments that led to Figures 3 and 4 is in preparation, which will be followed by a paper on the transport theory itself.



**Figure 4.** *The average intensity of mode 8 for 50 realizations. The two treatments agree for the exponential decay of the initial mode, even though they differ for individual realizations.*

## IMPACT/APPLICATIONS

Work in transport theory propagation and reverberation modeling should lead to improved simulation capability for shallow water propagation and reverberation in which multiple scattering from rough boundaries is properly taken into account. This capability should be particularly important in the mid-frequency range where multiple scattering effects can be important, yet where a modal description can be used. Transport theory propagation and reverberation modeling has the potential to be even faster than ray tracing, yet be able to account for scattering effects outside the scope of other efficient modeling methods.

## RELATED PROJECTS

1. PMW-120 (Marcus Speckhahn) is supporting work on developing a model (TOTLOS) that can approximately account for effects of surface forward scattering in ray-based (such as CASS/GRAB) or mode-based propagation and reverberation models. Results for transport theory are now being used to aid in TOTLOS development, which has become an important component of the present project.
2. The ONR OA project “Mid-Frequency Reverberation Measurements with Full Companion Environmental Support, DJ Tang (PI) is the parent project for TREX13, in which reverberation data have been obtained for verifying the accuracy of transport theory predictions for reverberation.

## REFERENCES

- [1] E. I. Thorsos, F. S. Henyey, W. T. Elam, B. T. Hefner, S. A. Reynolds, and J. Yang, "Transport theory for shallow water propagation with rough boundaries," *Shallow-Water Acoustics*, Proceedings of the Second International Shallow-Water Acoustics Conference, Shanghai, China, September 16-20, 2009, AIP Conference Proceedings 1272, pp. 99-105.
- [2] A. P. Rosenberg, "A new rough surface parabolic equation program for computing low-frequency acoustic forward scattering from the ocean surface," *J. Acoust. Soc. Am.* **105**, 144-153 (1999).
- [3] [ftp://ftp.ccs.nrl.navy.mil/pub/ram/RevModWkshp\\_II](ftp://ftp.ccs.nrl.navy.mil/pub/ram/RevModWkshp_II).

## PUBLICATIONS

E. I. Thorsos, J. Yang, W. T. Elam, F. S. Henyey, F. Li, and J. Liu, "Comparison of transport theory predictions with measurements of the decrease in shallow water reverberation level as the sea state increases," in Proceeding of the ICA 2013, Montreal, Quebec, Canada, 2-7 June 2013 [published].