Optimal Combining Data for Improving Ocean Modeling

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LONG-TERM GOALS

The long range scientific goals of the proposed research comprise: (1) developing rigorous approaches to optimal combining different kinds of data (images, CTD, HFR, glider, drifters , and possibly output of regional circulation models) for accurate estimating the upper ocean velocity field, subsurface thermohaline structure, and mixing characteristics (2) constructing computationally efficient and robust estimation algorithms based on alternative parameterizations of uncertainty and comprehensive testing them on synthetic data (3) processing real data in the Adriatic and Ligurian Sea via new techniques

OBJECTIVES

The objectives for the third year of research were:

(1) Further developing and testing methods for fusing glider data with ship CTD observations with focus on reconstructing frontal zones and identifying three-dimensional termohaline structures. (2) Developing parametric methods for estimating finite-size Lyapunov exponent (FSLE) from sparse drifter data on the base of theoretical studies of FSLE for anisotropic circulation patterns .

APPROACH

We develop theoretical approaches to the data fusion problem in context of the possibility theory (fuzzy logic) and in the framework of the classical theory of random processes and fields covered by stochastic partial differential equations. We also design computational algorithms derived from the theoretical findings. A significant part of the algorithm validation is their testing via Monte Carlo simulations. Such an approach provides us with an accurate error analysis. Together with my collaborators from Rosenstiel School of Marine and Atmospheric Research (RSMAS), Consiglio Nazionale delle Ricerche (ISMAR, LaSpezia, Italy), University of Toulon (France), Observatoire Oceanologique de Villefranche sur Mer (France), and Naval Postgraduate School (Monterrey, CA) we implement the algorithms in concrete ocean models such as HYCOM, NCOM, MFS, and NEMO as well as carry out statistical analysis of real data sets by means of new methods

WORK COMPLETED

1. Further developing and testing methods for fusing glider data with ship CTD observations with focus on reconstructing frontal zones and identifying three-dimensional termochline structures.

Underwater gliders are autonomous instruments that have become increasingly more common in oceanography during the last decade mostly because of their low consumption properties. They can cover areas of tens to thousands km during extensive monitoring missions of order of weeks to months [1-3], and are expected to play an increasingly significant role in future strategies for 3D monitoring of the ocean.

In order to use glider data in an optimal way, it is important to perform analyses that take into account the specific characteristics of the platform. In particular, gliders, because of their low consumption, move at a relatively low horizontal speed, of the same order as the propagation velocity of several phenomena at various scales in the ocean [1]. This implies that data along glider sections are influenced by both spatial and temporal variability, with time variations potentially appearing as folded into space variability. This phenomenon, often indicated as Doppler smearing, is not specific to glider data but it is common to all oceanographic measurements obtained from a moving platform including traditional ship based data and satellite data. The problem of time variability being projected onto spatial variability is indeed intrinsic, and once the data are collected cannot be completely removed. A number of methods

have been developed and applied to at least alleviate the problem, for instance using other independent measurements or additional assumptions on the flow to remove the uncertainty or taking advantage of specific sampling strategies [4,5].

The primary focus of our research was on better understanding the Doppler smearing and accounting for its effects using additional information such as ship CTD or/and model output in the same area. Another direction was to construct and test practical algorithms for separating spatial and temporal variability from glider observations. Finally, the problem of recovering 3D structures such as water intrusions from glider and CTD data has been addressed.

The main completed steps in the underlined research are as follows.

- A theoretical ground has been laid for the general problem of data fusion involving moving devices
- Numerical experiments have been carried out with artificial evolving intrusions and fronts to highlight and quantify the Doppler smearing effects
- Three approaches have been developed to recovering a front evolution from several glider transects over the same area combined with ship CTD data: first, direct deconvolution via an appropriate parameterization of the front, second, aggregating glider and CTD data using fuzzy regression, and, finally, a traditional polynomial regression applied to the glider data only with CTD observations serving as a control sample.
- An approach have been developed and tested for identifying 3D structures by combining glider and CTD data. In particular, equations were derived and analyzed connecting parameters of a moving water intrusion such as its dimensions and velocity to parameters of glider transects and CTD profiles.

- The developed techniques have been applied to real data at a frontal zone in the Ligurian Current in the North West Mediterranean Sea [6] and the results from the mentioned approaches were compared. The problem of retrieving the front evolution was challenging because of the presence of a number of competing scales of motion, some of which were of the same order as the glider sampling. As a consequence, glider data could be influenced by both space and time variability, and glider sections cannot be considered as snapshots in time.

2. Developing parametric methods for estimating finite-size Lyapunov exponent (FSLE) from sparse drifter data on the base of theoretical studies of FSLE for anisotropic circulation patterns.

The finite size (scale) Lyapunov exponent (FSLE) was originally introduced in [7,8] to measure the growth rate of finite size perturbations in turbulence on time scales preceding to diffusion regime. Since then FSLE has been applied to variety of problems in physical oceanography and physics of atmosphere. In particular, the approach was used for detecting barriers to transport [9-11], measuring stirring [9] and mixing[12,13], identifying Lagrangian coherent structures [14], and studying biological activity in upwelling systems [15].

In majority of the cited papers FSLE maps were derived from experiments with high resolution circulation models rather than from real data. The problem is that usually drifter data which potentially can be directly used for estimating FSLE are too sparse and short in time to ensure reliable estimates.

Our approach to estimating FSLE from sparse drifter observations short in time is based on linearization of the Eulerian velocity field in a vicinity of the point where FSLE is estimated. Then the unknown parameters of the velocity field such as divergence (γ), curl (σ), and stretching (s), can be efficiently computed from a few drifter trajectories even if they did not start close one to another. Finally, FSLE can be estimated using our theoretical findings relating FSLE to γ, σ, and s .

Notice that up to now theoretical works addressed primarily the scaling of FSLE denoted by $\lambda(\delta)$ as a function of the initial separation magnitude δ for flows close to isotropic, e.g. [7,8,16,17], while the most important anisotropic circulation patterns did not attract much attention, probably because it is a more challenging problem from the analytical viewpoint.

Regarding to FSLE studies the following tasks have been completed during the reported period.

- Large diffusivity asymptotics of FSLE have been investigated for a class of anisotropic models encompassing saddle points, shear flows, and other circulation patterns
- An exact expression for FSLE has been found in vicinity of a saddle point perturbed by noise and investigated its asymptotic for small diffusivity
- The theoretical results have been tested via Monte Carlo simulations
- A parametric method for estimating FSLE from drifter observations has been developed and tested. The method is based on the Maximum Likelihood approach in estimating parameters of stochastic differential equations. An error analysis was also provided.

RESULTS

To formulate our principal theoretical results on Doppler smearing let us present a rigorous formulation of the problem. Assume that one is interested in estimating the boundary of a certain 2D region G varying in time, from repeated glider transects through the region. In applications G could be an intrusion or the cold (warm) side of a front. Let the boundary be described by equation $f(z,x,t) = 0$ where z, x, t are the vertical coordinate , horizontal coordinate (along the glider transect), and time respectively.

First, we derived the following equation for the glider image (screening) of that boundary during a single intersect

$$f\left(\bar{x}_j - (-1)^j \Delta x_j \left(\frac{1}{2} - \frac{z}{D}\right), z, \frac{x_j}{v}\right) = 0 \qquad (1)$$

It is supposed that the glider moves along a saw-tooth trajectory with horizontal velocity v, $\bar{x}_j$ is the center of interval (x_j, x_{j+1}) where $x_j$are starting points of downcasts and upcasts of the same depth D with even j corresponding to downcasts and odd j corresponding to upcasts, and finally Δx_j is the length of interval (x_j, x_{j+1}).

Equation (1) was the basis for theoretical studies and numerical experiments addressing the Doppler smearing. One of the most important conclusions from such experiments is that under a relatively simple parameterization of the front $z = h(x,t)$ (the solution of $f(z,x,t) = 0$) one can observe complex and highly variable screenings. In particular, we experimented with a well known parameterization of fronts [19] via hyperbolic functions and different ratios of glider and front velocities. An example is provided in Fig.1 in the case of a front moving 15 percent faster than the glider along a transect .

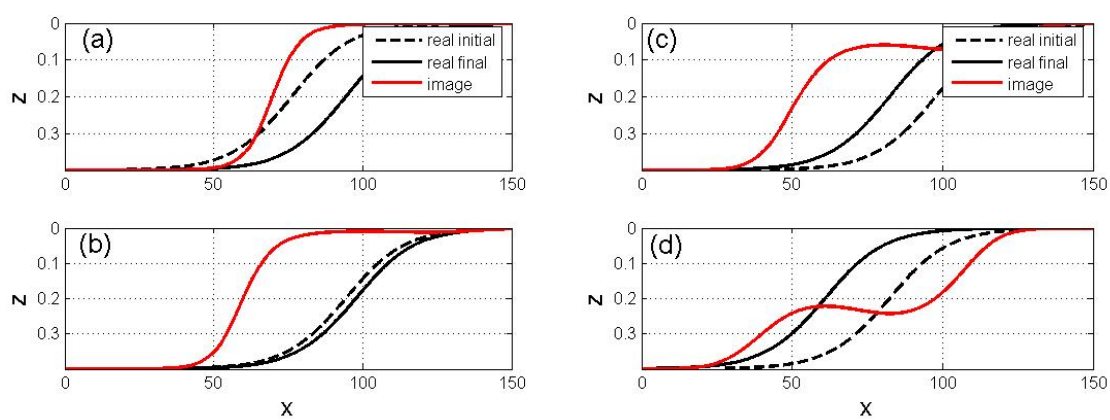

Figure 1 Example of difference between true front (black) and its glider image (red). Broken line shows the initial front positions and solid one shows its terminal position during the glider cruise. The four panels illustrate subsequent glider transects: transect 1 (a) and transect 2 (b), transect 3 (b) and transect 4 (d)

The glider moves twice back and forth along the transect, with the offshore (inshore) transects shown in the two upper (lower) panels. The corresponding screenings (red lines) appear significantly different from the actual frontal shapes (black lines), as a consequence of the complex relative motion of the glider and of the front. As the glider moves, also the front translates (without changing its shape) with an instantaneous velocity that is modulated in time, and changes direction in accordance with the sinusoidal time dependence. As a consequence, while both glider and front move offshore at initial time, they quickly become out of phase and at different times they move in opposite or concordant directions. The resulting screenings are quite variable, showing an alternation of steepening, typically indicative of motion in opposite direction, and flattening, typically associated with motion in the same direction.

We proved rigorously that local maxima (minima) on the glider image appear if and only if the front moves faster than the glider.

Next findings concern with advantages and drawbacks of three tested fusion algorithms. Even though we concentrated on glider/CTD data fusion the same approach can be applied to the glider/model output case.

The first method to retrieve $f(z,x,t)$ by combining glider and CTD data was based on the assumption that only the front location evolves in time while the curvature and its depth remains constant and can be estimated from CTD using non-linear regression. Thus, the problem was reduced to finding a single function of time (the position of the front center) from another single function of space variable x (glider screening) which is a well posed mathematical problem and can be easily solved by using (1). Unfortunately, this approach turned out to be unsatisfactory because of too restrictive assumptions, however the orders of estimated parameters were ealistic.

A more promising method we eventually applied is as follows.

First, an approximately uniform grid in space was choosen. Then $h(x,t)$ was approximated by a polynomial in t on that grid using observations neiboring to the corresponding grid points, and finally at each given t we used a space fitting function when appropriate.

An important issue is the choice of a polynomial approximation method. The most adequate approach is a fuzzy regression since its ability to combine data coming from different sources, e.g. [20]. However, in the present application the second data source is limited to only one CTD transect, and therefore it allows for estimating $h(x,t)$ in a relatively small vicinity of the time of that transect with the polynomial degree not higher than 2 (linear or quadratic). For this reason, we eventually stopped by a classical polynomial regression using the glider data only, while CTD observations are used as a control sample for validation of the interpolation method in use. In Fig.2 we show the result of estimating the front evolution obtained by this method and compare the result with satellite image.

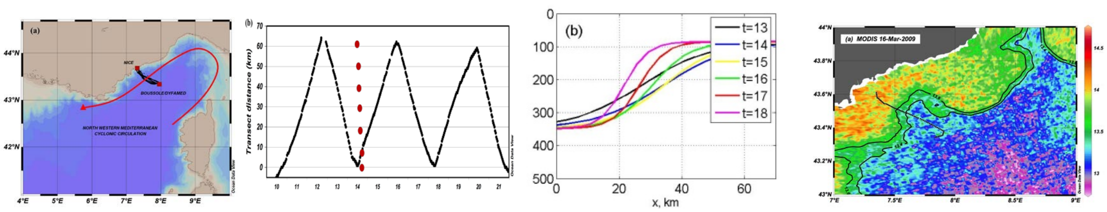

Figure 2. From left to right: 1) Geographical region of interest. 2) Glider mission in terms of cross-shore distance versus time with superimposed CTD casts (red dots). 3) The front spatial structure estimated for the period 18-22 March 2009. 4) SST images with superimposed the glider path during March 21

Overall, the obtained oscillation of the front location is in good agreement with the haracteristics of mesoscale variability of the Ligurian Current as reported in the literature [21], with a meandering activity scaling over 3-6 days and with amplitude of approximately two radii of deformation centered at 20-25 km offshore. This is also in agreement with qualitative indications from SST images, as shown in Fig.2 (last panel).

Regarding to the methodology we concluded that a traditional polynomial regression for retrieving the front evolution performed better than two other developed procedure (parametric estimation and fuzzy linear regression). In particular the polynomial regression of glider data showed a perfect agreement with CTD and allowed us to estimate the evolution for a longer period of time than other methods.

Finally, a method developed for reconstructing 3D structures by combining glider and CTD data turned out to be efficient when applying to synthetic deep sea intrusions. The problem was formulated as follows. What is the least number of glider transects and CTD stations should be performed to determine the dimensions and velocity of a moving intrusion and how to compute that parameters given glider 'screenings' and CTD profiles?

It was proven that two gliders transects (or a glider fleet of two) together with two CTD stations performed at different time and different locations allow for complete retreiving the dimensions and horizontal velocity components of a moving ellipsoidal intrusion. One of the experiments is demonstrated in Fig. 3

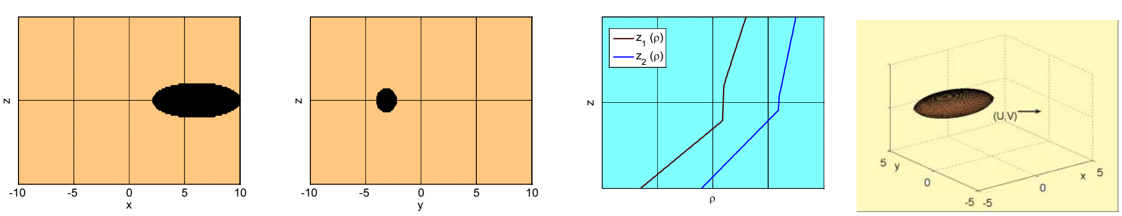

Figure 3. An experiment with synthetic data: 1) Gliders screenin along the x-axis. 2) Glider screening along the y-axis. 3) CTD profiles. 4) Reconstructed intrusion

2. Regarding to theoretical studies of FSLE we first obtained the exact dependence of $\lambda(\theta)$ on the polar angle as diffusivity D goes to infinity

$$\lambda_\infty = c_\alpha \left(\frac{(a+d)(1+\alpha^2)}{2} + \frac{2(a-d)cos2\theta}{3} + \frac{2(b+c)sin2\theta}{3} \right) \quad , \quad c_\alpha = \frac{\log \alpha}{\alpha^2 - 1} \tag{2}$$

where α>1 is a prescribed threshold, appearing in the FSLE definition, for the following model of Lagrangian turbulence

$$\dot{X} = aX + bY + \sqrt{2D}\dot{w}_1 \quad , \quad \dot{Y} = cX + dY + \sqrt{2D}\dot{w}_2 \tag{3}$$

where(X, Y) is the separation of two particles and w_1, w_2 independent Brownian motions. In Fig.4 we show exact expressions of $\frac{\lambda_\infty}{c_\alpha}$ for several typical circulation patterns: gyre, saddle, shear, divergence, and convergence.

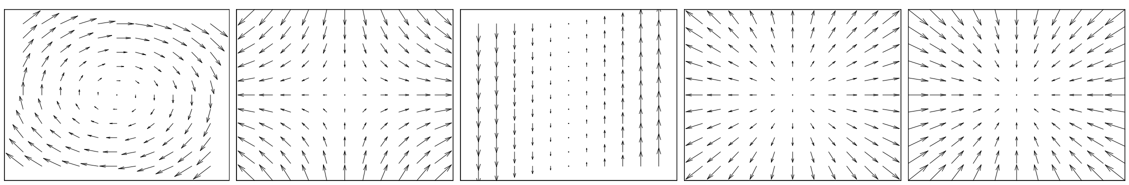

Figure 4. Normalized FSLE at infinite diffusivity for different types of circulation around a stagnation point.

Relation (2) was used to develop a parametric method for estimating FSLE from drifter observations. First, using (3) the following parameters were estimated by the Method of Maximum Likelihood: divergence $\gamma = a + d$, curl $\sigma = a - d$ and stretching $s = b + c$. Then γ, σ, s were recalculated in FSLE via (2). Monte Carlo experiments demonstrated a good accuracy of estimates even for 10 drifters travelling during 10 days under high intensity of turbulence D. One of the experiments with synthetic data is shown in Fig. 5 in which 10 drifters started from initial positions uniformly distributaed over a circle.

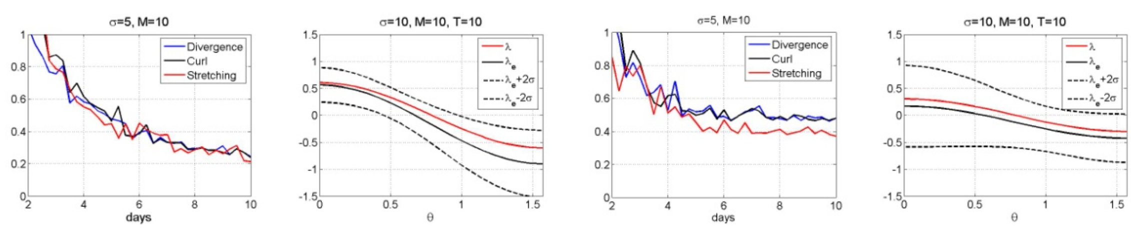

Figure 5. 1) Dependence of estimation error for circulation parameters in a vicinity of a saddle point, 2) 'True' FSLE (red) vs polar angle compared to the estimate (solid black) and confidence boundaries (broken black), 3) and 4) Same for a shear flow.

IMPACT/APPLICATIONS

The developed approach to separating space and time variability from glider and CTD observations provides the physical oceanography community with useful tools for adequate interpreting data collected by moving devices. As a consequence , that could lead to improving diagnosis and prediction of meso- and submesoscale processes in coastal frontal zones.

Our theoretical findings in studying finite-size Lyapunov exponent provide researchers with efficient tools for identifying different types of stagnation points in ocean circulation patterns such as shear flows, gyres, and hyperbolic circulation. The method of estimating FSLE developed on the base of that findings would be a competitive alternative to the existing estimation procedures from Lagrangian data especially in the case of sparse short time series .

RELATED PROJECTS

"Ocean 3D+", MURI Project, ONR N00014-11-1-0087, PIs: A. Griffa, T. Ozgokmen, I. Mezic, C. Jones , I. Rypina, S. L. Smith, L. Pratt, D. Kirwan

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3. L.I. Piterbarg, (2013), Finite size Lyapunov exponent at a saddle point, *Applied Mathematical Modeling*, submitted