

## **Fundamental Research to Support Direct Phase-Resolved Simulation of Nonlinear Ocean Wavefield Evolution**

Dick K.P. Yue

Center for Ocean Engineering  
Department of Mechanical Engineering  
Massachusetts Institute of Technology  
Cambridge, MA 02139

phone: (617) 253- 6823 fax: (617) 258-9389 email: [yue@mit.edu](mailto:yue@mit.edu)

Yuming Liu

Center for Ocean Engineering  
Department of Mechanical Engineering  
Massachusetts Institute of Technology  
Cambridge, MA 02139

phone: (617) 252- 1647 fax: (617) 258-9389 email: [yuming@mit.edu](mailto:yuming@mit.edu)

Award Number: N00014-10-1-0693

<http://www.mit.edu/~vfr/>

### **LONG-TERM GOAL**

The long-term goal is to develop a new generation of wave prediction capability, which is called **SNOW** (simulation of nonlinear ocean wavefield), for the evolution of large-scale realistic ocean wavefields using direct phase-resolved simulations. Unlike the available phase-averaged approaches, SNOW models the key physical mechanisms such as nonlinear wave-wave, wave-current, wave-wind and wave-bottom interactions and wave-breaking dissipation in a direct physics-based context.

### **OBJECTIVES**

The specific scientific and technical objectives are to:

- Develop and improve physics-based phenomenological modeling for wind forcing input and wave breaking dissipation.
- Speed up the computational algorithms underlying SNOW simulations on massively-parallel high-performance computing (HPC) platforms.
- Extend current capabilities to handle high sea states and very steep local waves while maintaining near linear SNOW operational count.
- Extend SNOW simulations to allow more general initial/boundary conditions based on wave spectral characteristics or hybrid (multiple-point and/or whole-field) wave measurements. Investigate and understand uniqueness and compatibility issues of such input to phase-resolved reconstruction and forecasting of directional broadband wavefields.

- Characterize and quantify the effects of noise, uncertainty, incompleteness, and incompatibility in hybrid wave data on phase-resolved wavefield reconstruction and prediction.
- Perform direct validation and quantitative cross-calibration of SNOW simulations with phase-averaged wave model predictions and field/laboratory measurements.
- Extend SNOW to general finite water depth by including effects of fluid stratification, variable current, and changing bathymetry.

## APPROACH

SNOW employs direct physics-based phase-resolved simulations for predicting the nonlinear evolution of large-scale ocean wavefields. SNOW is fundamentally different from the existing phase-averaged models in that, under SNOW, key physical mechanisms such as wave-current, wave-wind and wave-bottom interactions and wave-breaking dissipation are modeled, evaluated and calibrated in a direct physics-based context. In SNOW, detailed phase-resolved information about the wavefield is obtained, from which the statistical wave properties are derived.

SNOW is based on an extremely efficient high-order spectral (HOS) approach for direct computation of nonlinear ocean wavefield evolution. HOS is a pseudo-spectral-based method that employs Zakharov equation and mode-coupling idea and accounts for nonlinear wave-wave, wave-current, and wave-bottom interactions to an arbitrary high order ( $M$ ) in wave/bottom steepness. This method obtains exponential convergence and (approximately) linear computational effort with respect to  $M$  and the number of spectral wave/bottom modes ( $N$ ). SNOW is an ideal tool for phase-resolved prediction of realistic ocean wavefield evolution.

By incorporating point and/or area wave measurements into the simulation, SNOW provides a capability of reconstructing and forecasting nonlinear evolution of phase-resolved ocean wavefields. The objective of wave reconstruction is to obtain detailed specifications (including phase) of a nonlinear wavefield, which matches given (directly or remotely) sensed wave data or specified wave spectra. Nonlinear wave reconstruction is achieved based on the use of optimizations with multiple-level (theoretical and computational) modeling of nonlinear wave dynamics. Using the reconstructed wavefield as the initial condition, SNOW simulation would provide a (short-time) deterministic forecasting of the phase-resolved wavefield evolution (Wu 2004; Yue 2008).

SNOW computations can now be routinely performed for nonlinear ocean wavefields in an domain of  $O(10^{4-5})$  km<sup>2</sup> with an evolution time of  $O(1)$  hours. Such large-scale SNOW simulations are normally performed on advanced high-performance computing platforms using up to  $O(10^{3-4})$  processors (e.g. Xiao, Liu & Yue 2012) under the DoD challenge project: “Large-Scale Deterministic Predictions of Nonlinear Ocean Wavefields”.

## WORK COMPLETED

Over the past year, we continued our research in support of extension of SNOW to the general situations including the presence of non periodic boundary conditions, broadband wave spectrum, steep waves, two-layer fluids, and finite water depth. In addition, we continued to make direct comparisons of the SNOW simulations with wave-basin/field measurements and phase-averaged model predictions, and to apply SNOW computations to investigate the nonlinear wave statistics and the occurrence and characteristics of rogue waves. Specifically, the major work completed includes:

- ***Extension of SNOW for broadband nonlinear wave-wave interactions.*** We modified HOS algorithm to effectively account for nonlinear long-short wave interactions. The algorithm has been integrated into SNOW to enable the simulation of general broadband wavefield evolution including long-wave interactions. This allows for a better understanding of the characteristics of short surface wave motions which are of importance to proper interpretation of remotely sensed ocean surface data.
- ***Speedup and applications of SNOW simulations:*** We continued to improve the computational speed, scalability and robustness of the SNOW code on HPC platforms for the simulation of large-scale nonlinear ocean wavefield evolution. We applied large-scale SNOW computations to study nonlinear wave statistics and occurrence and characteristics of extreme wave events in open seas (Liu, Xiao & Yue 2013).
- ***Evaluation and assessment of phase-averaged wave prediction models.*** We performed direct comparisons of SNOW simulations with phase-averaged model predictions and laboratory experimental measurements on nonlinear evolution of three-dimensional ocean wavefields. Based on the comparisons, we assessed the validity and limitations of the available phase-averaged wave prediction models, and identified the areas for further improvements. In particular, it is found that the modified nonlinear schrödinger equation (MNLS) (Dysthe 1979; Trulsen & Dysthe 1996) overpredicts the spreading of wave energy in the transverse direction due to incomplete consideration of the bandwidth effect. Thus, the MNLS may not be valid in the prediction of long time evolution of nonlinear directional ocean wavefields.
- ***Characteristics and occurrence statistics of rogue waves in open seas.*** Routine large-scale SNOW computations were performed to obtain a large catalogue of nonlinear directional ocean wavefields corresponding to broad range of wave spectral parameters. From these nonlinear wavefields, we identified rogue wave events and investigated the characteristics and dependence of rogue wave events on spectral wave parameters, including the presence of both swells and seas (Liu, Xiao & Yue 2013).

## RESULTS

We applied direct phase-resolved SNOW simulations to investigate the energy flux in turbulence of capillary waves. We considered the inertial range spectrum of capillary wave turbulence. Recent experimental measurements (Falcon *et al.* 2007) reported a linear scaling of surface elevation spectrum  $\langle |\eta_k|^2 \rangle$  with energy flux  $P$ . This is in apparent disagreement with weak turbulence theory (WTT) which predicts  $\langle |\eta_k|^2 \rangle \sim P^{1/2}$  (Zakharov and Filonenko 1967). We conducted a direct numerical investigation of the problem by using SNOW (with a modification for capillarity wave dynamics). By considering a range of  $P$  spanning two orders of magnitude, we showed that the square-root and linear scalings are realized at relatively low and high values of  $P$  (associated with low and high nonlinearity of the wave system) respectively, thus resolving the controversy.

In figure 1, we plotted the (normalized) energy of the inertial range  $E_i$  (as a reflection of magnitude of  $\langle |\eta_k|^2 \rangle$ ) with respect to the energy flux  $P$  evaluated in the numerical simulation. For small values of  $P$ , our results confirm the dependence  $\langle |\eta_k|^2 \rangle \sim P^{1/2}$  predicted by WTT. For large values of  $P$ , we found a linear scaling relation  $\langle |\eta_k|^2 \rangle \sim P$ , consistent with the experimental findings. Therefore, the results from WTT and the experimental measurements are not in disagreement but are in fact complementary and are realized at different levels of nonlinearity for this problem.

## IMPACT/APPLICATIONS

This work paves the way toward the development of a new generation of wave prediction tool using direct phase-resolved simulations. It augments the phase-averaged models in the near term and may serve as an alternative for wave-field prediction in the foreseeable future.

## RELATED PROJECTS

This project is related to the project entitled “High-Resolution Measurement-Based Phase-Resolved” (N00014-08-1-0610). This project focuses on the development of advanced algorithms and physics-based modeling for phase-resolved prediction of ocean wavefield evolution while the related project focuses on the practical application of the wave prediction capability to realistic ocean environments.

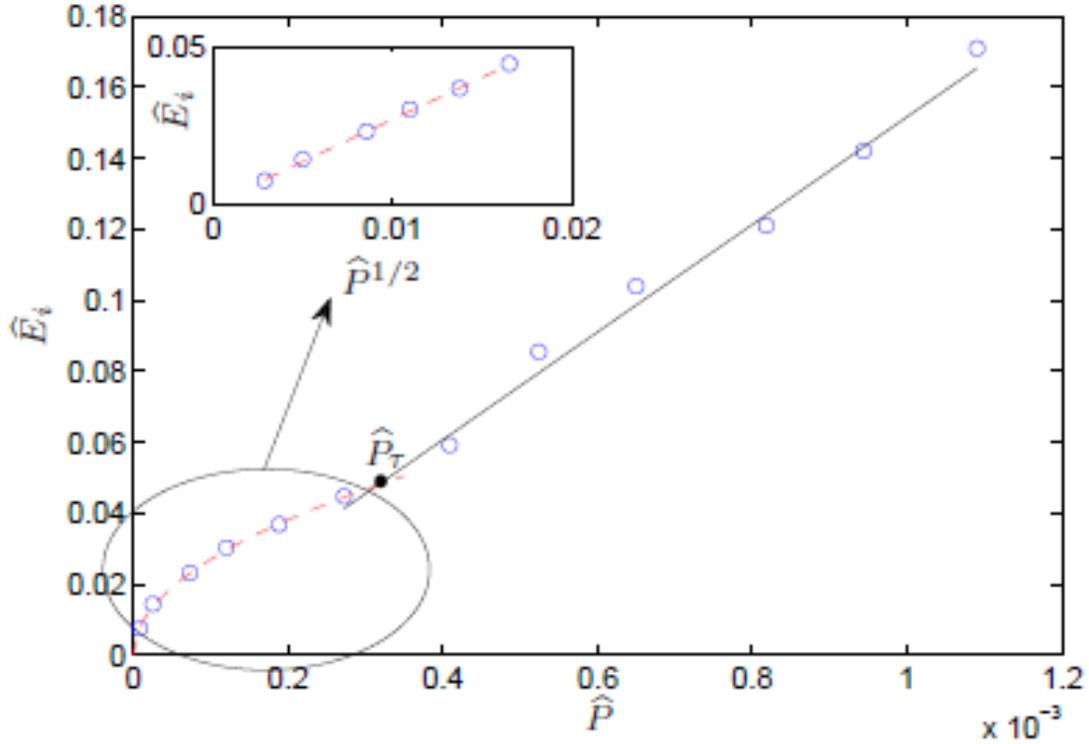
## REFERENCES

1. Dysthe, K.B. 1979 Note on a modification to the nonlinear schrodinger equation for application to deep water waves. *Proc. R. Soc. Lond. A*, 369:pp.105 – 14.
2. Falcon, E., Laroche, C. and Fauve, S. 2007 Observation of gravity-capillary wave turbulence. *Phy. Rev. Lett.* **98**, 94503.
3. Trulsen, K. and Dysthe, K.B. 1996 A modified nonlinear Schrodinger equation for broader bandwidth gravity waves on deep water, *Wave Motion*, Vol 24, pp. 281-189.
4. Wu, G. 2004 Direct simulation and deterministic prediction of large-scale nonlinear ocean wavefield. Ph.D Thesis, Massachusetts Institute of Technology, Cambridge, MA.
5. Xiao, W., Liu, Y. and Yue, D.K.P. 2012 Prediction of Rogue Waves by Large-Scale Phase-Resolved Nonlinear Wavefield Simulations, *Proceedings of the DoD HPCMP Users Group Conference 2012*, New Orleans, LA
6. Yue, D.K.P. 2008 Nonlinear Wave Environments for Ship Motion Analysis, *27<sup>th</sup> Symposium on Naval Hydrodynamics*, October 5 – October 10, 2008, Seoul, Korea.
7. Zakharov, V. and Filonenko, N. 1967 Weak turbulence of capillary waves. *Journal of Applied Mechanics and Technical Physics*, **8**, 37.

## PUBLICATIONS

1. Xiao, W., Liu, Y., Wu, G. & Yue, D.K.P. 2013 Rogue wave occurrence and dynamics by direct simulations of nonlinear wavefield evolution. *Journal of Fluid Mechanics*, **720**: 357-392. [published, refereed]
2. Liu, Y., Xiao, W. & Yue, D.K.P. 2013 Prediction of Rogue Waves in Open Seas by Phase-Resolved Nonlinear Wave-Field Simulations, *Proceedings of the DoD HPCMP Users Group Conference 2013*. [published, non-refereed]
3. Pan, Y. & Yue, D.K.P. 2013 Energy Flux in Turbulence of Capillary Wave, *Physics Review Letter*. [submitted]
4. Liu, Y., Wu, G., Qi, Y. & Yue, D.K.P. 2013 Phase-resolved nonlinear wave reconstruction and forecasting of irregular waves. Part I: theory. *Journal of Fluid Mechanics*. [to be submitted]

5. Liu, Y., Wu, G. & Yue, D.K.P. 2013 Phase-resolved nonlinear wave reconstruction and forecasting of irregular waves. Part II: comparisons to measurements. *Journal of Fluid Mechanics*. [to be submitted]



**Figure 1:** Energy of the inertial range  $E_i$  ( $\circ$ ) with respect to the energy flux  $P$  in the series of SNOW simulations, with square-root (---) and linear (—) fittings for small and large values of  $P$ . Inset: Values ( $\circ$ ) and fitting (---) of  $E_i$  with respect to  $P^{1/2}$  for lower values of  $P$ .