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Grant or Contract # N00014-14-1-0476

Long-duration Environmentally-adaptive Autonomous Rigorous Naval Systems

Progress Report for Period: May 1, 2014 – September 30, 2014

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Date Prepared: 09/25/2014

Section I: Project Summary

1. Overview of Project

Our long-term goal is to *develop and apply new theory, algorithms and computational systems for the sustained coordinated operation of multiple collaborative autonomous vehicles over long time durations in realistic multiscale nonlinear ocean settings, such that the integrated naval system optimally collects observations, rigorously propagates information backward and forward in time, and accurately completes persistent learning, environmental adaptation, machine metacognition and decision making under uncertainty.*

Specific Objectives:

- Derive, implement and evaluate rigorous and efficient Bayesian smoothing theory and schemes that respect nonlinear dynamics and capture non-Gaussian statistics, for robust persistent inference and learning, integrating information backward and forward in time over long durations in large-dimensional multiscale fluid and ocean dynamics.
- Derive and develop adaptive sampling theories and methods that predict the types and locations of the observations to be collected that maximize information about the ocean system studied (e.g. about its model state variables, parameters and/or formulations)
- Merge and refine our reduced-order DO stochastic equations with our path planning methods, to obtain new stochastic schemes for time-, coordination-, energy-, dynamics- and swarm- optimal path planning that efficiently account for ocean forecast uncertainties.
- Develop efficient onboard routing and high-level adaptation schemes that utilize observations collected by vehicles to autonomously adapt optimal plans (e.g. for paths, sampling strategies, collaboration or decision making process).
- Apply these schemes to simulated fluid and ocean dynamics, from idealized to realistic settings, and integrate these schemes for real sea exercises of opportunity involving distributed computations across components of the autonomous naval sensing systems.

2. Activities this period

Time-optimal Path Planning: Our previous path planning results include the development of an exact theory and an efficient algorithm for time-optimal routing of vehicles operating in strong, dynamic flows.

We have successfully extended this level-set based methodology to account for: obstacles, forbidden regions (both static and dynamic), uncertainty predictions of the flow-field, coordination among vehicles and on-board routing. Having completed various types of studies in both ideal and complex realistic ocean fields, our efforts during this period have been mainly directed towards publishing all the above results (Lermusiaux et al., 2014; Lolla et al., 2014a,b; Lolla and Lermusiaux, 2014; Lolla et al., 2014c).

Time-optimal Path Planning for Anisotropic Vehicles (Sailboats): We generalized our level-set methodology for time-optimal path planning to the case of vehicles with anisotropic motion constraints. Without increasing the computational complexity, our algorithm now predicts the fastest paths of vehicles such as sailboats, whose speed depends on the direction and magnitude of the wind that drives them. Fig. 1a depicts a typical polar diagram of the speed of a sailboat relative to the external flow, assuming the wind blows eastward at unit speed. This plot indicates that the sailboat achieves the optimal speed when it tacks at a non-zero angle relative to the wind. This anisotropic property seamlessly integrates with our existing framework and has been applied to several sailing-path planning scenarios, two of which are exemplified next.

Fig. 1b illustrates the optimal trajectory (in black) of a sailboat operating in the presence of a uniform 2D wind, directed northward. The sailboat starts at coordinates (0.2, 0.5) (marked by a red circle) and must be steered to the red star at coordinates (0.5, 0.5), located east of the start point. The optimal trajectory resembles an inverted 'V' shape, whose steepness directly depends on the tacking angle that leads to the maximum sailboat speed. Hence, the sailboat optimally utilizes the wind during both legs of its journey to minimize its travel time.

Fig. 1c considers the case of a sailboat operating in (i) an unsteady ocean flow past a circular island and (ii) a spatially uniform unsteady sinusoidal wind blowing in the NE-SW direction. The intent is to simulate the qualitative behavior of the coastal breeze, whose direction reverses during the course of any given day. The sailboat needs to be steered from the west of the domain (marked as a circle) to the far-east (marked as a star) in minimum time. The lower panel of Fig. 1c depicts, in blue, the intermediate level sets (reachability fronts) computed using our generalized path planning methodology. The optimal sailboat trajectory, computed using a modified backtracking procedure, is shown in red.

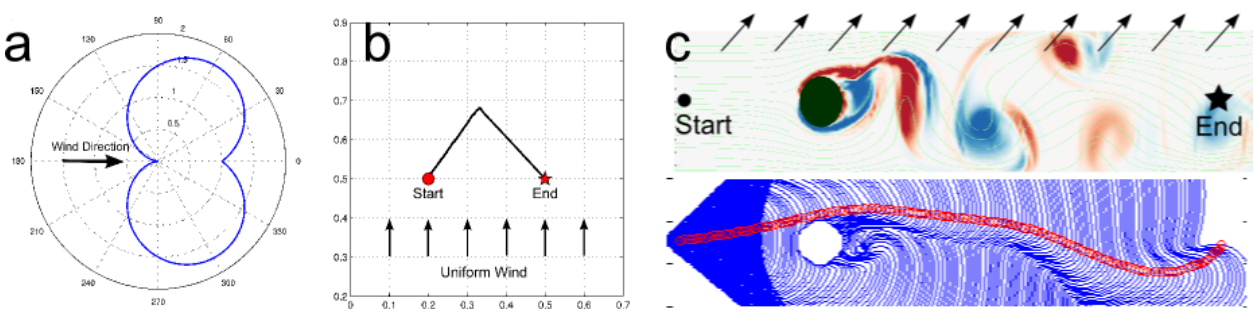


Figure 1: Time-optimal path planning for sailboats: (a). A typical polar diagram of a sailboat, for wind of unit magnitude blowing eastward. The blue curve shows the speed of the sailboat at different angles relative to the wind direction. The lack of azimuthal symmetry enforces anisotropic constraints on the sailboat motion. (b) The inverted 'V' shaped time-optimal trajectory of a sailboat navigating from (0.2, 0.5) to (0.5, 0.5) in a uniform wind directed northward. (c) Motion of a sailboat in an unsteady flow past a circular island, with sinusoidal wind in the SW-NE direction. The intermediate level set contours are shown in blue, while the optimal trajectory is depicted in red.

Energy-based Path Planning: We developed a novel stochastic optimization method to compute energy-optimal paths, among all time-optimal paths, for vehicles traveling in dynamic unsteady currents (Subramani, 2014; Subramani et al., 2014). The method is based on solving a stochastic level set equation using a dynamically orthogonal decomposition. The nominal vehicle speed is set-up as a to-be-optimized random variable in the level set PDE, and new dynamically orthogonal (DO) level set PDEs are derived. Our DO methodology is 100-1000 times faster than a classic Monte Carlo method for solving the stochastic level-set PDE. Global optimization is possible when all acceptable nominal vehicle velocity functions are sampled during the stochastic simulation step. The sampling can be done from a probabilistic distribution (e.g. uniform distribution), or a stochastic process (e.g. random-walk or a more general suitable Markov process). The advantage of this method is that the optimum solution within the class can be found in a single stochastic optimization step. If the class within which the search is performed is complete, one can then obtain the true solution. If the class is not complete, one can utilize the optimal result obtained for a given class to hierarchically generate new classes from the existing ones, hence refining the optima at each stochastic optimization, aiming to obtain the true solution for a complete class iteratively.

To demonstrate the nuances and inner workings of our methodology, we first applied it to two steady flow test cases, one that simulates a steady front, and the other, a steady eddy. For the first steady front-crossing test case, we formulated a dual energy-time optimization problem, and solved it to obtain a "semi-analytical" solution. This semi-analytical solution was then compared to the optimal energy path obtained from our Stochastic DO Level-Set Optimization method. Figure 2 illustrates the test case setup. Table 1 give the optimal parameters for a pre-chosen time to reach of t= 0.26, as obtained from solving the dual optimization problem (in MATLAB) and from the DO methodology. The close agreement between the two optimal estimates validates our stochastic DO level-set optimization methodology.

Table 1: Comparison of optimal parameters obtained using dual optimization and using new stochastic DO level-set optimization method for the optimal front crossing test-case for the chosen time to reach, t=0.26.

Parameter	Using NonLinear Optimization	Using new stochastic DO level-set optimization
θ_1	23.5^o	22.4^o
θ_2	23.5^o	20.7^o
β	65.8^o	65.9^o
F_1	2.9	2.8
F_d	2.6	2.5
F_2	2.9	3.0

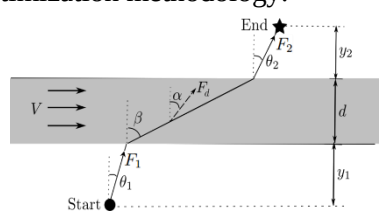


Figure 2: A uniform jet flowing from west to east between y=0.4 and y=0.6 simulates a steady front. The mission is to start the vehicle from the Start point (circle) and travel to the End point (star) by crossing the front.

Next, we applied our methodology to plan paths that are energy-optimal among all time-optimal paths in an idealized ocean simulation. Here, we used the wind driven double gyre flow field, which simulates the Gulf Stream in the Atlantic Ocean, and Kuroshio in the Pacific Ocean. Figure 3 shows the energy utilized by vehicles that are operated with different nominal engine speed time-functions.

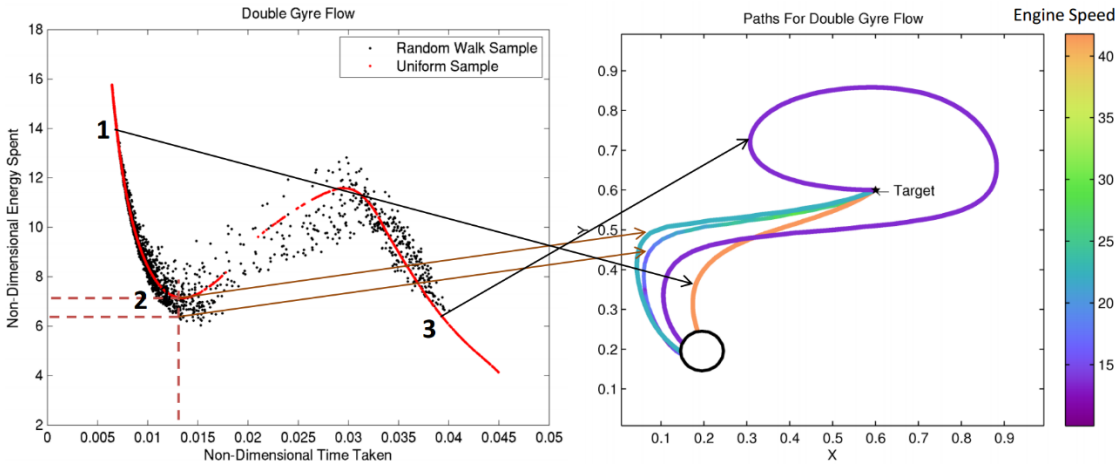


Figure 3 A non-dimensional wind driven double gyre flow field (Re=1000 and non-dimensional wind stress a=1000) is utilized to plan the heading and engine-speed time-histories of a vehicle starting from (0.2, 0.2) in the domain and heading to a target at (0.6, 0.6). **Left**: shows the exploration of time-to-reach and energy expended by vehicles whose nominal engine speeds have been sampled either by Uniform Sampling or by Random Walk Sampling. **Right**: shows 4 paths corresponding to points marked on the left panel. For example, for the given time to reach of t=0.013 (point 2), our results show that the minimum-energy path among all random-walk-engine-speed vehicles (vehicle varying its nominal speed according to a random walk) utilizes 15% less energy than the vehicle that uses a constant nominal speed. Another result shown is that of a vehicle which has a constant nominal speed of 10 (point 3) and executes a longer path by "riding the currents", but thereby utilizes less energy (but at the cost of arriving later) than a vehicle with non-dimensional speed of 40 (point 1).

Finally, we applied our methodology to plan paths in realistic simulated conditions. The mission considered was to start just offshore of Buzzard's Bay near WHOI and reach a target in the AWACS region, as shown in Fig. 4. Gliders that travel at relative horizontal velocities between 10 cm/s and 30 cm/s are assumed to be released on Aug 28, 2006 at 00 UTC. The flow data is obtained from the MSEAS free-surface primitive-equation model utilized in an implicit two-way nested computational domain set-up, with both tidal and atmospheric forcings. These simulated ocean flows assimilate real ocean data and correspond to a reanalysis of the real-time AWACS and SW06 exercises (Aug.-Sep. 2006) in the Middle Atlantic Bight and shelfbreak front region.

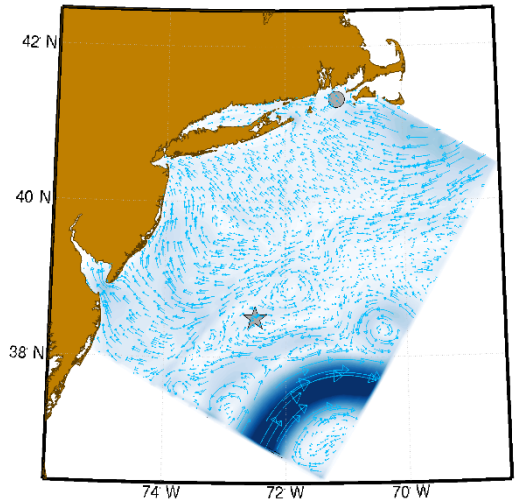


Figure 4 The start point is marked as a circle and the end point is marked as a star. The initial flow on Aug 28, 00 UTC is shown on the color axis in cm/s.

All gliders are assumed to follow the same yo-yo pattern in the vertical and the effects of the small vertical ocean velocities are assumed to be accounted for in the forward motions of the vehicles. We consider yo-yo patterns from the near surface to either the local near bottom or 400 m depth, whichever is shallower (for the mission considered, a large portion of the paths occurs on the shelf, within about 20 to 100 m). The horizontal currents that a glider encounters during its yo-yo motion are then the horizontal currents integrated along its path. Of course, it is the path to-be-determined that specifies the currents that are actually encountered. Our new stochastic DO level-set based energy optimal path planning method is employed to determine the time-optimal level sets for the class of relative glider speeds considered. Within that class, the evolution of the level sets corresponding to the minimum energy is obtained by sorting and the energy-optimal paths are computed by backtracking. We note that our method computes a large set of energy optimal paths, for a range of arrival times. Only a few of such paths are shown in Fig. 5, three of which are energy-optimal solutions, the other is also a time-optimal but constant-speed path shown for reference.

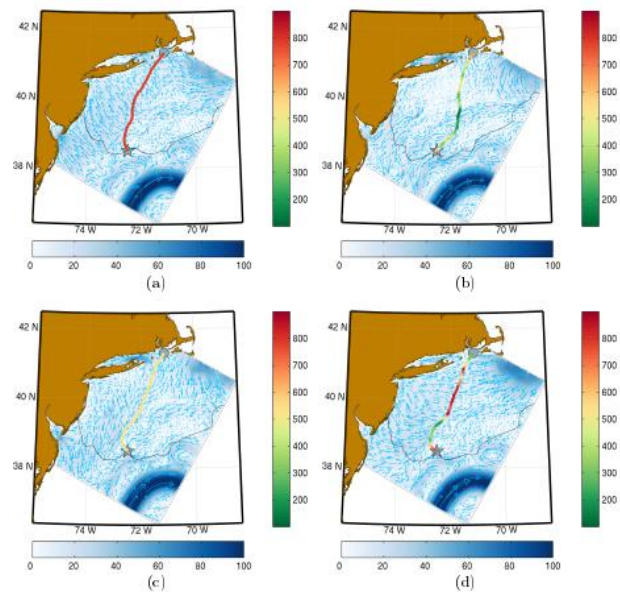


Figure 5: (a) Path that reaches in the shortest time, 12.96 days, but consumes the highest energy. (b) Path that takes 6 more days to reach the end point (18.78 days), but utilizes 40% less energy. (c) Path that reaches in 16 days using a constant speed. (d) Path that also takes 16 days but is energy optimal: it utilizes about 10% less energy than the path at constant speed. The vehicle speed along the path is plotted in color.

Adaptive Ocean Sampling and the GMM-DO Smoother: Our research in this area is focused on the development of an optimal non-Gaussian Bayesian smoothing scheme for high dimensional stochastic systems, such as ocean flows, that are governed by nonlinear dynamics. Smoothing enables inference of the system state, both backward and forward in time. This includes the accurate learning of initial conditions, which is an integral component of reanalysis studies. Smoothing also allows one to assess the impact of candidate future observations on present states through the metric of mutual information. This can efficiently utilize the limited oceanic sampling resources by deploying them at locations that maximize the mutual information between the observations and the forecast quantities of interest.

We developed a novel GMM-DO smoother, building on concepts from our GMM-DO filter. The smoother uses the DO equations for uncertainty prediction and the GMM-DO scheme for filtering. Smoothing is performed using a state augmentation procedure in which the past and the present states are first appended to form the prior distribution of a larger state vector. Observations are then assimilated by efficiently carrying out Bayes law in the reduced DO subspace of the augmented vector, using our GMM-DO filter. The smoothed distribution is then read off from the posterior of the augmented state vector. We implemented this new smoother and tested it using the example of a stochastic flow exiting a strait or an estuary. Results are illustrated next.

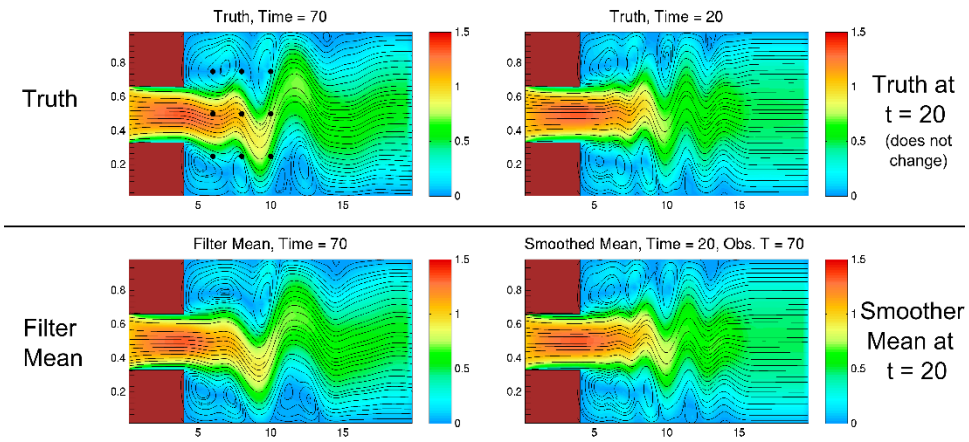
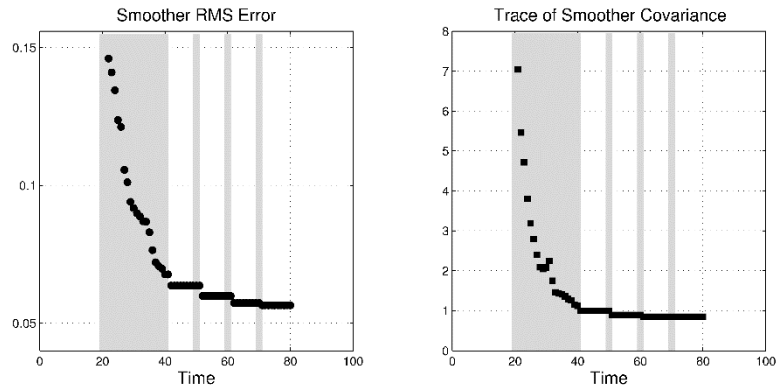


Figure 6: Flow exiting a strait or an estuary: **Top-left**: True flow field at time t = 70. Nine static sensors are arranged in the form of a 3x3 grid near the mouth of the constriction. **Top-right**: True solution at time t=20. **Bottom-left**: GMM-DO filter mean estimate at time t=70. **Bottom-right**: GMM-DO smoother mean estimate at time t=20. All panels depict the flow streamlines overlaid on a color plot of the flow magnitude

Figure 7: Left: RMS error (black markers) in the GMM-DO smoother mean estimate at time t=20. The first black dot indicates the RMS error of the filter (i.e. assimilating observations made only at t=20). Assimilating subsequent measurements (indicated by gray bars) reduces the RMS error. **Right:** Trace of smoother covariance matrix at time t=20. Reducing values of the trace indicates greater confidence in smoother estimates, as more data is gathered. Ideally, this curve (right) should resemble the observed RMS error (left)



The top-left panel of Fig. 6 shows the true solution, a noisy observation of which is recorded by the nine static sensors marked in black and arranged in the form of a 3x3 grid. Observations are made at all integer times between t=20 and t=40, and at times 50, 60 and 70. The top-right panel shows a snapshot of the true solution at time t=20. The bottom panels shows the posterior mean of the flow-field, computed by our GMM-DO smoother. The bottom-left panel shows the mean of the posterior flow-field at time t=70, while the bottom-right one shows the mean smoothed flow at time t=20, both determined after assimilating all observations until t=70. The qualitative performance of the smoother can be judged by comparing the similarity between the lower panels and their true solution counterparts in the upper panels. We see that the smoother estimates resemble the true solutions very well, indicating that the smoother is a powerful estimator of the true flow. Figure 7 provides quantitative metrics to analyze the smoother performance. The left panel of Fig. 7 shows the RMS error between the smoother mean and the true solution at time t=20, as observations are assimilated (observation times shown in gray). It is clear that as more observations are made, the error in the smoother estimate reduces, providing a significant improvement over the filtered estimate. Moreover, as depicted in the right panel of Fig. 7, the trace of the smoother covariance matrix (that indicates the level of uncertainty in the posterior estimates) also diminishes as observations are made. These results suggest that the smoother estimates globally converge in a Bayesian sense to the true solution, as more observations are collected.

3. Significance of Results

The rapidly increasing usage of mobile units such as AUVs in naval exploration and sampling missions demands rigorous and efficient methodologies to minimize operational costs. Energy optimal path planning of underwater vehicles still remains an open problem and poses formidable challenges, arising mainly due to (*i*) the lack of a closed-form solution, (*ii*) the continuous nature of the problem, and (*iii*) the resultant infinite dimensional control space. Our DO-based stochastic level-set optimization approach allows us to efficiently search this control space, while retaining the core advantages of the underlying level set methodology for path planning (e.g. rigorous, efficient, direct collision avoidance). This novel technique, therefore, is a significant advancement over existing energy based path planning methods. Our research results in adaptive oceanic sampling and smoothing are also significant. Smoothing enables accurate learning of the central stochastic system by propagating information from observations both forward and backward through time. Additionally, it facilitates the computation of the information content of candidate observations, which may be used to derive optimal adaptive sampling strategies for the operating naval vehicles, thereby maximizing their utility.

4. Plans and upcoming events for next fiscal year

We first plan to validate the GMM-DO smoother by applying it to high dimensional linear problems and systems with reversible dynamics, in which cases true posterior distributions can be computed. We further plan to analyze the GMM-DO smoother by comparing its performance with other smoothers in the literature, both Gaussian and non-Gaussian. We then plan to implement a novel adaptive sampling scheme, using the GMM-DO smoother to compute the mutual information. We intend to develop theory and schemes on “adaptive sampling swarms” and “artificial intelligence for collaborative swarms”. We plan to account for uncertain stochastic ocean predictions in our planning schemes, both for single paths and for coordinated paths maintaining vehicle formations. We also plan to initiate research towards other optimality criteria such as dynamics-optimal and swarm-optimal. We plan to start integrating our novel smoothing, adaptive sampling and path planning to enable long-duration environmentally-adaptive autonomous rigorous naval systems. We plan to continue to transfer the methods and algorithms to NRL. We expect to continue to apply our work to four-dimensional realistic ocean fields and/or participate to sea exercises, aiming to couple ocean-acoustic predictions, uncertainty prediction, autonomous strategies for learning and swarming, with all feedbacks. We will continue to report our findings and enable knowledge transfer through publications and participation in technical conferences.

5. Recommended reading

Lermusiaux P.F.J, T. Lolla, P.J. Haley. Jr., K. Yigit, M.P. Ueckermann, T. Sondergaard and W.G. Leslie, 2014. *Science of Autonomy: Time-Optimal Path Planning and Adaptive Sampling for Swarms of Ocean Vehicles*. Chapter 11, Springer Handbook of Ocean Engineering: Autonomous Ocean Vehicles, Subsystems and Control, Tom Curtin (Ed.), In press.

Lolla, T., Lermusiaux, P. F. J., Ueckermann, M. P. and Haley Jr, P. J. (2014a). *Time-optimal path planning in dynamic flows using level set equations: theory and schemes*. Ocean Dynamics, 64(10), 1373-1397. DOI: 10.1007/s10236-014-0757-y

Lolla, T., Haley Jr, P. J. and Lermusiaux, P. F. J. (2014b). *Time-optimal path planning in dynamic flows using level set equations: realistic applications*. Ocean Dynamics, 64(10), 1399-1417. DOI: 10.1007/s10236-014-0760-3

Subramani, D.N., Lolla, T., Haley, Jr, P.J., Lermusiaux, P.F.J. (2014). *A Stochastic Optimization Method for Energy-based Path Planning.* The Dynamic Data-driven Environmental Systems Science Conference, Cambridge MA, In press.

Sondergaard, T. and P.F.J. Lermusiaux, 2013a. *Data Assimilation with Gaussian Mixture Models using the Dynamically Orthogonal Field Equations. Part I. Theory and Scheme*. Monthly Weather Review, 141, 6, 1737-1760, doi:10.1175/MWR-D-11-00295.1.

6. Transitions/Impact

We met with (and provided theory and software to) different NRL researchers. We transfer results to ONR-supported PIs. The aide of Rear Admiral Titley, Mrs. Jen Landry, LCDR USN, successfully completed her SM with our group in Aug, 2014. We continue to work with Steve Rutherford (OPNAV N2/N6E) and NR-Stennis for transition possibilities. We maintain a software web-page for the distribution of our results. MIT undergraduates are involved in this research. They are sponsored by MIT's Undergraduate Research Opportunities Program (UROP). Undergraduates completed research and their senior thesis with us on the science of autonomy. Material from this project is used in MIT courses. Companies (e.g. air transports, shipping) and research labs (e.g. MIT Lincoln Lab) contact us for our methods, software and ongoing collaborations.

7. Collaborations

We collaborate with several ONR-supported PIs and had meetings with other PIs in the Science of Autonomy program. Collaborations occurred with our related ONR project "Stochastic Forcing for Ocean Uncertainty Prediction" (N00014-12-1-0944) and Naval Research Laboratory – Stennis project (N00173-13-2-C009). Visitors from the NATO CMRE research center and Pisa/Bologna Universities were also given methods and software.

8. Personnel supported

Principal investigator: Dr. Pierre F.J. Lermusiaux

Graduate Students: Tapovan Lolla, Deepak Subramani

Research staff: Dr. Patrick Haley Jr.

Undergraduate Students: Ben Hessels, Quantum Wei (both for free to this grant and ONR)

List of any students previously supported by the program who have taken positions performing DoD relevant research and where they have gone

Jen Landry - LCDR U.S. Navy

9. Publications

Publications resulting from this project (some of these publications started as part of N00014-09-1-0676):

Journal Articles

Lolla, T., Lermusiaux, P. F. J., Ueckermann, M. P. and Haley Jr, P. J. (2014a). *Time-optimal path planning in dynamic flows using level set equations: theory and schemes*. Ocean Dynamics, 64(10), 1373-1397. DOI: 10.1007/s10236-014-0757-y

Lolla, T., Haley Jr, P. J. and Lermusiaux, P. F. J. (2014b). *Time-optimal path planning in dynamic flows using level set equations: realistic applications*. Ocean Dynamics, 64(10), 1399-1417. DOI: 10.1007/s10236-014-0760-3

Lermusiaux P.F.J, T. Lolla, P.J. Haley. Jr., K. Yigit, M.P. Ueckermann, T. Sondergaard and W.G. Leslie, 2014. *Science of Autonomy: Time-Optimal Path Planning and Adaptive Sampling for Swarms of Ocean Vehicles.* Chapter 11, Springer Handbook of Ocean Engineering: Autonomous Ocean Vehicles, Subsystems and Control, Tom Curtin (Ed.). In press.

Haley, P.J., Jr., A. Agarwal, P.F.J. Lermusiaux, 2014. Optimizing Velocities and Transports for Complex Coastal Regions and Archipelagos. Ocean Modeling, sub-judice.

Lolla, T. and Lermusiaux, P.F.J (2014). *Time-optimal path planning in strong dynamic flows.* Submitted to SIAM Journal of Control and Optimization.

Lolla, T., Haley Jr., P. J., and Lermusiaux, P. F. J (2014c). *Path planning in multiscale ocean flows: coordination, pattern formation and dynamic obstacles.* Submitted to Ocean Modeling.

Conference Papers

Subramani, D.N., Lolla, T., Haley, P.J. Jr. and Lermusiaux, P.F.J., 2014. *A Stochastic Optimization Method for Energy-based Path Planning*. 2014 Dynamic Data-driven Environmental Systems Science (DyDESS) Conference. In press.

Other Publications

Subramani, D.N.: Energy Optimal Path Planning Using Stochastic Dynamically Orthogonal Level Set Equations. Master's thesis, School of Engineering, Massachusetts Institute of Technology (September 2014)

Hessels, B.D. Time-optimal Path Planning for Sea-surface Vehicles under the Effects of Strong Currents and Winds. BS in Mechanical Engineering. Massachusetts Institute of Technology (May 2014).

Cumulative List of Journal Articles

- N/A

10. Point of Contact in Navy

Jen Landry - LCDR U.S. Navy, 08/26/2014

Ruth Preller (NRL Stennis), 09/15/2014
Gregg Jacobs (NRL Stennis), 07/01/2014
Charlie Barron (NRL Stennis), 09/20/2014
Ira Schwartz (NRL - DC), 08/13/2014
Steve Rutherford (OPNAV N2/N6E), 11/01/2012

11. Acknowledgement/Disclaimer

This work was sponsored by the Office of Naval Research, ONR, under grant/contract number N00014-14-1-0476. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Office of Naval Research, or the U.S. government.

Section II: Project Metrics

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Progress Report for Period: May 1, 2014 – September 30, 2014

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5-207B; 77 Mass. Avenue; Cambridge, MA 02139-4307

Date Prepared: 09/25/2014

12. Metrics

[Please include each of the following metrics. If none, please indicate N/A.]

Number of faculty supported under this project during this reporting period: 1 (0.6 month)

Number of post-doctoral researchers supported under this project during this period: 0

Number of graduate students supported under this project during this reporting period: 2

Number of undergraduate students supported under this project during this period: 0 (2 for free)

Number of refereed publications during this reporting period for which at least 1/3 of the work was done under this effort: 3

Number of publications (all) during this reporting period: 9

Number of patents during this reporting period: 0

Number of M.S. students graduated during this reporting period: 1

Number of Ph.D. students graduated during this reporting period: 0

Awards received during this reporting period:

- Tapovan Lolla, Wunsch Foundation Silent and Hoist Crane Award for Excellence in Research, MIT, 05/15/2014 (student full time on this proposal)

13. 1-2 paragraph summary of all accomplishments for the entire grant

Several refereed publications on our time-optimal path planning were published or completed and submitted. We developed a novel stochastic optimization methodology based on Dynamically Orthogonal

(DO) level set PDEs for identifying energy-optimal paths among all time-optimal paths in complex, time varying flow fields. The paths so planned utilize less energy by intelligently making use of favorable spatiotemporal currents while avoiding adverse currents. We demonstrated the methodology first for a simple, yet important steady flow field and validated the solution through a semi-analytical classic optimization solution. Then, we applied our methodology to plan paths in a wind driven double gyre flow, which is an idealized ocean simulation for the Gulf Stream in the Atlantic Ocean, and Kuroshio in the Pacific Ocean. Finally, we demonstrated that the method could be successfully applied to real ocean flows by planning paths for completing a mission in the Hudson Canyon/ Middle Atlantic Bight region.

We also generalized our level-set methodology for time-optimal path planning to the case of vehicles with anisotropic motion constraints. Without increasing the computational complexity, our algorithm and codes can predict the fastest paths of vehicles such as sailboats whose speed depends on the direction and magnitude of the wind that drives them. This also applies to more detailed models of underwater robots and propelled surface-crafts which account for direction-dependent form drags and drags due to surface and internal waves.

We developed the GMM-DO smoother, an optimal non-Gaussian smoothing scheme that respects nonlinear dynamics and retains non-Gaussian statistics of the system state. It is applicable to high-dimensional stochastic systems such as ocean flows and enables accurate inference of the system state, both backward and forward through time. It allows one to assess the impact of candidate future observations on past states through the metric of mutual information. It also facilitates optimal adaptive sampling, allowing the efficient use of observational platforms by deploying them at locations and at times that provide maximum information about the fields of interest. The GMM-DO smoother uses the DO equations for uncertainty prediction and the GMM-DO scheme for filtering. Smoothing is performed using a state augmentation procedure in which the past and the present states are first appended to form the prior distribution of a larger state vector. Observations are then assimilated by efficiently carrying out Bayes rule in the reduced DO subspace of the augmented vector, using our GMM-DO filter. The smoothed distribution is read off from the posterior distribution of the augmented state vector. This smoother was successfully implemented and tested for the case of stochastic flow exiting a strait or an estuary, achieving global convergence to the true solution in the limit of sufficient observations.

14. A list of which items on the SOW will be worked on during FY14 (Oct 2014 to Sept 30 2015). Please give this to me as narrative text and not just as a list of numbers from your proposal. Please divide by base and potential option if you have both.

We first plan to validate the GMM-DO smoother by applying it to high dimensional linear problems and systems with reversible dynamics, in which cases true posterior distributions can be computed. We further plan to analyze the GMM-DO smoother by comparing its performance with other smoothers in the literature, both Gaussian and non-Gaussian. We then plan to implement a novel adaptive sampling scheme, using the GMM-DO smoother to compute the mutual information. We intend to develop theory and schemes on “adaptive sampling swarms” and “artificial intelligence for collaborative swarms”. We plan to account for uncertain stochastic ocean predictions in our planning schemes, both for single paths and for coordinated paths maintaining vehicle formations. We also plan to initiate research towards other optimality criteria such as dynamics-optimal and swarm-optimal. We plan to start integrating our novel

smoothing, adaptive sampling and path planning to enable long-duration environmentally-adaptive autonomous rigorous naval systems. We plan to continue to transfer the methods and algorithms to NRL. We expect to continue to apply our work to four-dimensional realistic ocean fields and/or participate to sea exercises, aiming to couple ocean-acoustic predictions, uncertainty prediction, autonomous strategies for learning and swarming, with all feedbacks. We will continue to report our findings and enable knowledge transfer through publications and participation in technical conferences.

15. If you are in your final year, will you require a no-cost extension to your period of performance? If so, until when?

- N/A

16. 1 summary PowerPoint slide of your entire project in any format. This should be something I can use to brief your effort to an external audience at a professional society meeting or to explain the significance of your work to my management in a few minutes when I am overviewing my entire program.

See attached