# Dynamics of Nearshore Infragravity Waves

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## LONG-TERM GOALS

The proposed work will investigate the dynamics of directional infragravity waves (IGW) in the nearshore. The long-term goal of the project is to formulate the theoretical framework for directional IGW propagation, and revisit existing field observations (e.g., Duck '97 experiment, e.g., Sheremet et al. 2002; Sheremet et al. 2005) from the perspective of IGW directionality. The work will benefit our understanding of wave current-sediment interaction in the nearshore and improve numerical forecasting models of nearshore hydrodynamics.

## OBJECTIVES

Because the scope of the problem of IGW dynamics involves theoretical and experimental work well beyond the reach of a 2-year project, the short term objective of the proposed work is to establish a theoretical framework, identify goals that are reachable in the short term, and formulate strategies for long-term goals. Main short-term objectives are:

(1) Study the dynamics of the three IGW types: leaky, edge and trapped Riemann waves. The first step will be to investigate the nonlinear interaction mechanism mechanism within the edge-wave family.

(2) Development a unified nonlinear shoaling model that incorporates the edge-wave dynamics into the general description of the IG generation processes.

## APPROACH

The proposed work will focus on establishing the theoretical background for understanding the coupled dynamics of the three IGW types. The short term objective is to identify goals that are reachable in the short term, and formulate strategies for long-term goals. The work will focus on formulating a comprehensive description of the generation, propagation and dissipation mechanisms for the three wave types, considered separately.

**IGW generation and nonlinear interaction.** The IGW excitation process that occurs in the shoaling zone, where energy is transferred through nonlinear interactions form swell to IGW components. Active interaction processes include IGW self- and swell-interaction. The analysis is complicated by the existence of edge waves, (Guza and Davis, 1974; Guza and Bowen, 1976) that involves the interaction between the continuum of swell components and the discrete edge-wave spectrum. A preliminary analysis based on the shallow water equations forced by swell-induced radiation stresses was carried out by Henderson and Bowen (2002) using Green's function for the linear problem. The next step will be to study the effects of near resonant the triad interaction mechanism in the IGW spectrum, in the context of a given directional shoaling swell. The nonlinear mild slope equation (NMSE) model developed by (Agnon et al., 1993; Agnon and Sheremet, 1997) will be used to investigate this problem.

**Dissipation mechanisms.** Dissipation mechanisms are essential to understand the IG energy balance. Several basic dissipation processes and any combinations of these are a priori possible:

1) Bottom friction, including surf turbulence, e.g., Feddersen et al. (1998; 2003), Feddersen (2012), and others. Bottom friction and Riemann-type breaking are the best understood and their integration into a theoretical framework is straightforward. Breaking appears to play a role in laboratory experiments but it is unlikely to be significant in the field due to strong directional spread.
2) Reverse nonlinear shoaling was investigated by Henderson et al. (2006)) based on the Duck '97 observations. This is a reverse generation mechanism, which is already included in theoretical and numerical simulations.
4) Percolation is an intriguing process that is not well understood (Longuet-Higgins, 1983; Packwood and Peregrine, 1980). Percolation through the porous sandy bed should play a role similar to bottom friction in the shoaling and surf zones, but the effect of possible IGW-induced oscillations of the water table at the shoreline, have not been considered.

**Numerical modeling and data analysis.** The spectral-domain modeling effort will be based on the nonlinear mild slope model for triad interactions in water of finite depth developed by Agnon et al. (1993); Agnon and Sheremet (1997), in both phase-resolving and phase-averaged forms. The unidirectional form of the model has been tested successfully for laboratory data (Sheremet et al., 2011). A directional version of the model is under development and will become operational as part of a separate effort funded by NOPP. The model is particularly well suite for the study of IGW generation. It will be expanded to a wave-action model to include mean water level (wave set-down/up) and Stokes drift currents, as well as nearshore currents with mild vorticity (e.g., McKee, 1974).

## WORK COMPLETED

**Edge-wave dynamics.** A theoretical model for edge-edge wave interaction was developed based on Zakharov's (1999) Hamiltonian formulation for a cylindrical (straight-isobath) beach under the assumption of shallow water. The governing equations for irrotational water waves can be written as the Laplace equation

$$\Phi_{xx} + \Phi_{yy} + \Phi_{zz} = 0 \text{ in } -h \leq z \leq \eta,$$

with the bottom boundary condition

$$\Phi_z = -h_{xx}\Phi_{xx} - h_{yy}\Phi_{yy}, \text{ on } z = -h,$$

and the surface boundary condition given in the Hamiltonian form as

$$\eta_t = \mathrm{H}_\phi, \quad \phi = -\mathrm{H}_\eta, \text{ on } z = \eta$$

where

$$\mathcal{H} = \iint H\, dxdy, \quad H = T + V,$$
$$T = \frac{1}{2}\int_{-h}^{\eta} \left[(\Phi_x)^2 + (\Phi_y)^2 + (\Phi_z)^2\right] dz, \quad V = \frac{1}{2} g\eta^2,$$

with H the hamiltonian, *H* the hamiltonian density, and *T* and *V* the kinetic and potential energy, respectively. Equations of both dynamical (phase-resolving) and a kinetic (phase averaged) type were derived. The evolution of the IGW field is represented as the time-evolution of shoreline amplitudes of a generalized Fourier-Sturm-Liouville orthogonal basis of function (harmonic time-alongshore representation; Sturm-Liouville cross-shore eigenfunctions). In the Fourier frequency-alongshore wave-number space $(\omega, \kappa)$, the free surface elevation is represented as a superposition of modes

$$\eta = \sum_{\omega,\kappa} \eta_{\omega,\kappa}(x) = \sum_{\omega,\kappa} A_{\omega,\kappa} F_{\omega,\kappa}(x) \exp i(\kappa y - \omega t),$$

where $F(x)$ is the cross-shore structure of the mode. For edge-waves, the near-shore localization and the constraint of low frequency (necessary for strong shoreline reflection) allow one to use the shallow water approximation, which leads, for $F(x)$, to the classical problem

$$(hF_x)_x + \left(\frac{\omega^2}{g} - h\kappa^2\right) F = 0,$$

with boundary conditions at the shoreline and infinity. For an edge-wave structure, for which a forbidden domain $x > x_0$ exists such that $\omega^2 - gh(x)\kappa^2 < 0$, the boundary conditions are

$$F = 1 \quad \text{at } x = 0,$$
$$F \to 0 \text{ as } x \to \infty.$$

The problem given by the equation for *F* with the above boundary conditions is an eigenvalue problem of the Sturm-Liouville kind, where one can think of the along-shore wave numbers as known parameters, while the frequency $\omega(\kappa)$ is an eigenvalue. The solutions $\mathrm{F}_{\omega,\kappa}(x)$ form a discrete orthogonal basis of function for the cross-shore structure functions. Thus, a complete Fourier-Sturm-Liouville decomposition is given by

$$\eta = \sum_{n,\kappa} \eta_{n,\kappa}(x) = \sum_{n,\kappa} A_{n,\kappa} F_{n,\kappa}(x) \exp i(\kappa y - \omega_n(\kappa) t),$$

where $n$ is the index of the eigenvalue mode $\omega_n(\kappa)$. Substituting this expansion into the governing equations, the hamiltonian becomes

$$\mathcal{H} = \mathcal{H}^{(EE)} + \mathcal{H}^{(EEE)}$$

where $\mathcal{H}^{(EE)}$ and $\mathcal{H}^{(EEE)}$ are the linear, and nonlinear quadratic parts, respectively, and the superscript $E$ denotes the edge wave component. The evolution equation can be obtained in the standard Zakharov variables as

$$\frac{da^E_{j,n}}{dt_1} = \sum_{m,l} \int_{-\infty}^{\infty} V^{EE}_{j,p,q,n,m,l} a^E_{p,l} a^E_{q,m} \delta^{\kappa}_{j,p,q} \delta^{\omega}_{n,l,m} d\kappa_{p,q},$$

where the deltas represent selection criteria for interaction.

The equations describe the nonlinear evolution of the shoreline amplitude of infragravity edge-wave due to resonant triad edge-edge wave interactions. The resonance manifold is identified as the zeros of the function

$$\mathrm{G}_{l,m,n}\left(\kappa_p, \kappa_q\right) = \omega_l\left(|\kappa_p|\right) + \omega_m\left(|\kappa_p|\right) - \omega_n\left(|\kappa_p + \kappa_q|\right),$$

e.g., Figure 1. An example of numerical simulations showing the nonlinear interactions between three edge wave modes is shown in Figure 2.

**Directional shoaling: TRIADS, a hyperbolic phase-resolving model.** A directional spectral model (Agnon and Sheremet 1997) was developed and tested. The model implementation includes: incorporation of offshore wave spectra; remapping of frequency-directional spectra into frequency-alongshore wave number grids; and snapshots of the free surface elevation. At present the model is being tested with Duck AWAC (Acoustic Wave and Current) data. Coupling with the WAVEWATCH-III model is ongoing, and comparisons with nonlinear parabolic models and phase-averaged nearshore models are being planned.

**Directional shoaling: A generalized swell-edge wave model based on the Fourier-Sturm-Liouville representation.**

While both approaches described above do include various aspects of edge-wave dynamics, the models resulting from these are incomplete. The edge-edge interaction model does not include explicitly the interaction with the leaky waves, which constitute the main source of energy for edge waves. The TRIADS model is hyperbolic, which means that it can only provide an estimate of the energy transferred to the edge-wave band, but cannot describe the evolution of the band itself.

A generalized model based on the Fourier-Sturm-Liouville representation can be derived, if one replaces for the leaky wave system the shallow-water approximation used to model shore-localized edge waves, with the mild-slope approximation (e.g., Agnon and Sheremet, 1997),

$$\nabla \cdot (cc_g) + \omega^2 \frac{c_g}{c} F = 0,$$

with

$$F < \infty \text{ as } x \to \infty,$$

reflecting the fact that leaky waves are finite far from the shoreline. The system will accept waves propagating in both directions along the cross-shore axis. A shoreline condition can be specified in therms of reflection coefficients, for example, zero reflection (breaking) for swell and seas, and total reflection for leaky infragravity waves. The resulting leaky and edge-mode cross-shore structures are also orthogonal, thus providing a general decomposition basis. The algebra, albeit tedious, is similar to that used for the edge-wave interaction. The hamiltonian becomes

$$\mathcal{H} = \mathcal{H}^{(EE)} + \mathcal{H}^{(EEE)} + \mathcal{H}^{(LL)} + \mathcal{H}^{(EEL)} + \mathcal{H}^{(ELL)} + \mathcal{H}^{(LLL)},$$

where $\mathcal{H}^{(LL)}$ and $\mathcal{H}^{(LLL)}$ are the linear, and nonlinear quadratic parts corresponding leaky modes (superscript *L*), and various combinations of *E* and *L* denote different interaction patterns. For example, the subharmonic mechanism identified by Guza and Davis (1974) is contained in the $\mathcal{H}^{(EEL)}$ compponent of the hamiltonian. Zakharov-type of equations can be then derived for both leaky- and edge-wave components.

## RESULTS

Both dynamical and kinetic models for nonlinear edge-wave evolution have been fully implemented numerically The equations agree with previous derivations by Kirby et al. (1998) and Kurkin and Pelinovsky (2002). The resonant manifolds have been identified. An example of the process for a constant-slope, plane beach is given in Figures 1. For realistic bathymetries the resonance manifolds have to be calculated numerically. Numerical simulation with single triads (Figure 2) have been verified against analytical solutions (e.g., the Jacobi elliptic function solution for a single triad, Swaters, 1988; Craick 1988).

The generalized swell-edge wave model has been developed and tested on theoretical cases. For example, it was shown to include the subharmonic resonance mechanism that transfers energy from one swell component to two edge wave components of half the frequency and propagating in opposite directions. Other edge-wave excitation mechanisms are identified and discussed in Tian et al (in preparation). The edge-swell model is in the process of being generalized to circular bathymetry (Discenza et al., in preparation).

The TRIADS model has been tested field observations collected at FRF Duck NC (Sheremet et al., in review), and during the wave-mud MURI 2008 experiment on the Atchafalaya shelf (Safak et al., in review). Figure 3 shows an example of numerical simulations (Davis et al. 2013) of the evolution of a directional spectrum based on the nonlinear mild slope equation (Agnon and Sheremet, 1997), highlighting the generation of infragravity waves (Figure 4c) as well as nonlinear effects in swell propagation (e.g., the development of asymmetry, the steepening of the frontal wave slope).

## IMPACT/APPLICATIONS

The present research aim at extending the predictive capability of the Navy's wave forecasts in the nearshore to account for the IGW processes. The dynamics of IGW are important for the energy balance in the nearshore, with has significant consequences for inundation, erosion and nearshore sediment transport. Despite the amount of work dedicated to the study the coupled dynamics of nearshore waves, currents, and sediment transport, our understanding of these processes is still rudimentary. The current paradigm is unidirectionality: waves propagate perpendicular to the shoreline; currents are flow alongshore; sand moves up and down the beach profile. The Navy goal of developing reliable forecasting models for arbitrary nearshore weather conditions cannot be achieved if a significant portion of the physics is ignored or misrepresented.

## RELATED PROJECTS

None.

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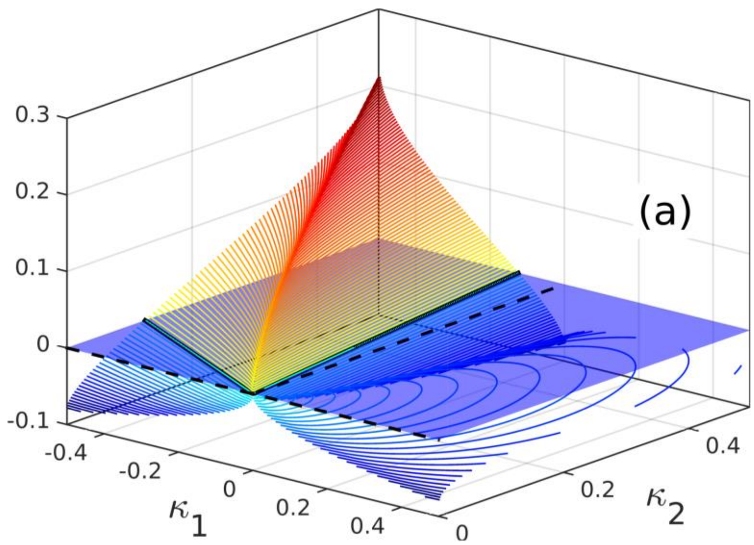


***Figure 1: An example of surfaces generating the resonance curves for edge-edge wave triad interactions for a plane-beach (constant slope beach profile). The resonance manifolds are the intersection of the such surfaces with the horizontal plane passing through the origin (gray). Represented are surfaces for triads involving Sturm-Liouville modes (n,m,l) = (0,0,0), (0,0,1), (0,1,1), and (1,1,1).***

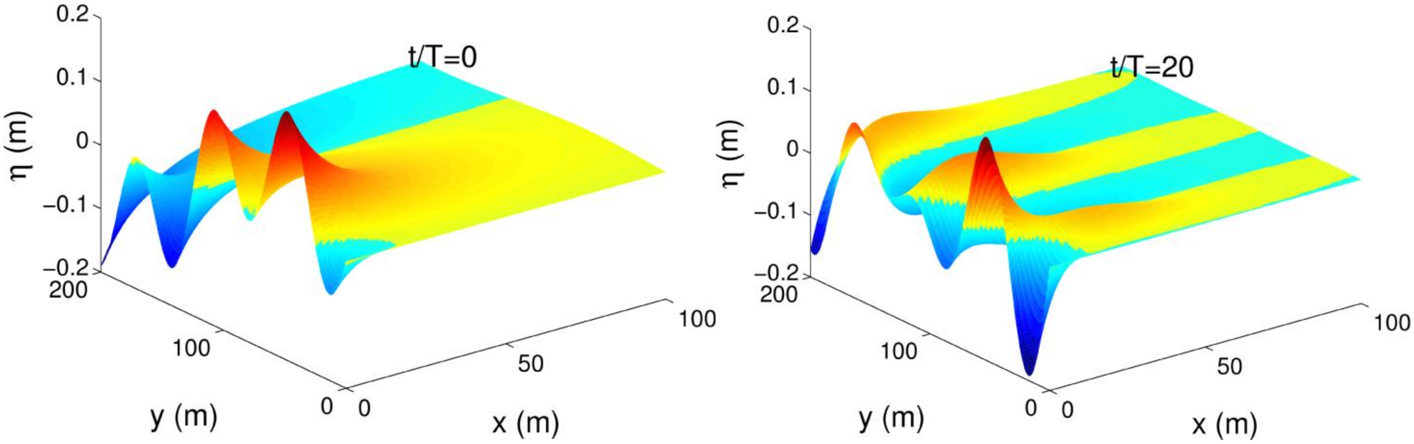


***Figure 2: Numerical simulation illustrating the generation of a mode-1 edge wave (one cross-shore zero) through the interaction of two mode-0 edge waves (no cross-shore zeros). Left: Initial free surface elevation, showing the superposition of the mode-0 edge waves. Right: Mode-1 edge wave becomes apparent.***

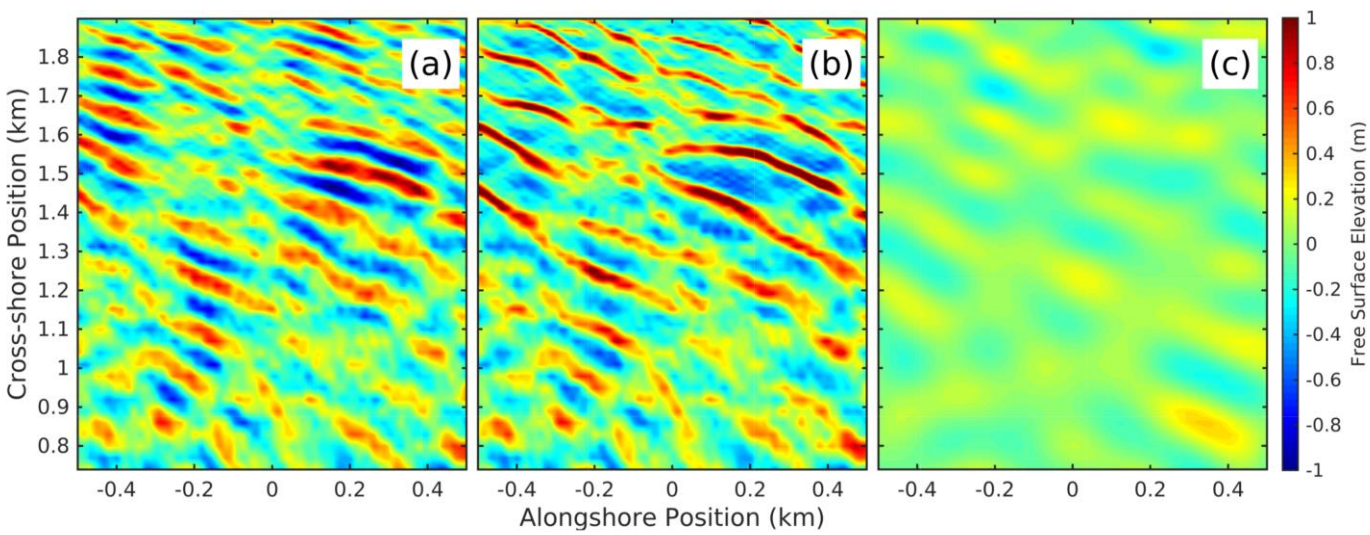


***Figure 3: Example of a realization of the free surface elevation generated by the phase-resolving nonlinear directional TRIADS model Sheremet et al. (2014), for waves observed at FRF Duck, NC during Hurricane Bill (2009). a) Linear model; b) Nonlinear model result. c) Infragravity wave field generated during shoaling. The nonlinear effects are also visible in panel b) in the development of wave asymmetry. The shore is at the top of the figure.***