Probabilistic Geoacoustic Inversion in Complex Environments

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Grant Number: N000141512019

LONG-TERM GOALS

Propagation and reverberation of acoustic fields in shallow water depend strongly on the spatial variability of seabed geoacoustic parameters, and lack of knowledge of seabed variability is often a limiting factor in acoustic modeling applications. However, direct sampling (e.g., coring) of vertical and lateral variability is expensive and laborious, and long-range inversion methods can fail to provide sufficient resolution. For proper quantitative examination of variability, parameter uncertainty must be quantified first which can be particularly challenging for large data sets, and in range-dependent and/or dispersive seabed environments.

This project aims to advance probabilistic geoacoustic inversion methods for complex ocean environments for a range of geoacoustic data types. The work is based on previous contributions to Bayesian inversion[1–5] which allow inferences on complex environments (arbitrary and unknown layering) and advanced physical theories (acoustics of dispersive media and spherical reflection coefficients). The focus is on (1) improving automated parameterizations to capture seabed variability and uncertainties consistent with data resolution, and (2) improved numerical methods for parameter and uncertainty estimation for greater reliability and efficiency (computational cost is currently a limiting factor in uncertainty estimation). These improvements will expand our ability to apply inversion to problems of increasing complexity while reducing the subjective assumptions made in the quantitative inversion. Fundamental advancements in inversion/inference methods are required since science goals of the ocean acoustics community are increasingly focused on more complex environments and/or more complex wave propagation theories. The methods will be developed with simulated data and measured data (existing and new). In particular, data from the Ocean Acoustics Seabed Characterization Experiment will be considered once available.

OBJECTIVES

The objective of this research proposal is to carry out geoacoustic inversions with advanced sediment models for complex environments with spatial variability in geoacoustic parameters. In particular, highly-informative data which reduce/eliminate oceanographic effects are considered. The resulting geoacoustic models represent benchmarks for meso-scale variability and uncertainty estimation, and also allow the study of compressional- and shear-wave dispersion and attenuation-frequency dependence. Bayesian hierarchical models and trans-dimensional (trans-D) inversions allow for increasingly automated data analysis in challenging shallow-water environments. The problems are studied with existing data from Mediterranean Sea test beds. Studies will be extended to new data from upcoming shallow-water experiments, when available. Particular focus for new shallow-water data will be on rigorous variability and uncertainty estimation, attenuation-frequency dependence, sound-velocity dispersion, and shear-wave velocity structure of the seabed.

APPROACH

Meaningful inferences on seabed variability require estimation of geoacoustic parameter uncertainties. Rigorous uncertainty estimates can be based on probabilistic/Bayesian inference techniques, which are discussed here. Quantifying parameter uncertainty in geoacoustic inversion is achieved here with probabilistic sampling methods.[6] Probabilistic sampling requires assigning a model which is defined here to include the physical theory describing signal-system interaction, an appropriate parametrization, and a statistical representation of residual errors (the difference between predictions and observations). This model specification constitutes prior information that fundamentally impacts uncertainty estimates. The process of choosing a model parametrization that is consistent with the resolving power of the data is referred to as model selection for which the Bayesian framework provides a quantitative approach.

In particular, we focus on methods which relax subjective assumptions which are commonly required in inversion. One of the strongest assumptions in many studies is that of discretization for the geoacoustic parameters (e.g., number of seabed layers and/or range-dependent variation). Choosing a particular discretization can profoundly change the results of a study. Over the last two decades, more general parametrizations that abandon strong assumptions have emerged in the statistics literature and are coined trans-D models. A trans-D[7–9] formulation implicitly provides objective model-parametrization selection without practitioner decisions/interactions.[10,11] To date, we have developed several methods for trans-D inference[1,12–14] that benefit from cross-fertilization with developments in seismology.[2,15] Trans-D inversions treat the number of unknowns in the problem as unknown itself which is marginalized (integrated), resulting in a posterior probability density (PPD) that spans multiple subspaces of different dimensions. A key advantage of this formulation is that the posterior includes the uncertainty of the parameterization in the parameter uncertainty estimates.

In most acoustic/geophysical applications, a partition modeling approach is applied to parametrize the environment in a trans-D model (although other approaches exist[16]). For example, a 1D seabed model is given by the positions of interfaces and the geoacoustic parameters associated with the partition between the interfaces and the model extends to a maximum depth of interest. In addition, the number of interfaces is itself an unknown parameter. Inversion results are given by a large ensemble of parameter vectors with varying number of interfaces and inference is carried out via marginalization. Importantly, resolution in these models can adapt locally to the environmental variability and data information content such that complex structure is resolved where environmental variability and data information are high while simple structure is produced where little variability/information exists. This is of particular importance for models with two spatial dimensions, where parametrization is traditionally carried out via over-parametrization and global regularization (e.g., smallest or smoothest models). Trans-D models require no regularization but rather adapt locally to provide optimal model

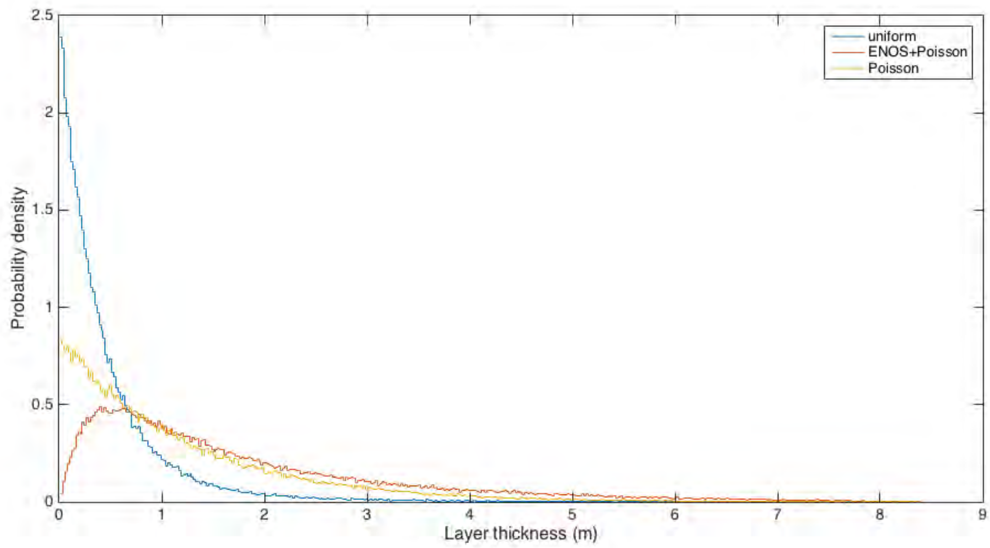


Figure 1: ***Marginal probability densities for layer thickness values for three choices of prior $p(k)$: uniform (blue), Poisson (red), ENOS and Poisson (yellow).***

complexity. Hence, no subjective regularization parameters or local breaks in the otherwise global regularization are required, resulting in less subjective inversion results.

WORK COMPLETED

We studied the utility of even-numbered order statistics (ENOS)[8] as a prior on layering complexity in geoacoustic inversion. Trans-D sampling is governed by the Metropolis-Hastings-Green probability which can be shown to lead to unbiased uncertainty estimates. The probability is given by

$$\alpha = \min\left[1, \frac{p(\mathbf{x}')}{p(\mathbf{x})}\left(\frac{L(\mathbf{x}')}{L(\mathbf{x})}\right)^{\beta}\frac{q(\mathbf{x}|\mathbf{x}')}{q(\mathbf{x}'|\mathbf{x})}|\mathbf{J}|\right], \quad (1)$$

where $\mathbf{x}$ is the parameter vector, p the prior, q the proposal, and L the likelihood function. From this equation, a Markov chain is simulated by proposing a new chain state $\mathbf{x}'$ based on the current state $\mathbf{x}$ and the proposal q. The proposed state is accepted or rejected based on α. In the following, we particularly focus on the prior ratio $p(\mathbf{x}')/p(\mathbf{x})$ and will use a prior that has previously not been considered for geoacoustic inversion but results in significant advantages for the inversion. For models where the number of seabed layers k is unknown, $\mathbf{x} = (k, \mathbf{m})$, and $p(\mathbf{x}) = p(k)p(\mathbf{m})$. Typically, $p(k)$ has been assumed to be uniform[1] under the premise that a uniform prior on k is to some degree uninformative.

While a uniform $p(k)$ treats all numbers of layers as equally probable (which may be desirable), it also has the effect of expressing strong preference for the presence of thin layers (not desirable). The effect is illustrated in Fig. 1 by carrying out an inversion without data (setting $L(\mathbf{x}')/L(\mathbf{x}) = 1$ in Eq. (1)) and then considering all layer thickness values that appear in the posterior (i.e. a normalized histogram of all layer thickness values in the inverse problem solution). Figure 1 clearly shows that a uniform $p(k)$ strongly favors thin layers. Note that the peak in the marginal is at zero thickness, such that the largest preference is given to models with layers that are most likely below the resolution power of the data. Hence, the uniform prior on k has the undesirable effect of strongly emphasizing models that the data are not sensitive to. This, in turn, increases uncertainty estimates for other parameters and decreases algorithm performance by requiring the sampling of models that have additional layers that the data are not sensitive to.

To avoid such strong preference, Green [8] applies the combination of a Poisson prior and ENOS to the coal-mine disaster data set. The Poisson distribution is given by p(k)

$$p(k) = \exp(-\lambda)\frac{\lambda^k}{k!}\,, \qquad (2)$$

where λ is a width parameter (chosen to be 6 in this work) and the distribution is conditioned on $k < k_{\mathrm{max}}$. The Poisson prior substantially reduces the preference for thin layers (Fig. 1) but still has a peak near zero. To avoid this and create a less informative prior, the Poisson distribution is combined with the ENOS to avoid very thin layers that are not supported by data (Fig. 1). The ENOS for k interfaces is given by $2k + 1$ uniformly distributed positions where the k even-numbered ones are selected. This statistic results in a reasonable suppressing of very thin layers (Fig. 1)

In addition, we acquired and installed a new high performance computer consisting of 160 CPU cores and 16 GPUs in August 2015 to support this project (Fig. 2). This hybrid computing infrastructure is ideally suited for the computationally intensive work in this project and is maintained and administrated by Dettmer. The computer is jointly funded by ONR (15%) and the Natural Sciences and Engineering Research Council (NSERC, 50%) of Canada and the strategic environmental research and development program (SERDP; DOD, EPA, DOE; 35%). Several of the inversion algorithms in this project have been developed to take full advantage of the massively parallel hybrid architecture of the cluster. In particular, the recently developed Levin integration algorithm for spherical reflection coefficient computation[5] was specifically developed for this computer. In addition, we still maintain the older ONR/NSERC funded HPC cluster.

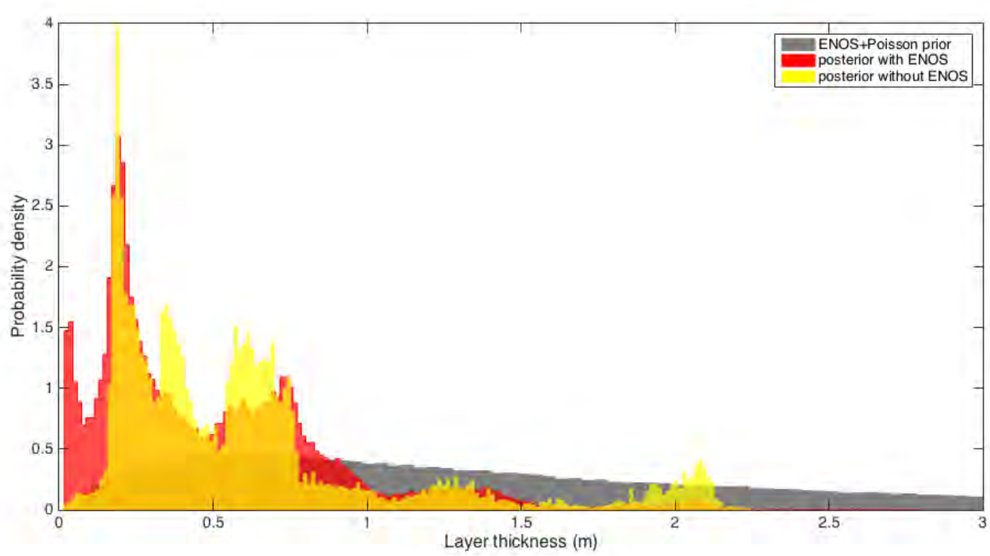
RESULTS

Results presented in this section focus on some of the research carried out this year to better specify prior information in spherical reflection coefficient inversion. All computations use the Levin integration approach and are carried out on hybrid CPU-GPU computers.

The new prior was applied to a reflection-coefficient data set off the coast of Elba (site 2). Figure 3 shows the ENOS+Poisson prior for layer thickness, and the posterior marginals for two inversions, one with ENOS and one without (both apply the Poisson prior). The figure illustrates that the ENOS significantly reduces the presence of thin, unresolved layers in the posterior.

Figure 4 shows a comparison of geoacoustic profile probabilities for inversions with and without the ENOS. The inclusion of the ENOS prior results in a simpler inversion result with reduced uncertainty and better resolved interfaces. In addition, parametrization complexity (number of layers) is substantially reduced and less uncertain in the ENOS case (Fig. 5). This reduction lead to $\sim$30% less computer time required for the inversion.

Figure** 2: **The newly installed hybrid computer combines CPUs and GPUs in a dense configuration and already has allowed us to consider complicated data sets that were previously difficult to address.



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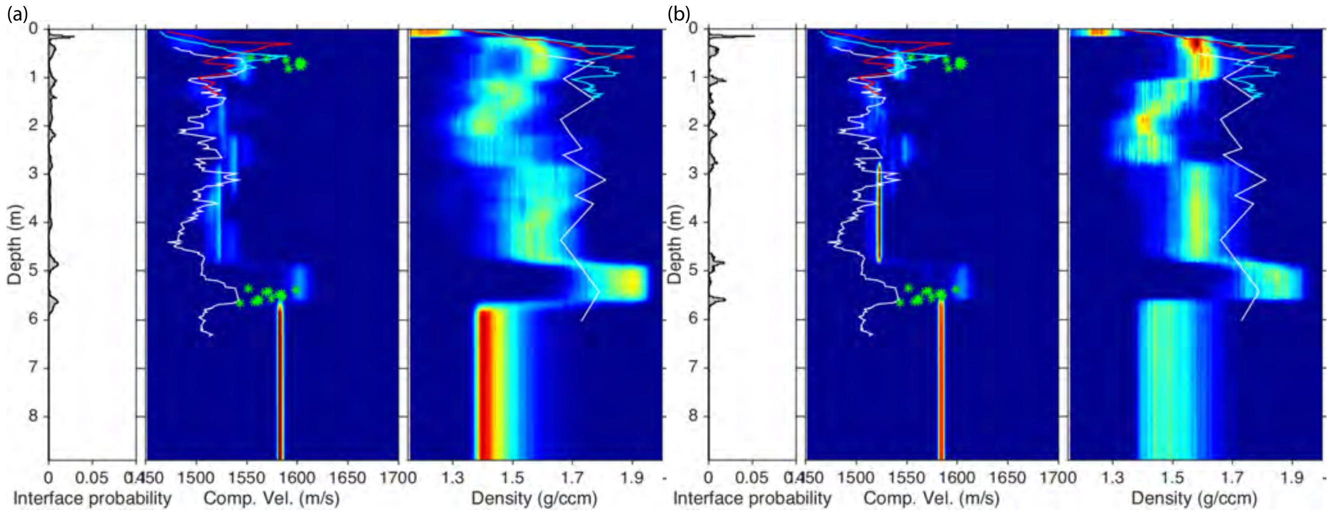

Figure 4: Comparison of geoacoustic profiles for sound velocity and density for two inversions of the same data (a) without and (b) with the ENOS prior. Several core measurements are also shown for comparison.

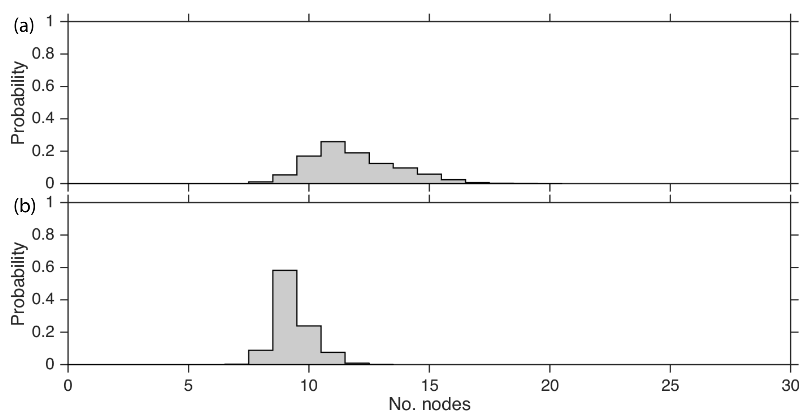

Figure 5: Comparison of parametrization complexity (No. layers) for two inversions of the same data (a) without and (b) with the ENOS prior.

IMPACT/APPLICATIONS

The ability to obtain *in-situ* seabed parameter estimates remotely (i.e., without direct sampling) has important geoscience implications (e.g., understanding sediment processes) and in some cases can be the only feasible way of obtaining such inferences (e.g., low-frequency dispersion). Further, variability estimates for seabed parameters are important for understanding the physics of acoustic-seabed interaction. Since variability can only be estimated if uncertainties are understood, uncertainty estimation is crucial. Important applications also include improved Navy databases (for ASW and MCM), as well as many commercial applications (pipeline or cable laying). A particular strength of this work is rigorous geoacoustic uncertainty estimation. These geoacoustic uncertainty models impact reliability and quality of transmission loss prediction.

RELATED PROJECTS

- Dettmer and Holland recently started a SERDP project (15 MR01-056) to quantify geoacoustic uncertainty in very shallow water (<20 m) to aid in understanding munitions burial, transport and detection.
- Broadband Clutter JRP project (NURC, ARL-PSU, DRDC-A, NRL)
- Dosso's NSERC Discovery Grant "Geoacoustic Inversion" (2009-2014) at the University of Victoria: Dosso, Quijano and Dettmer work closely together on advancing Bayesian inference applications.
- Dettmer's work at Australian National University in seismology is closely related to some of the methods developed in this program resulting in strong mutual benefits (e.g., [15, 17]).

PUBLICATIONS

Refereed Journal Articles

4. J. E. Quijano, S. E. Dosso, J. Dettmer, and C. W. Holland. Geoacoustic inversion of seabed spherical reflection data using a fast forward model. *J. Acoust. Soc. Am.*, 2015 [in press, refereed].

3. G. A. Warner, S. E. Dosso, J. Dettmer, and D. E. Hannay. Bowhead whale localization using asynchronous hydrophones in the Chukchi Sea. *J. Acoust. Soc. Am.*, 2015 [submitted, refereed].

2. S. E. Dosso, J. Dettmer, and M. J. Wilmut. Efficient localization and spectral estimation of an unknown number of ocean acoustic sources using a graphics processing unit. *J. Acoust. Soc. Am.*, 2015 [submitted, refereed].

1. J. Dettmer, S. E. Dosso, T. Bodin, J. Stipčević, and P. R. Cummins. Direct-seismogram inversion for receiver-side structure with uncertain source-time functions. *J. Acoust. Soc. Am.*, 2015 [in press, refereed].

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