

Development of a Two-Equation Turbulence Model for Mean Shear- and Internal Wave-Driven Mixing

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LONG-TERM GOALS

The long-term goal of the project is to develop realistic, functional parameterizations of stratified turbulent mixing usable in numerical circulation models of oceans and coastal seas. Specifically, we aim at developing models which explicitly allow for the coexistence and three-way interaction of three major components of stratified geophysical flows: "mean" currents, internal inertia gravity waves (IGW) and smallscale, 3D turbulence. Here, "mean" loosely refers to any current that - in contrast to IGWs and turbulence - can be explicitly resolved in a variety of operational numerical models. Traditional turbulence closures, which our approach extends, only acknowledge the existence of two of the three flow components, turbulence and sheared mean currents. They are thus ignorant of the direct energy flux from "breaking" IGWs to turbulence. As to its importance, we note that this direct energy flux from IGWs powers the turbulent mixing in the bulk of the depths of the world ocean.

OBJECTIVES

The objectives of the project are to (i) develop a two-equation, closure-like turbulence model with turbulent kinetic energy (TKE) production and turbulence length and time scales resulting from mean shear *and* IGWs simultaneously and (ii) to test the new algorithm through comparisons with observed flows. Our chosen problem is rather challenging. Within the current project we intend to provide a "proof of concept."

APPROACH

This project is a joint effort of the PI and of *Helmut Baumert* of the Institute for Advanced Marine and Limnic Studies (IAMARIS), Hamburg, Germany (baumert@iamaris.org). Dr. Baumert is being funded by ONR Global / NICOP (Grant Number: N62909-10-1-7050). We have been collaborating for over 10 years on the topic of stratified turbulent mixing, combining the skills in theory and modeling of Baumert with the observational experience of Peters.

The current project is based on a series of published studies of stratified mixing by Baumert and Peters (*Baumert and Peters*, 2000, 2004, 2009; *Peters and Baumert*, 2007). Over the course of this work it became increasingly clear that, to be realistic, turbulence models *cannot* ignore IGWs. This viewpoint

is explicitly supported by the path-breaking analysis of Lagrangian stratified flow spectra of *D'Asaro and Lien* (2000a, b).

The base from which this project starts is contained in our latest joint publication, *Baumert and Peters* (2009). It presents a two-equation model that contains the two limits of internal wave-driven mixing without mean shear and mean shear without internal waves. Our logical first step thus is to set up a physically sound two-equation system capable of handling mean shear and waves simultaneously. This requires theoretical development. At the same time, analyses of existing observations need to be examined and prepared for the second step in our approach, testing the new model.

WORK COMPLETED

This project just started last April/May. In order to get jump-started, PI Peters traveled to Hamburg and worked with Baumert on a work plan from 7/12 to 7/19/2010. Work on theory and observations as outlined above has begun.

RESULTS

Despite the recent start of the project we are happy to report significant new findings. Independent from ONR funding, Baumert has been working on a theoretical foundation for two-equation turbulence models based on vortex dynamics and vortex geometry. *Baumert* (2009) shows this approach to be fully consistent with the earlier k - Ω model of *Baumert and Peters* (2009). Von Kármán's constant is derived as $(2\pi)^{-1/2} \cong 0.399$. More recently, inspired by *Herrmann's* (1990) dynamic packing of circles, the theory could be extended to wavenumber spectra and Lagrangian frequency spectra. In Kolmogorov's $\alpha \varepsilon^{2/3} k^{-5/3}$ and $\beta \omega^{-2}$, the new theory predicts values of α and β consistent with experimental values including those from the oceanographic observations of *D'Asaro and Lien* (2002a,b) and *Lien and D'Asaro* (2000). Numerical simulations of the geometrical formulation show wavenumber exponents consistent with the familiar $-5/3$.

IMPACT/APPLICATIONS

Turbulence models that explicitly allow for the presence and effect of IGWs have the potential to make the representation of turbulent mixing in operational and research circulation models more realistic and thus to improve the model forecasts. For climate research and prediction this is essential.

RELATED PROJECTS

Numerous past and present projects either pursue traditional, wave-ignorant turbulence closure for stratified shear flows or internal wave-driven turbulent mixing based on observations and internal wave interaction theory. Links between mixing and waves in shear flows have been analyzed in the Equatorial Undercurrent [e.g., *Moum et al.* (1992), *Lien et al.* (1996)] and in Knight Inlet [*D'Asaro and Lien*, 2000a,b]. More recently, J. Nash, H. Peters and J. Pelegrí conducted [pilot observations on the role of internal waves in the outflow from the Mediterranean in the Gulf of Cádiz](#). In 2009 IAMARIS has sponsored an internal research project on fundamental questions of turbulence theory and modeling which has served as a preparatory phase of the present NICOP-supported engagement.

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