Capturing Uncertainty in the Common Tactical Environmental Picture

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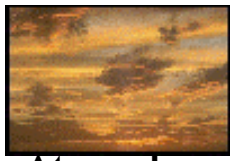
Improved Environments for M&S

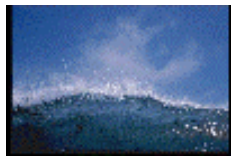
- Synthetic Natural Environments
  - Fleet Battle Experiment Hotel
  - Global 2001
- Critical Issues:
  - How Much is Enough?
  - Temporal and Spatial Scales (natural, req'd)
  - Uncertainty Propagation / Communicating it
  - For the Warfighter

Maritime Environment for FBE's

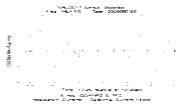
Assemble Archived Data Sources



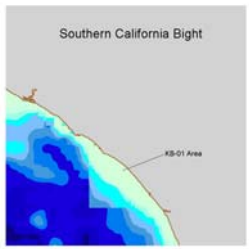
Atmosphere



Ocean



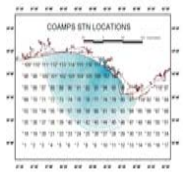
Wave Height



Bathymetry

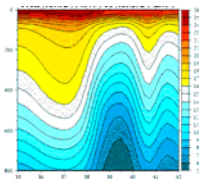
Initialize Model

Assimilate near Real time data



COAMPS Data

Buoy Data



MODAS and POM For water column

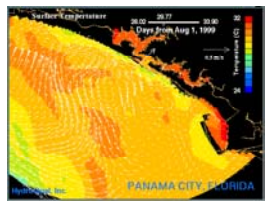
Daily Forecast/Nowcast Process

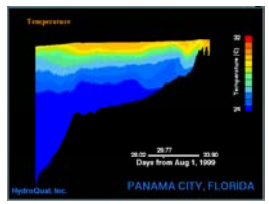
Creates Data for JSAF Simulation

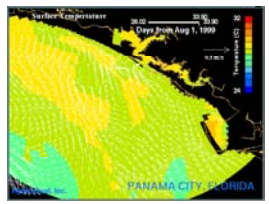
0000: Receive COAMPS Forecasts
Run ECOM for each Forecast
Compute 48 hours of ECOM data

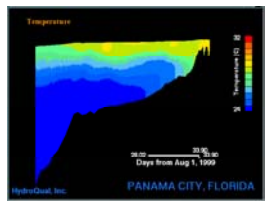
0600: SERVE Maritime Environment to FBE
Replace prior forecasts with new data

2330: Prepare to repeat process









Days/hours before FBE

Daily during FBE

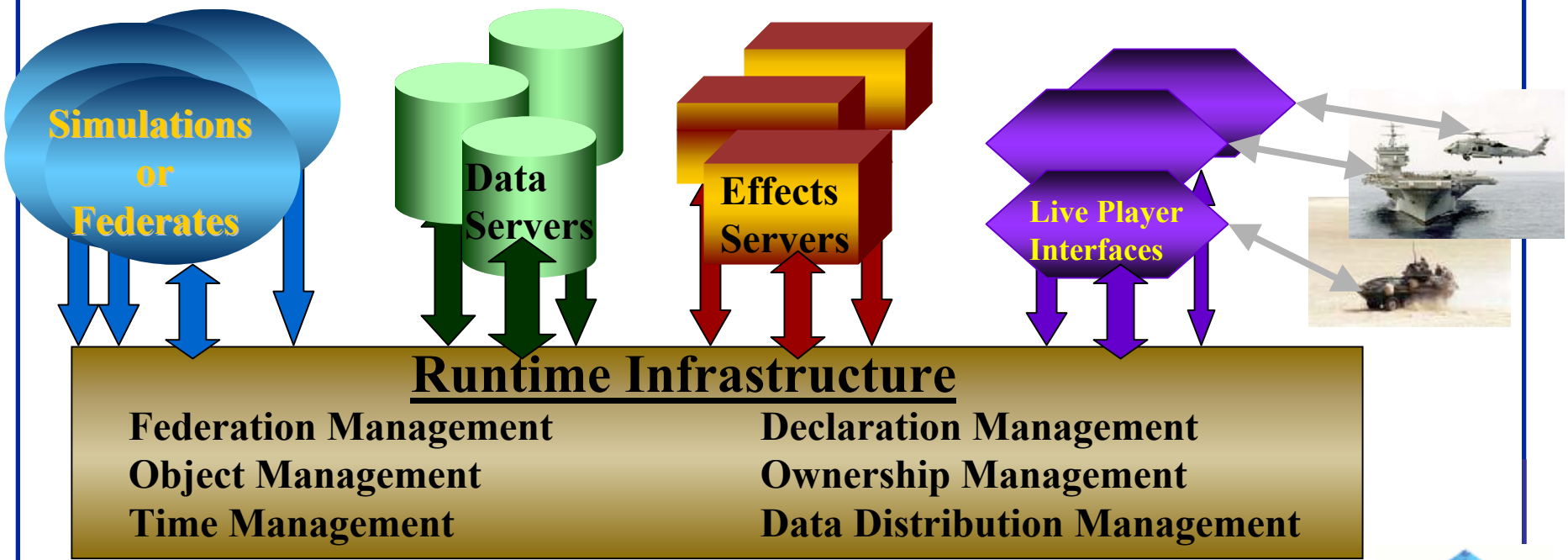
Sources: NAVO, NRL/SSC, MEL

Sources: NAVO, NRL/SSC, any

Required Work: Design Methodology, Modify Server, Enable Simulated Sensors

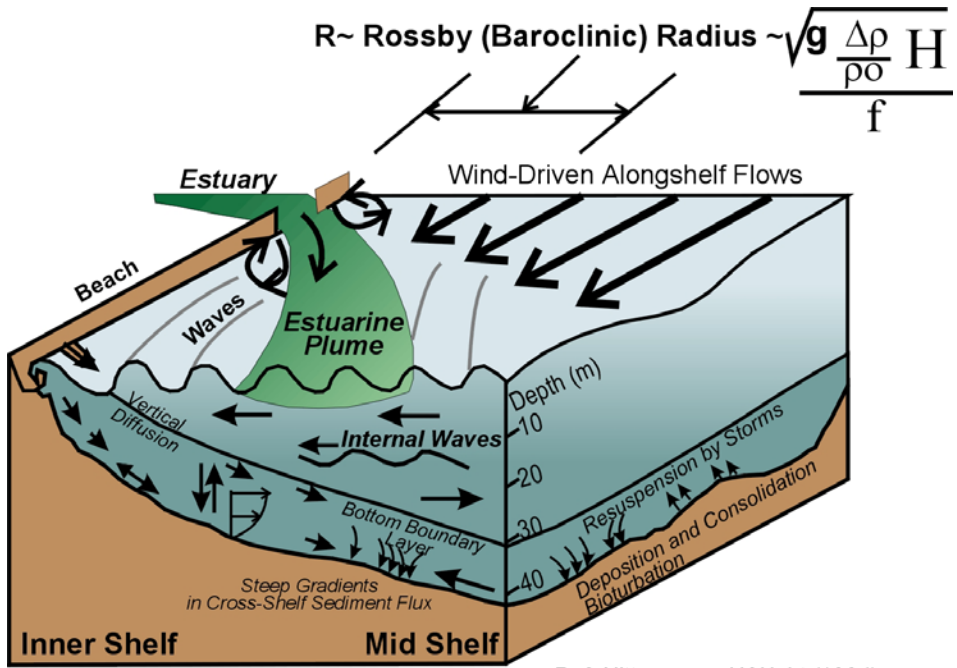
HLA - An Architectural Mandate

- Challenge: Exploit the High Level Architecture (HLA) to use environment server and develop effects server as federates



What Must Models Include?

- A Problem of Time: Refresh rates.
  - Littoral Ocean Regions are characterized by long and short term factors, but unlike the deep water, short term effects are more significant
- A Problem of Space
  - Rapid changes in bottom slopes create the need for non-uniform grids



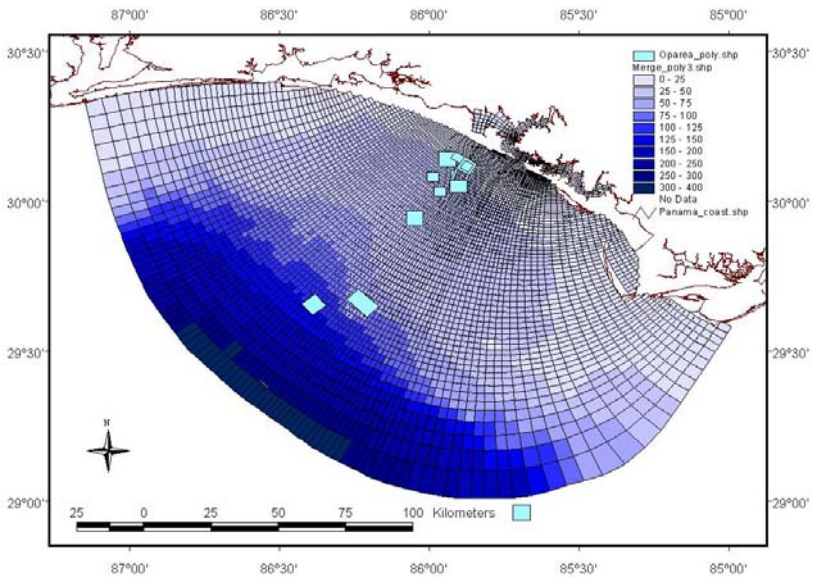
Ref: Nittrouer and Wright (1994).

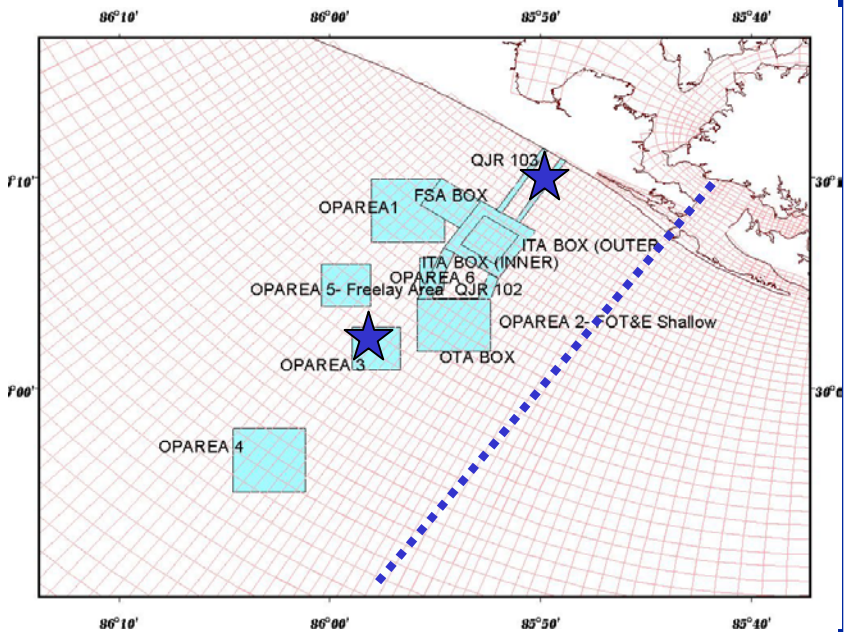
Minehunting

- Minehunting is going organic
- Experiments and Training must reflect
  - PC-SWAT (Sammelmann) to model sonar
- System Performance Evaluation

Model Results

Littoral Dynamics

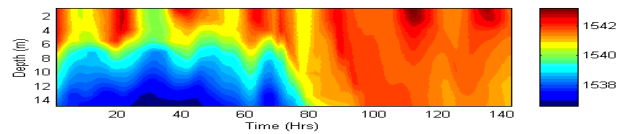


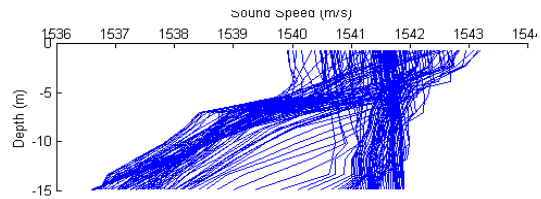


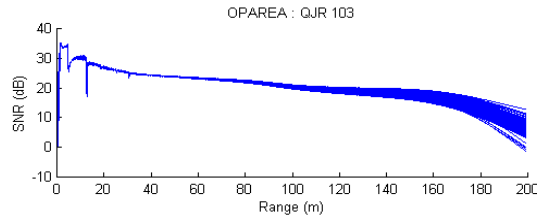
Tactical Impact

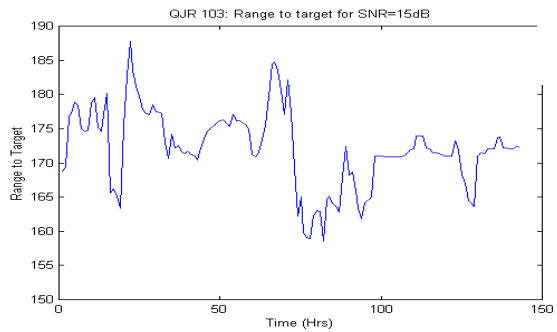
- Shallow Water
  - Subject to surface and mixing effects
  - More responsive to atmospheric forcing
- Deeper Water
  - Profiles more stable
  - System response consistent
- System Impact
  - System response reflects changeable nature of vsw environment
- SNE must reflect this behavior or result in false training

Very Shallow Water (QJR 1-3)

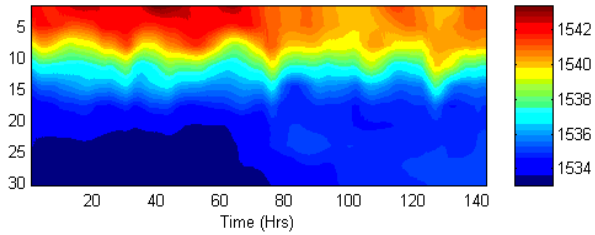


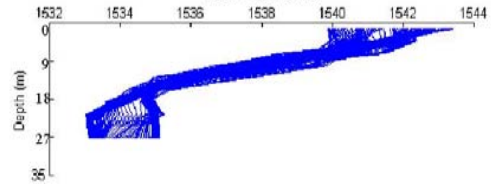


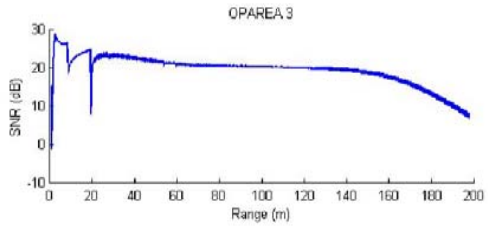


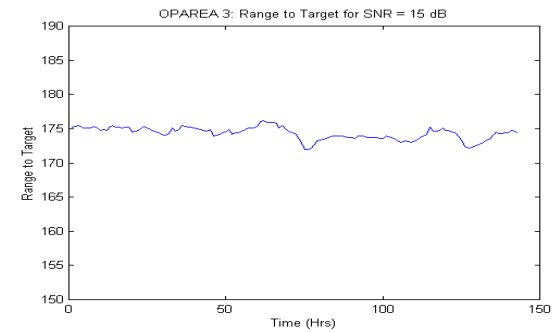


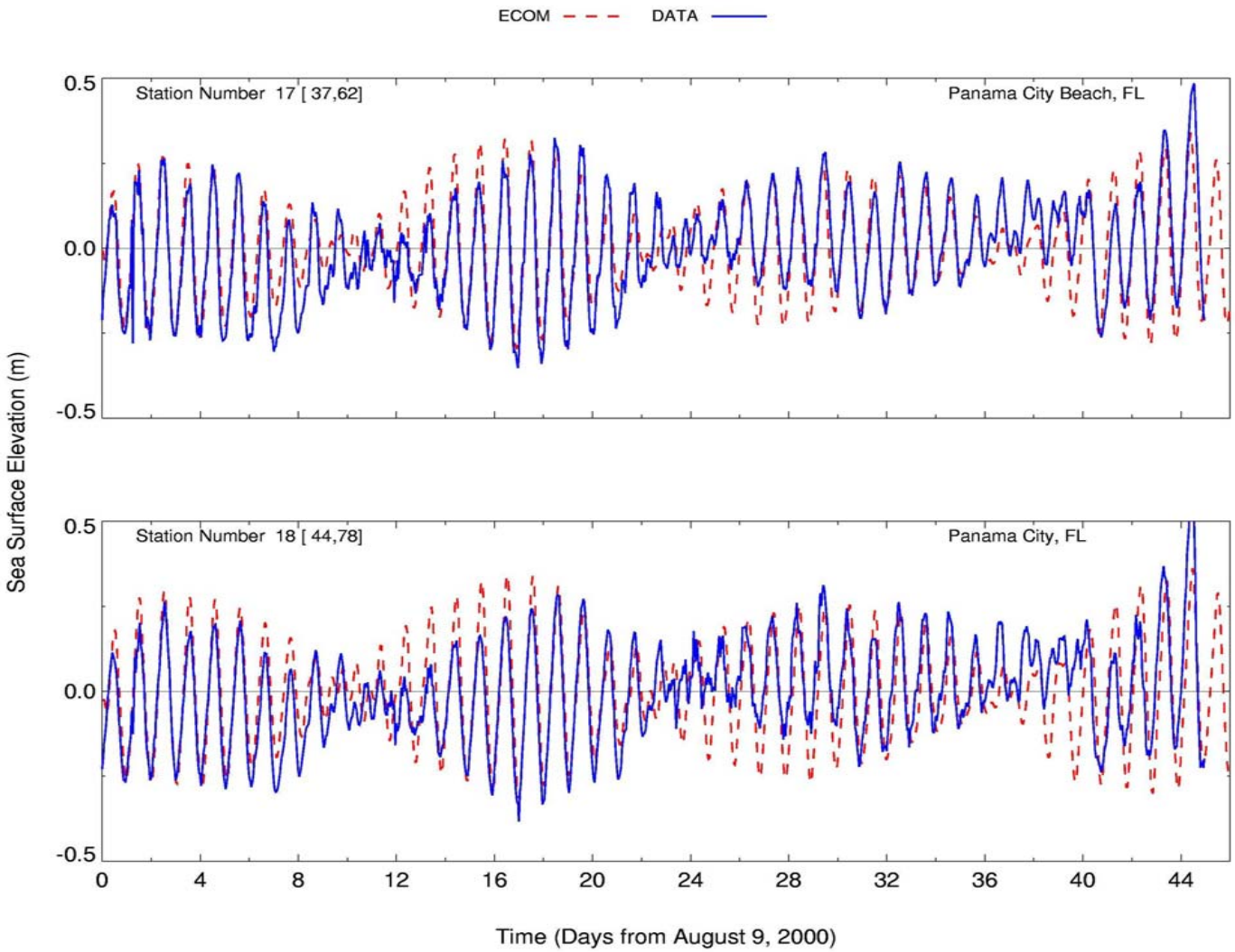
Deeper Water Op Area 3










ECOM
DATA
Station Number 17 [37,62]
Panama City Beach, FL
Station Number 18 [44,78]
Panama City, FL
Sea Surface Elevation (m)
0.5
0.0
-0.5
0
4
8
12
16
20
24
28
32
36
40
44
Time (Days from August 9, 2000)

Sources of Uncertainty

Can originate from:

- Uncertainty in input data
- Model equation approximations
- Model resolution: spatial, time scales
    - (discretization “error”)
- Stochastic elements of model

Systematics of Uncertainty

- System of "Black Boxes" (Models)
  - Model tenders aren't plug compatible
- Common representation of uncertainty
  - Otherwise you have confusion
- Mean, Covariance is minimum necessary
  - For P(detect), P(fa)
- How does uncertainty propagate?

Major Challenges

- Application of Filtering to Ocean Model
- Dimensional Reduction
- Evaluation of Effects of Oceanographic Uncertainty on Acoustic Propagation

Kalman Filtering

- Kalman / Extended Kalman Filtering
  - Systems are non-linear
- Straightforward approach has deficiencies
  - Assumption of independence
  - EKF: not best capture of nonlinear behavior
- Consider Modified Approach
  - Covariance Intersection for dependence
  - U-Transform for nonlinear tracking
  - Determine dimensional reduction

Covariance Intersection (CI) Update Equations

To combine two state estimates, $\{\mathbf{a}, \mathbf{A}\}, \{\mathbf{b}, \mathbf{B}\}$, to get $\{\mathbf{c}, \mathbf{C}\}$

$$\mathbf{C} = \left(\omega \mathbf{A}^{-1} + (1-\omega)\mathbf{B}^{-1}\right)^{-1}$$

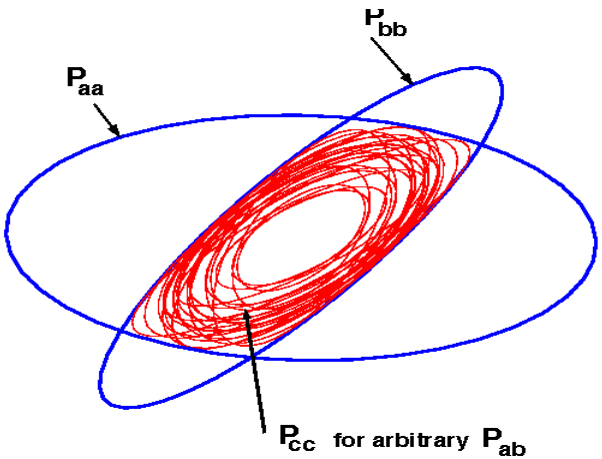
$$\mathbf{c} = \mathbf{C}\left(\omega \mathbf{A}^{-1}\mathbf{a} + (1-\omega)\mathbf{B}^{-1}\mathbf{b}\right)^{-1}$$

Which can be compared to the Kalman update

$$\mathbf{C} = \left(\mathbf{A}^{-1} + \mathbf{B}^{-1}\right)^{-1} = \tfrac{1}{2}\left(\tfrac{1}{2}\mathbf{A}^{-1} + \tfrac{1}{2}\mathbf{B}^{-1}\right)^{-1}$$

$$\mathbf{c} = \mathbf{C}\left(\mathbf{A}^{-1}\mathbf{a} + \mathbf{B}^{-1}\mathbf{b}\right)^{-1}$$

Covariance Intersection (CI)
How it works (geometrically)

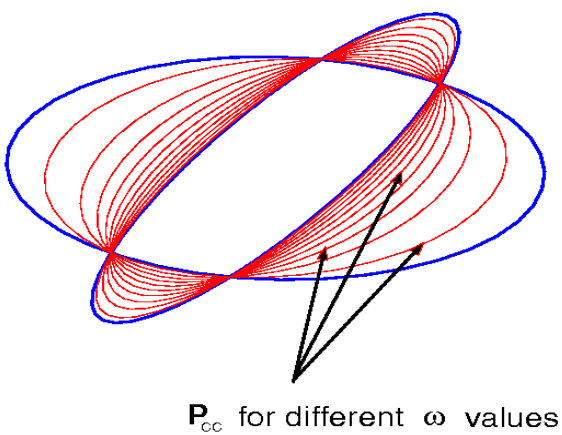


Blue:

- input 1 sigma uncertainty contours

Red:

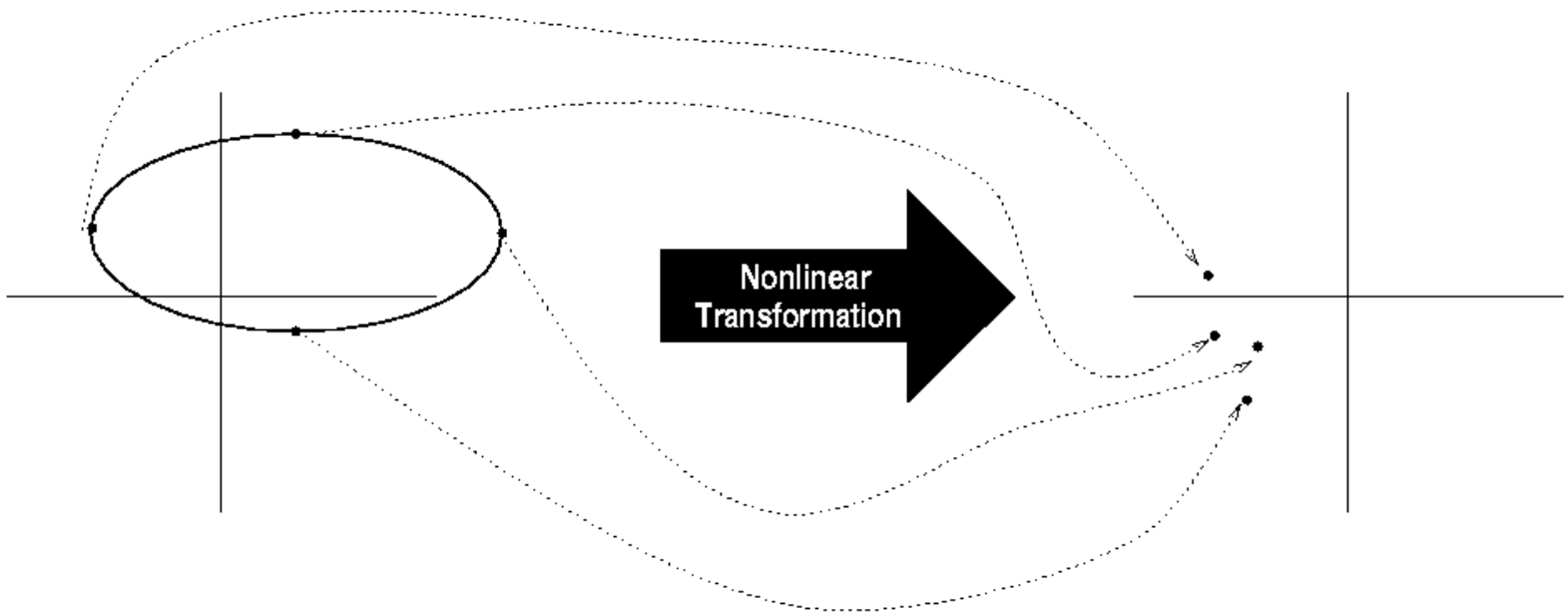
- fused 1 sigma uncertainty contours for arbitrary cross-covariance



Red:

- CI update uncertainty contours
- select based on optimality criterion
- Places a bound on true uncertainty

U-Transform



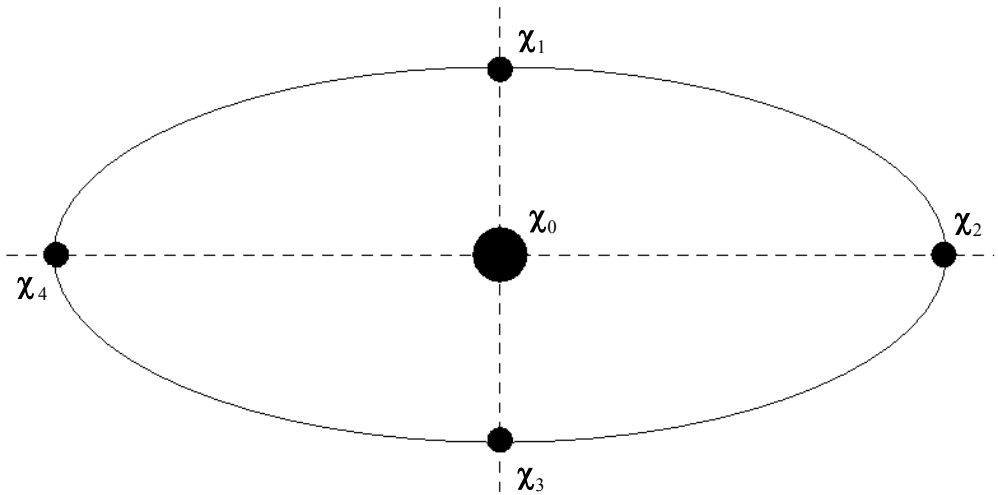
- Choose a weighted set of points with the same mean and covariance as the prior distribution
- Apply nonlinear transformation to each point in set
- Advantages: requires no change to computational model, more accurate than linearization, same order of calculation as extended Kalman Filter

U-Transform, cont'd

- Symmetric χ - points:

$$
\begin{aligned}
\boldsymbol{\mathcal{X}}_0(k \mid k) &= \hat{\mathbf{x}}(k \mid k), \\
W_0 &= \kappa/(n+\kappa), \\
\boldsymbol{\mathcal{X}}_i(k \mid k) &= \hat{\mathbf{x}}(k \mid k) + \left(\sqrt{(n+\kappa)\mathbf{P}(k \mid k)}\right)_i, \\
W_i &= 1/\{2(n+\kappa)\}, \\
\boldsymbol{\mathcal{X}}_{i+n}(k \mid k) &= \hat{\mathbf{x}}(k \mid k) - \left(\sqrt{(n+\kappa)\mathbf{P}(k \mid k)}\right)_i, \\
W_{i+n} &= 1/\{2(n+\kappa)\}.
\end{aligned}
$$

- In 2D, these simply look like:



U-Transform Updates

1. Determine the set of projected χ - points from

$$\boldsymbol{\mathcal{X}}_i(k+1 \mid k) = \mathbf{f}\left[\boldsymbol{\mathcal{X}}_i(k \mid k), \mathbf{u}(k), k\right]$$

2. Calculate the mean as

$$\hat{\mathbf{x}}(k+1 \mid k) = \frac{1}{n+\kappa}\left\{\kappa \boldsymbol{\mathcal{X}}_0(k+1 \mid k) + \frac{1}{2}\sum_{i=1}^{2n} \boldsymbol{\mathcal{X}}_i(k+1 \mid k)\right\}$$

3. Calculate the covariance as

$$\begin{aligned}\mathbf{P}(k+1 \mid k) &= \frac{1}{n+\kappa}\Big\{\kappa[\boldsymbol{\mathcal{X}}_0(k \mid k) - \hat{\mathbf{x}}(k+1 \mid k)][\boldsymbol{\mathcal{X}}_0(k \mid k) - \hat{\mathbf{x}}(k+1 \mid k)]^T \\ &+ \frac{1}{2}\sum_{i=1}^{2n}[\boldsymbol{\mathcal{X}}_i(k+1 \mid k) - \hat{\mathbf{x}}(k+1 \mid k)][\boldsymbol{\mathcal{X}}_i(k+1 \mid k) - \hat{\mathbf{x}}(k+1 \mid k)]^T\Big\}\end{aligned}$$

Dimensional Reduction

- State space representation is redundant
- Evidence shows significant reduction is possible
- Reduction is necessary for filtering (algorithmic scaling)

Acoustic Problem Statement

Given a mean and covariance estimate of water column conditions, predict the uncertainty associated with a mean acoustic performance estimate in a region of interest

Issues

Are we trying to:

- Support an individual detection?
- Support a mission plan?
- Study the acoustic effects of oceanographic events?

What is the scenario?

- ASW – low frequency, deeper waters, long range propagation
- MIW – high frequency, shallower waters, short range propagation

What is the allotted processing time?

- Dictates acoustic model complexity
- Dictates uncertainty propagation approach
  (e.g. Monte Carlo simulation vs. table-look-up)

How much information can be usefully passed to the warfighter?

Generating Realizations of Environment Based on Environmental Mean and Covariance Estimate

Given: Mean and covariance estimate of environment

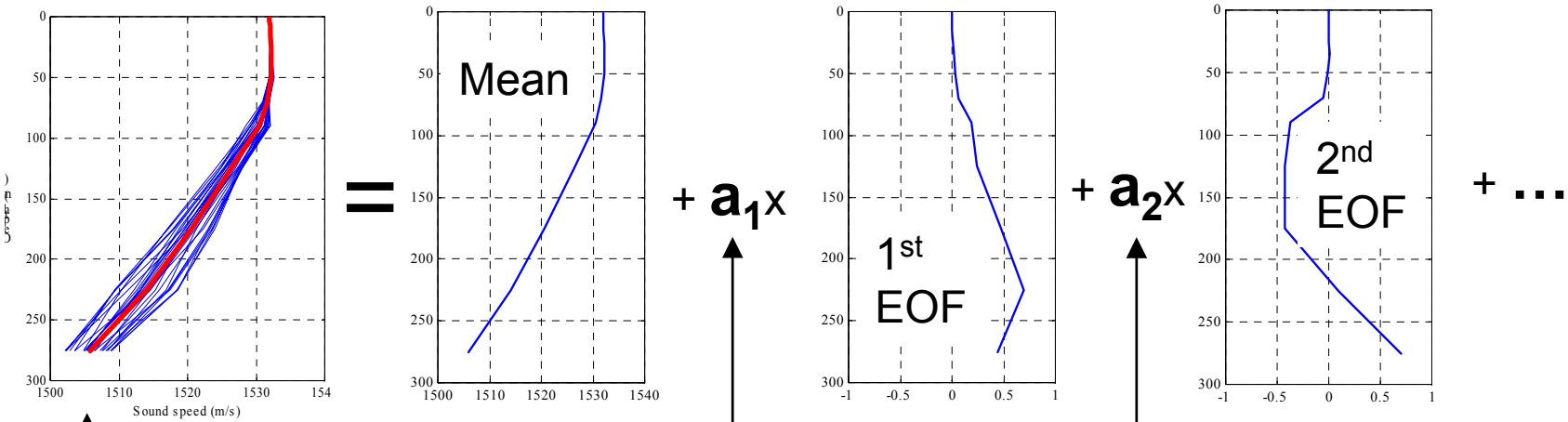
Generate: Realizations of environment to carry environmental uncertainty through to acoustic uncertainty

Process: Decompose Environmental Variability into Eigenfunctions

- Reduces and Orthogonalizes Environmental Parameter Space
- Analytical vs. Empirical?... Question for research
- Analytical decomposition allows for phenomenological description
- Empirical decomposition allows for accurate description of variability
- Question: Is acoustic model sensitivity too great for general phenomenological description alone?

➢ Realizations of variable environment are used to run acoustic models.
➢ Statistical analysis of acoustic output used to determine acoustic uncertainty.

Example: Orthogonal Function Expansion of SVP

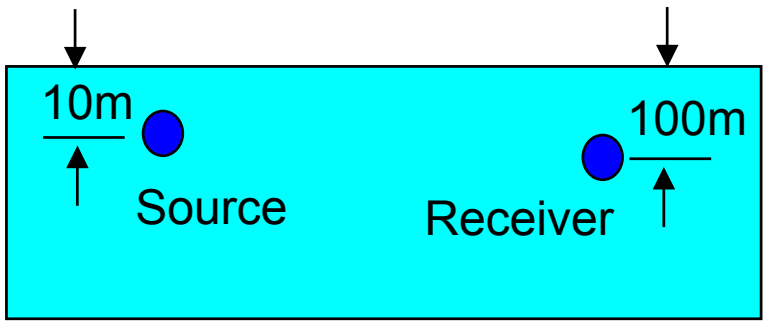


- Empirical Orthogonal Functions (EOF) coefficients
- Zero mean, assumed Gaussian, with known variance ($D_a = V^T D V$)
- Case shown: $\sigma_1 = 4.3$ (m/s), $\sigma_2 = 1.5$ (m/s)

- One-day simulated svp variations due to tidal fluctuations only
- Provides reasonable estimate of svp covariance to use for example

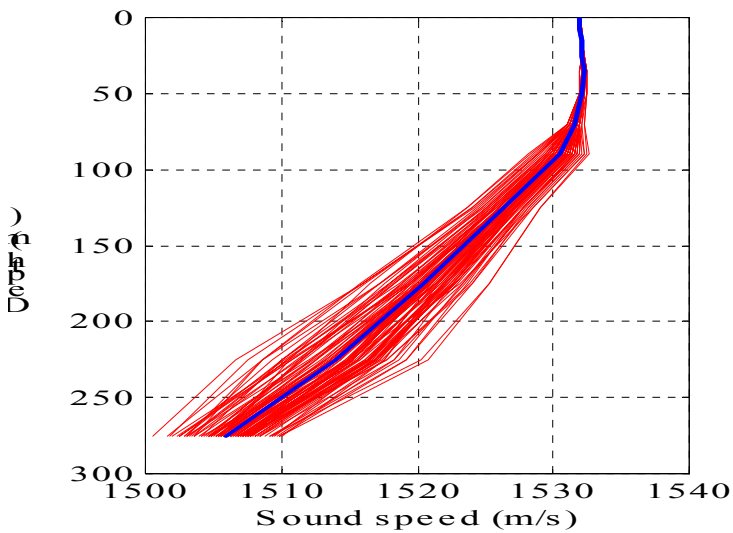
Computing Acoustic Uncertainty using Monte Carlo Simulations

Experiment

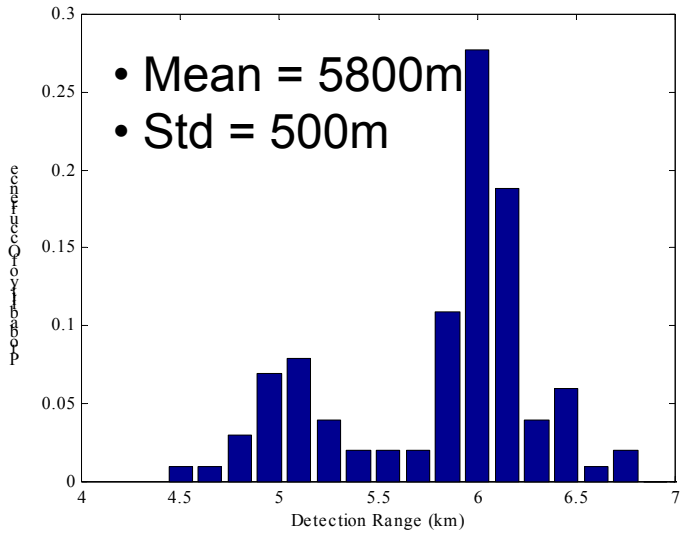


- Model: OASES-TL, 3.5kHz, omni-source, range-independent
- Acoustic Metric: Detection Range
- 100 Realizations

100 realizations of SVP



Distribution of Detection Range



- MCS requires assumption of pdf
- MCS may be too time-consuming for real-time

Linearized Covariance Transfer

Step 1: Generate Acoustic Sensitivity Matrix

$$\tilde{S} = \left.\begin{bmatrix} \frac{\partial Y_1}{\partial X_1} & \frac{\partial Y_1}{\partial X_2} & \cdots & \frac{\partial Y_1}{\partial X_N} \\ \frac{\partial Y_2}{\partial X_1} & \frac{\partial Y_2}{\partial X_2} & \cdots & \frac{\partial Y_2}{\partial X_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Y_M}{\partial X_1} & \frac{\partial Y_M}{\partial X_2} & \cdots & \frac{\partial Y_M}{\partial X_N} \end{bmatrix}\right|_{X=X_o}$$

$\tilde{S} =$ **Sensitivity matrix**

$\bar{Y} =$ **Acoustic Model Output Parameters**

$\bar{X} =$ **Acoustic Model Input Parameters (sound-speed)**

- **Input parameters must be independent (EOF)**
- **Possible output parameters: detection range, point TL, coefficients of curve-fitted TL**

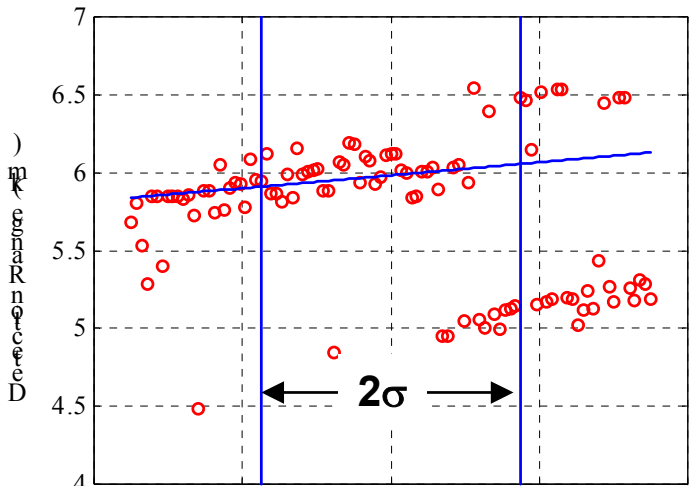
Step 2: Transfer Input Covariance

$$dY = \tilde{S} dX$$

Covariance of Input Parameters

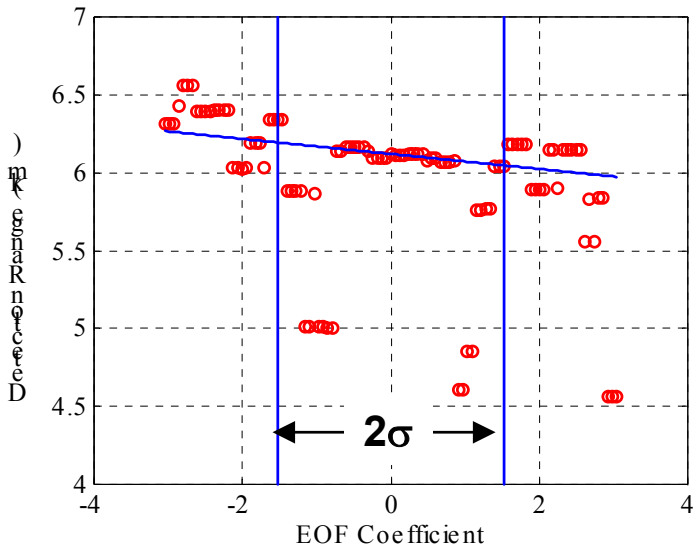
$$C_{dY} = E\left[(dY)(dY)^T\right] = \tilde{S} D_a \tilde{S}^T$$

Linearized Covariance Transfer Results



Detection Range vs Variations in a_1

$S_{11} = \partial R_D / \partial a_1$ = ??



Detection Range vs Variations in a_2

$S_{12} = \partial R_D / \partial a_2$ = ??

Conclusions

- **Need to linear acoustic model**
- **For fast transfer of uncertainty**
- **Conventional Sonar Metrics are**
- **Not well behaved**
- **Need better behaved acoustic measures**
- **(I.e., eigenstructure)**