Capturing Uncertainty Review: June 16, 2004

Tools for Connecting Acoustic and Oceanographic Sensitivity

Bruce Cornuelle for

E. Di Lorenzo, W. Hodgkiss, P. Hursky, K. Kim, W. Kuperman, M. Porter, and A. Thode

Theme: Sensitivity

- Acoustic observables to sound speed
  - PE adjoint (Hursky)
  - Other observables: intensity, Bartlett ambiguity (Thode, Kim), peak arrival time
- Ocean to initial conditions and forcing
  - PE adjoint (again!)
- Combined systems (theory)

Outline (Acoustics)

Propagation modeling and the Green's function

Medium and Green's function perturbations

Numerical examples:

- Conversion to more linear observables: intensity, …
- Peak arrival time sensitivity kernel (E. Skarsoulis*)

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Propagation Modeling (Frequency Domain)

The pressure $p_r(w)$ at the receiver in the frequency domain

$$p_r(w;c) = P_s(w) G(\vec{x}_r \mid \vec{x}_s; c; w)$$

$P_s(w)$: the emitted (source) signal in the frequency domain

$G(\vec{x} \mid \vec{x}_s; c; w)$: the *Green's function* of the acoustic channel

ω : the circular frequency

$c(\vec{x})$: the sound-speed distribution

$\vec{x}_s, \vec{x}_r$: the source / receiver locations

Perturbation of Green's Function

$$c: \quad \left[\nabla^2 + \frac{\omega^2}{c^2(\vec{x})}\right] G(\vec{x} \,|\, \vec{x}_s) = -\delta(\vec{x} - \vec{x}_s) \quad + \text{B.C.} + \text{I.C.} + \text{R.C.}$$

$$c + \Delta c: \left[\nabla^2 + \frac{\omega^2}{\left(c(\vec{x}) + \Delta c(\vec{x})\right)^2}\right] \left[G(\vec{x} \,|\, \vec{x}_s) + \Delta G(\vec{x} \,|\, \vec{x}_s)\right] = -\delta(\vec{x} - \vec{x}_s)$$

Linearization (first Born approximation)

$$\Delta G(\vec{x} \,|\, \vec{x}_s) = -2\omega^2 \iiint_V G(\vec{x}\,' \,|\, \vec{x}_s) G(\vec{x} \,|\, \vec{x}\,') \frac{\Delta c(\vec{x}\,')}{c^3(\vec{x}\,')} dV(\vec{x}\,')$$

Green's Function or Pressure Sensitivity

Green's Function

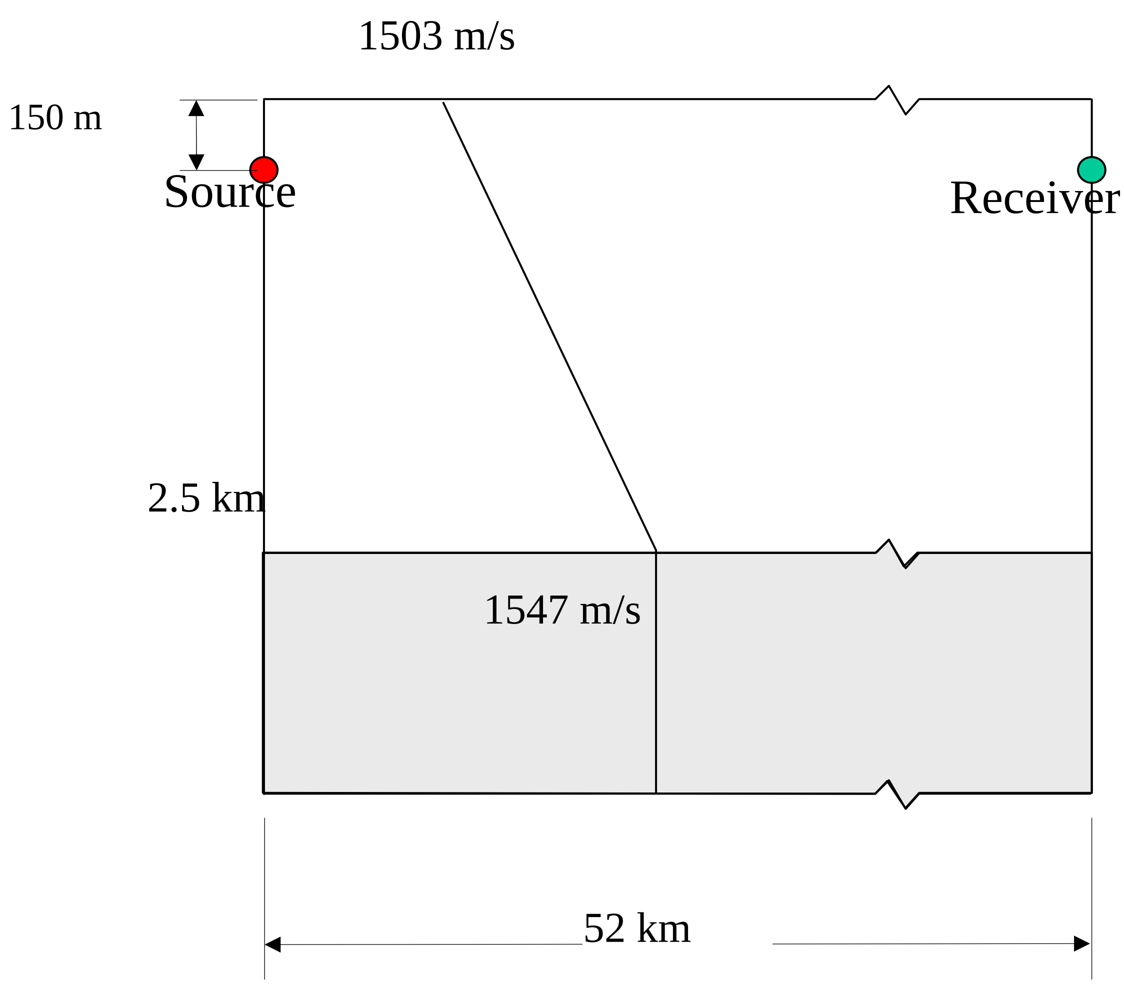
$$\frac{\Delta G(\vec{x}\,|\,\vec{x}_s)}{\Delta c(\vec{x}\,')} = -2\omega^2 \iiint_V G(\vec{x}\,'\,|\,\vec{x}_s)\,G(\vec{x}\,|\,\vec{x}\,')\,\frac{1}{c^3(\vec{x}\,')}\,dV(\vec{x}\,')$$

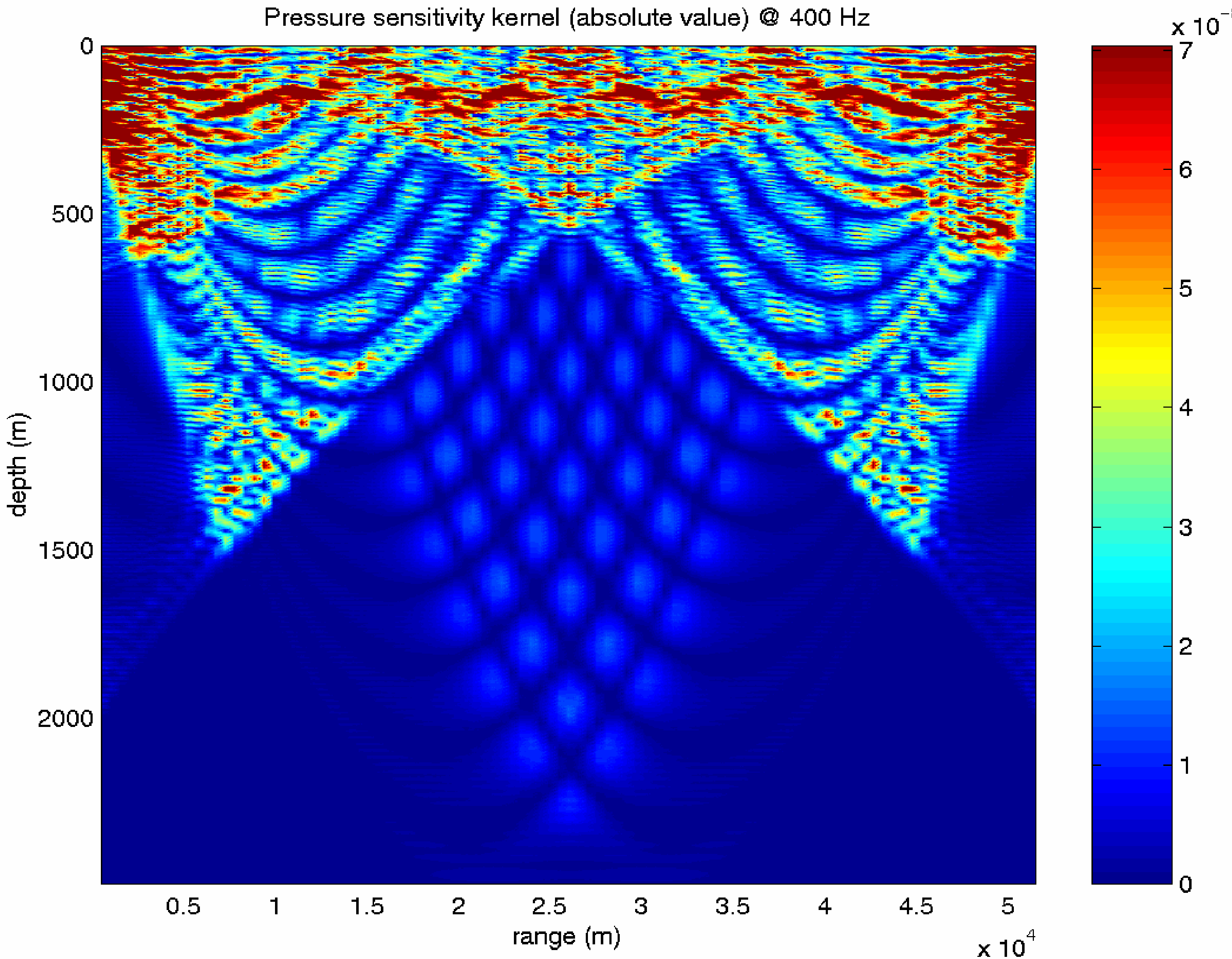
Pressure

$$\frac{\partial p(\vec{x}\,|\,\vec{x}_s)}{\partial c(\vec{x}\,')} = -2\omega^2 \iiint_V G(\vec{x}\,'\,|\,\vec{x}_s)\,G(\vec{x}\,|\,\vec{x}\,')\,\frac{P_s(\omega)}{c^3(\vec{x}\,')}\,dV(\vec{x}\,')$$

which is comparable to the adjoint of the PE model.
Note: Reciprocity needed for efficient computation.

Numerical Example




Pressure sensitivity kernel (absolute value) @ 400 Hz
depth (m)
0
500
1000
1500
2000
range (m)
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x 10^4
x 10^-8
7
6
5
4
3
2
1
0

Intensity Sensitivity

$$A(\omega) = p(\omega)p(\omega)^*$$

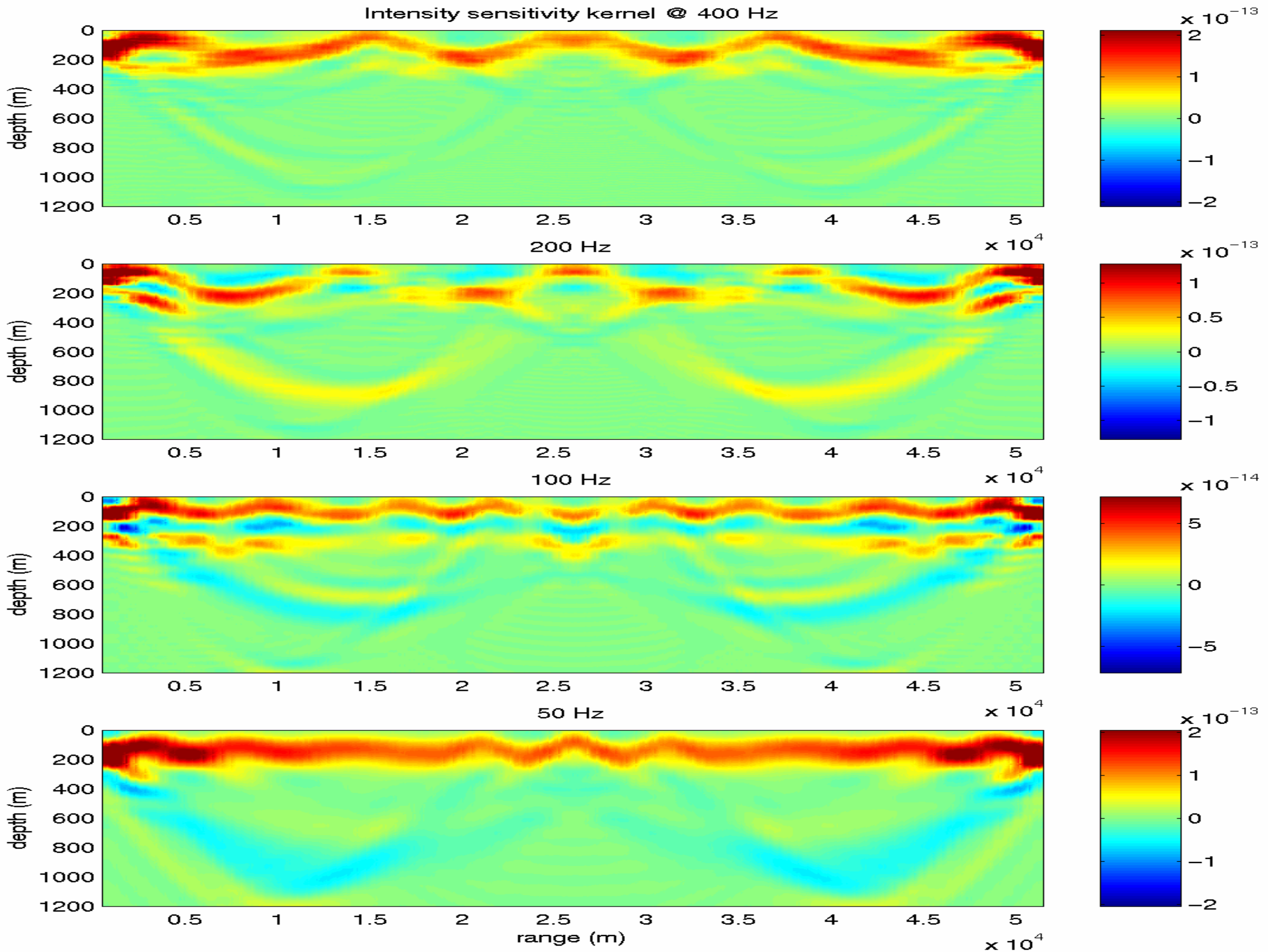
Chain rule derivative:

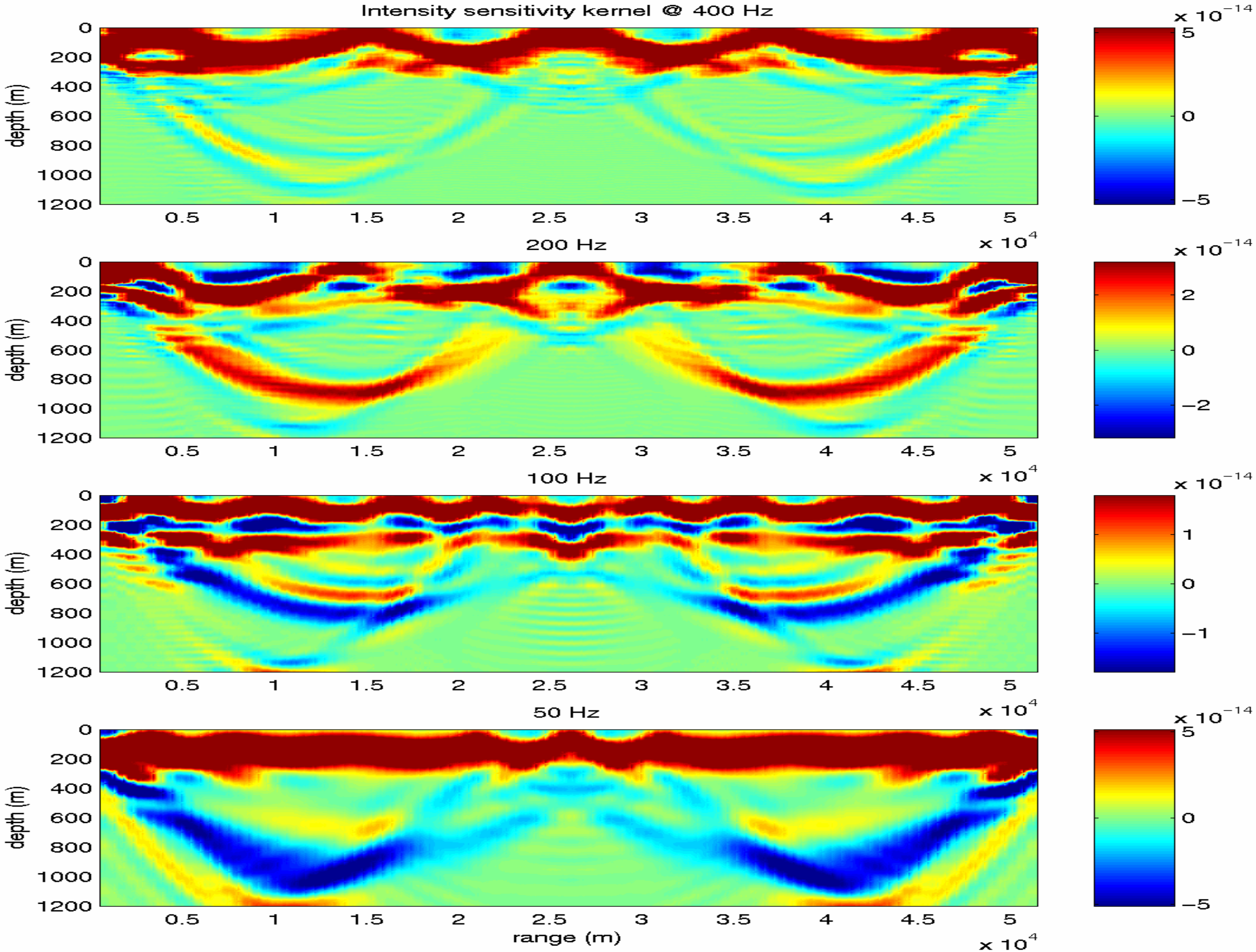
$$\frac{\partial A}{\partial c} = \frac{\partial p(\omega)}{\partial c} p^* + p \frac{\partial p(\omega)^*}{\partial c}$$

Broadband intensity:

$$I = \sum_i p(\omega_i) p(\omega_i)^*$$

$$\frac{\partial I}{\partial c} = \sum_i \frac{\partial p(\omega_i)}{\partial c} p(\omega_i)^* + p(\omega_i) \frac{\partial p(\omega_i)^*}{\partial c}$$


Intensity sensitivity kernel @ 400 Hz
200 Hz
100 Hz
50 Hz
depth (m)
range (m)
x 10^4
x 10^-13
x 10^-14


Intensity sensitivity kernel @ 400 Hz
200 Hz
100 Hz
50 Hz
depth (m)
range (m)
x 10^4
x 10^-14
0
200
400
600
800
1000
1200
0.5
1
1.5
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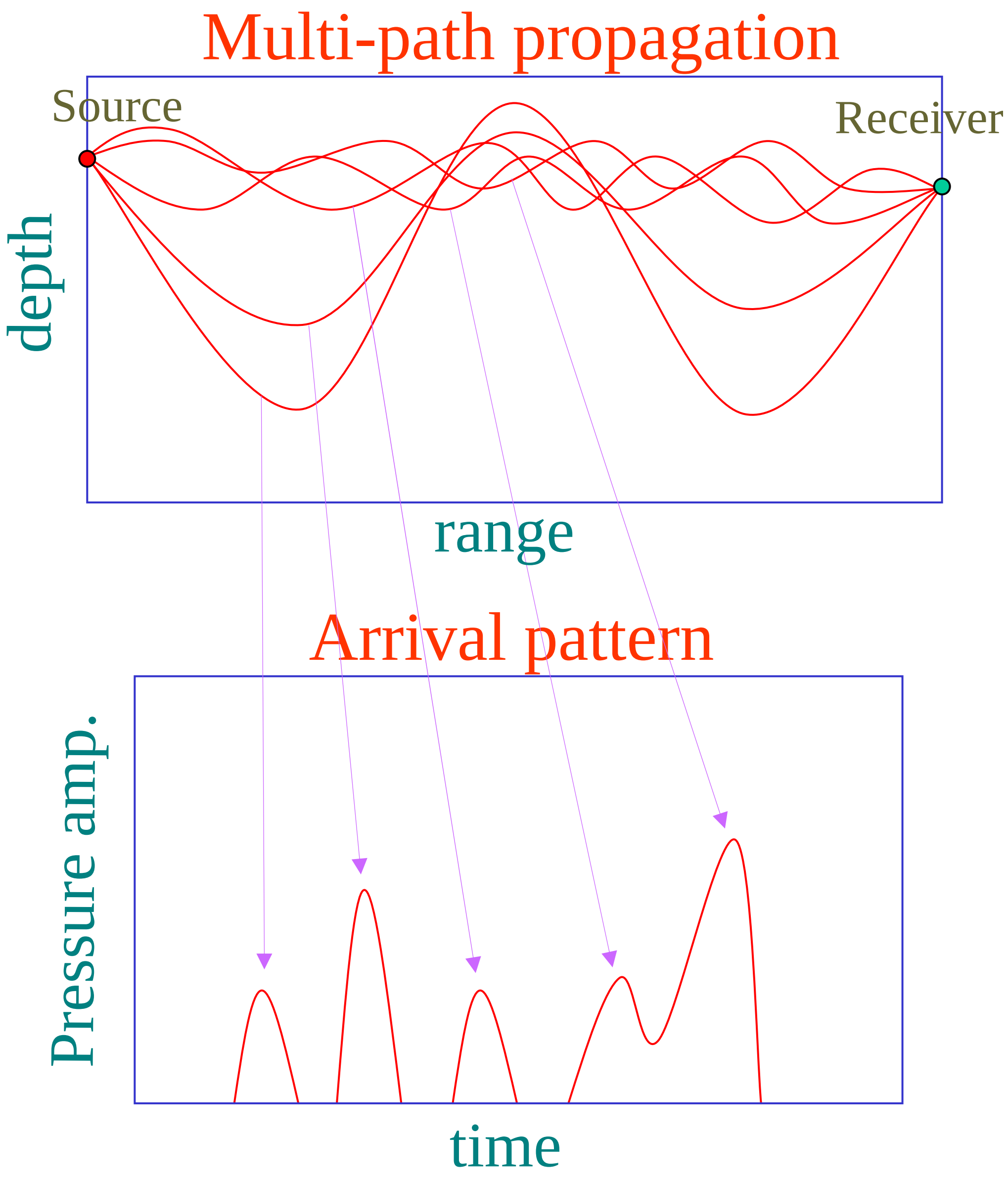
−5
−2
2
−1



Ray theory is a high-frequency asymptotic approximation.

What is the spatial sensitivity of finite-frequency travel times ?

Propagation Modeling (Time Domain)

The pressure p_r at the receiver in the time domain

$$p_r(t;c) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underbrace{P_s(\omega)G(\vec{x}_r \mid \vec{x}_s;c;\omega)}_{P_r(\omega;c)} e^{i\omega t}\,d\omega$$

$P_s(\omega)$: the emitted (source) signal in the frequency domain

$G(\vec{x} \mid \vec{x}_s;c;\omega)$: the *Green's function* of the acoustic channel

ω : the circular frequency

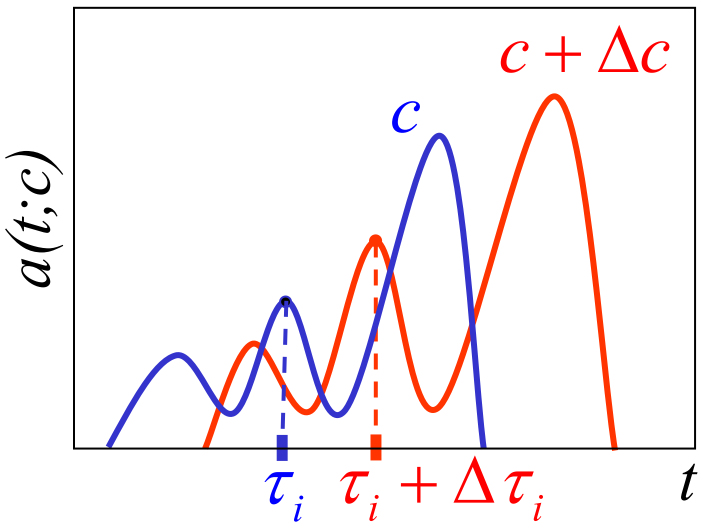
$c(\vec{x})$: the sound-speed distribution

$\vec{x}_s, \vec{x}_r$: the source / receiver locations

Modeling of Arrival Time Observables

Peak-arrival times (local maxima of arrival pattern $a(t;c) = \left| p_r(t;c) \right|$):

$$\dot{a}(\tau_i;c) = 0$$



Arrival time perturbations (due to sound-speed perturbations):

$$\dot{a}(\tau_i + \Delta\tau_i; c + \Delta c) = 0$$

Linear approximation: $\Delta\tau_i = -\dfrac{\dot{u}_i \Delta u_i + u_i \Delta\dot{u}_i + \dot{w}_i \Delta w_i + w_i \Delta\dot{w}_i}{\dot{u}_i^2 + u_i \ddot{u}_i + \dot{w}_i^2 + w_i \ddot{w}_i}$

where $p_r = u + jw$, $\Delta p_r = \Delta u + j\Delta w$, $u_i = u(\tau_i;c)$, …

Arrival Time Sensitivity Kernel

The perturbation sequence

$$\Delta c \;\Rightarrow\; \Delta G \;\Rightarrow\; \Delta p_r \,,\; \text{i.e. } (\Delta u_i, \Delta w_i) \;\Rightarrow\; \Delta \tau_i$$

Representation of $\Delta \tau_i$ in terms of Δc

$$\Delta \tau_i = \iiint_V K_i(\vec{x}\,' \,|\, \vec{x}_s; \vec{x}_r; c)\, \Delta c(\vec{x}\,')\, dV(\vec{x}\,')$$

The travel-time sensitivity kernel K_i describes the effect that a localized sound-speed perturbation anywhere in the medium has on arrival time τ_i .

Arrival Time Sensitivity Kernel

$$K_i(\vec{x}'\,|\,\vec{x}_s;\vec{x}_r;c) = \Re\left\{\frac{u_i - jw_i}{2\pi b_i}\int_{-\infty}^{\infty} j\omega Q(\vec{x}'\,|\,\vec{x}_s;\vec{x}_r;\omega;c)e^{j\omega\tau_i}d\omega\right.$$

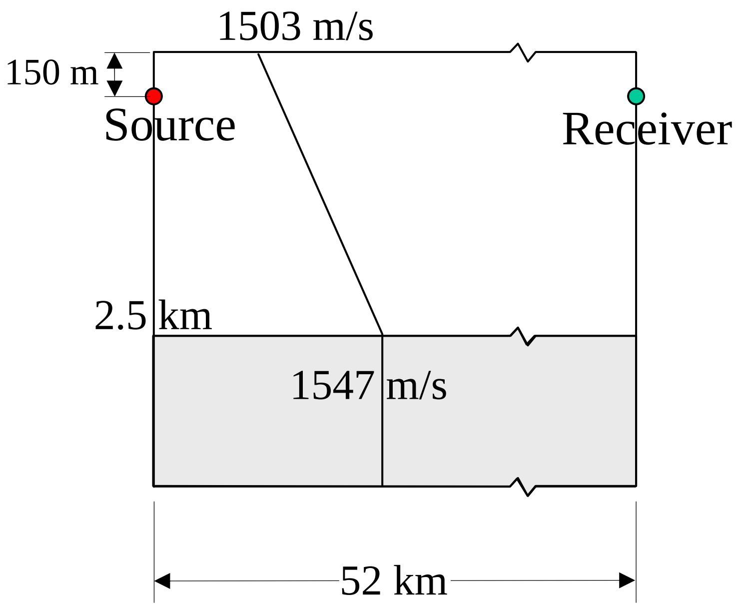
$$\left.+\frac{\dot{u}_i - j\dot{w}_i}{2\pi b_i}\int_{-\infty}^{\infty} Q(\vec{x}'\,|\,\vec{x}_s;\vec{x}_r;\omega;c)e^{j\omega\tau_i}d\omega\right\}$$

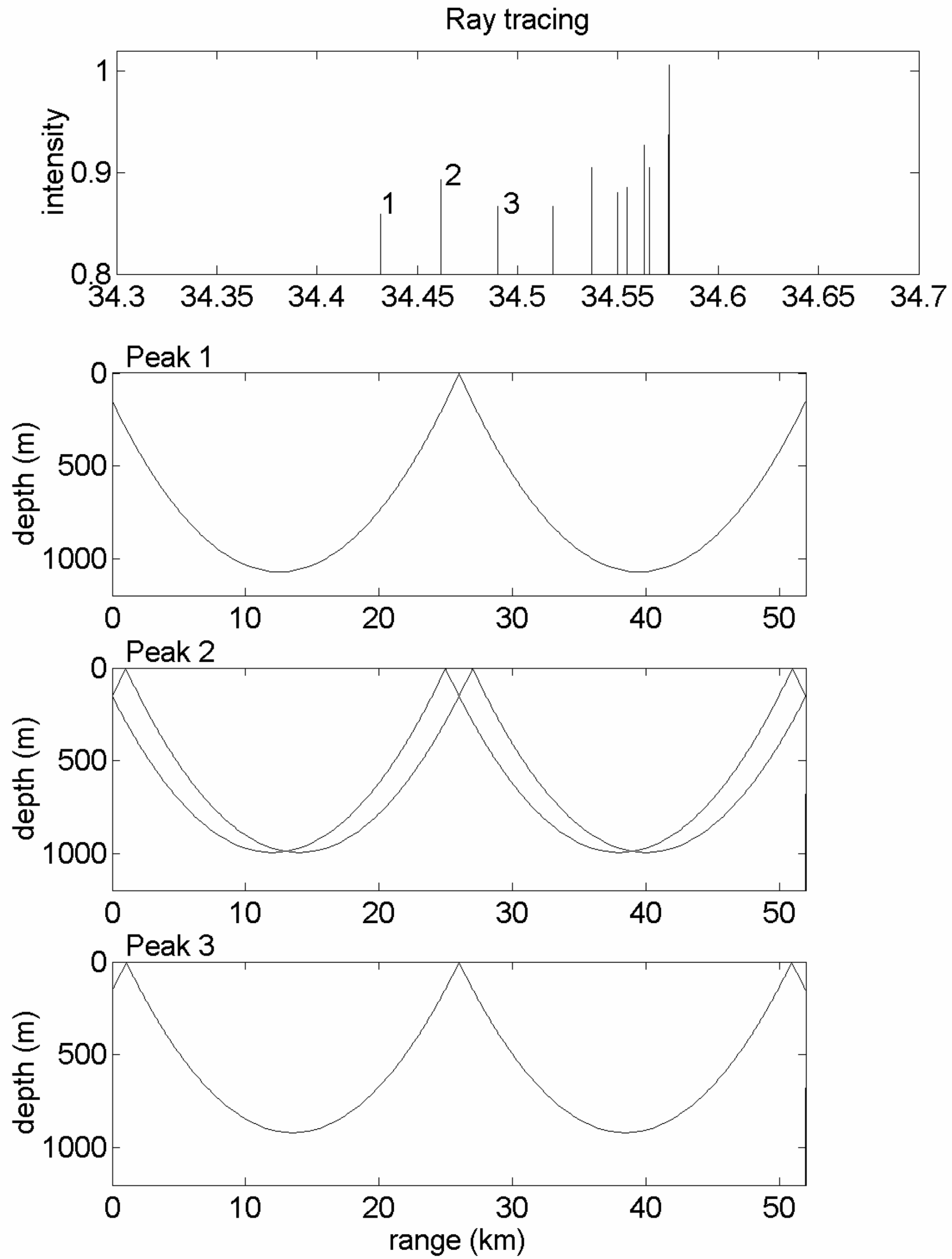
where $b_i = \dot{u}_i^2 + u_i\ddot{u}_i + \dot{w}_i^2 + w_i\ddot{w}_i$ and

$$Q(\vec{x}'\,|\,\vec{x}_s;\vec{x}_r;\omega;c) = \frac{2\omega^2 P_s(\omega)}{c^3(\vec{x}')}G(\vec{x}'\,|\,\vec{x}_s;\omega;c)G(\vec{x}_r\,|\,\vec{x}';\omega;c)$$

Numerical Example

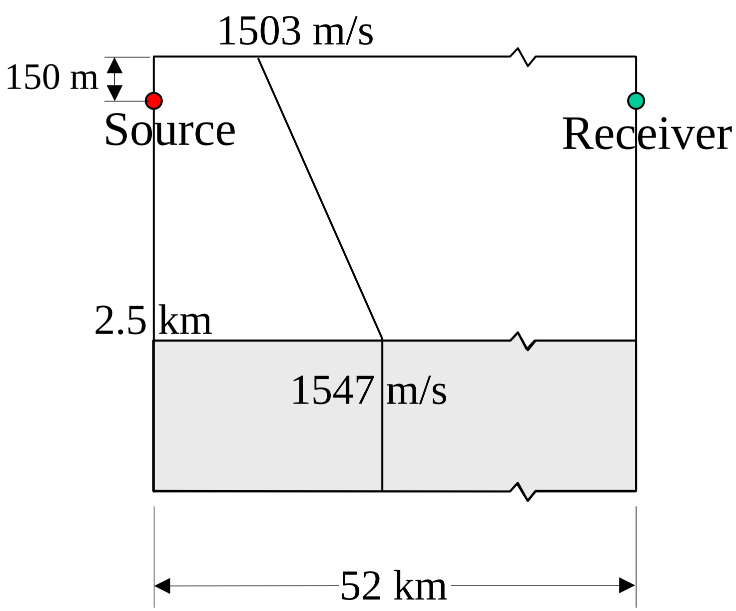
Ray Theory

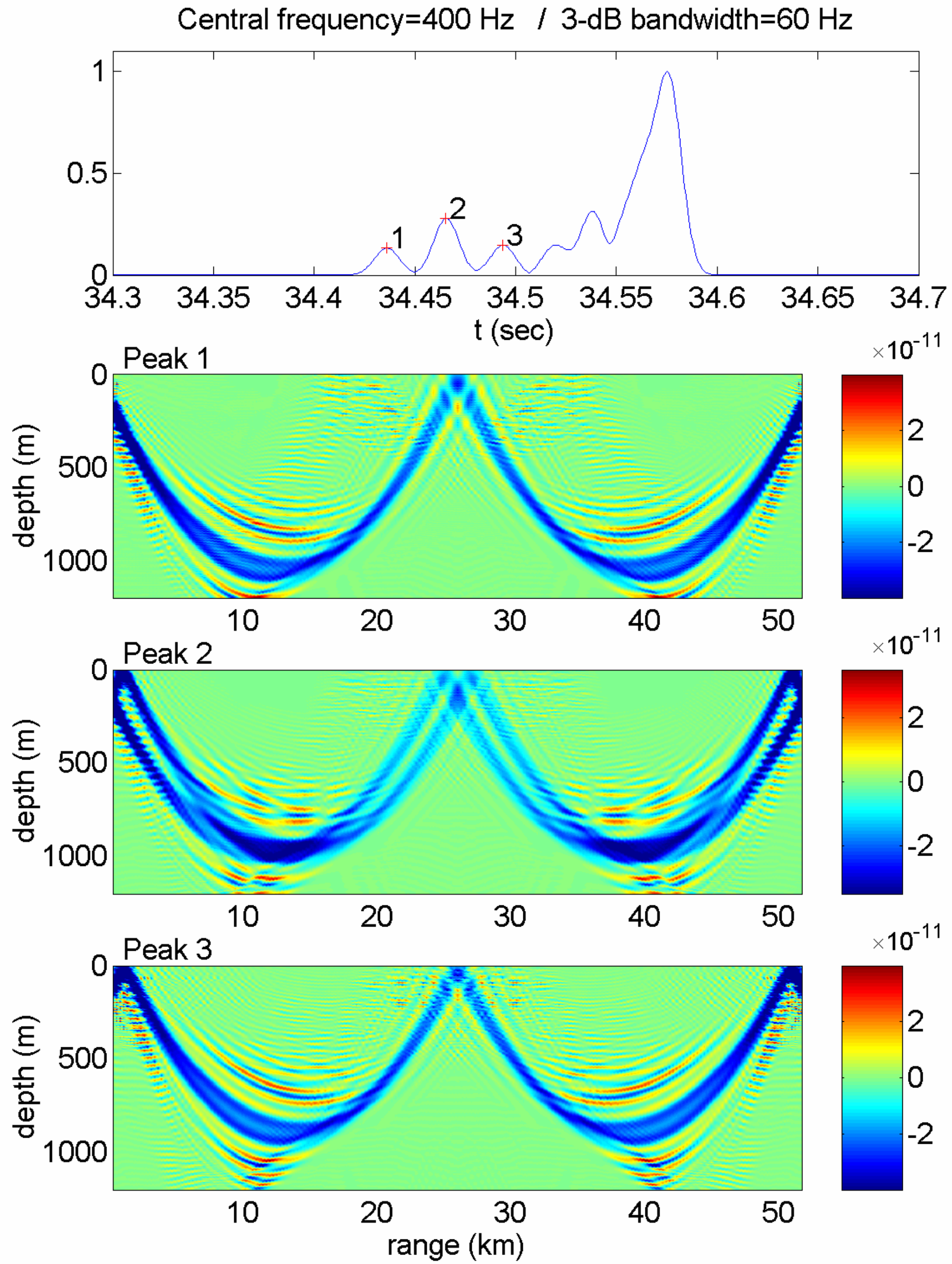




Numerical Example

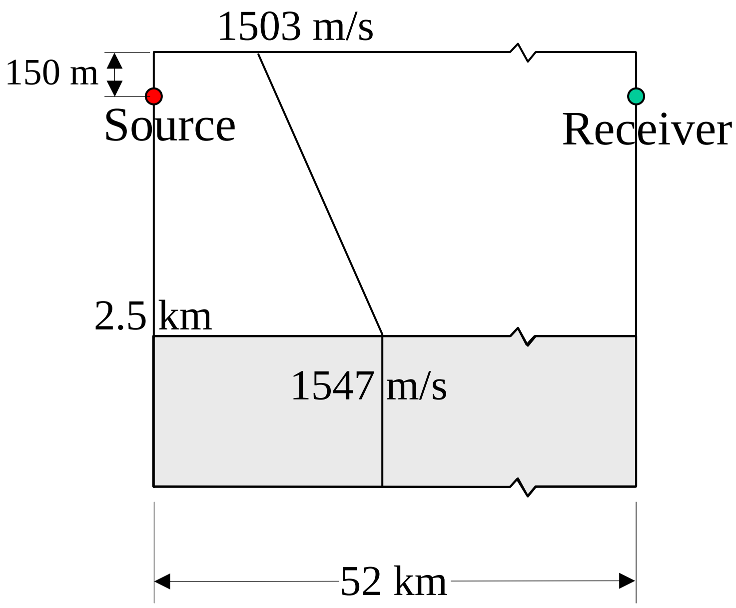
Wave Theory (400 Hz)

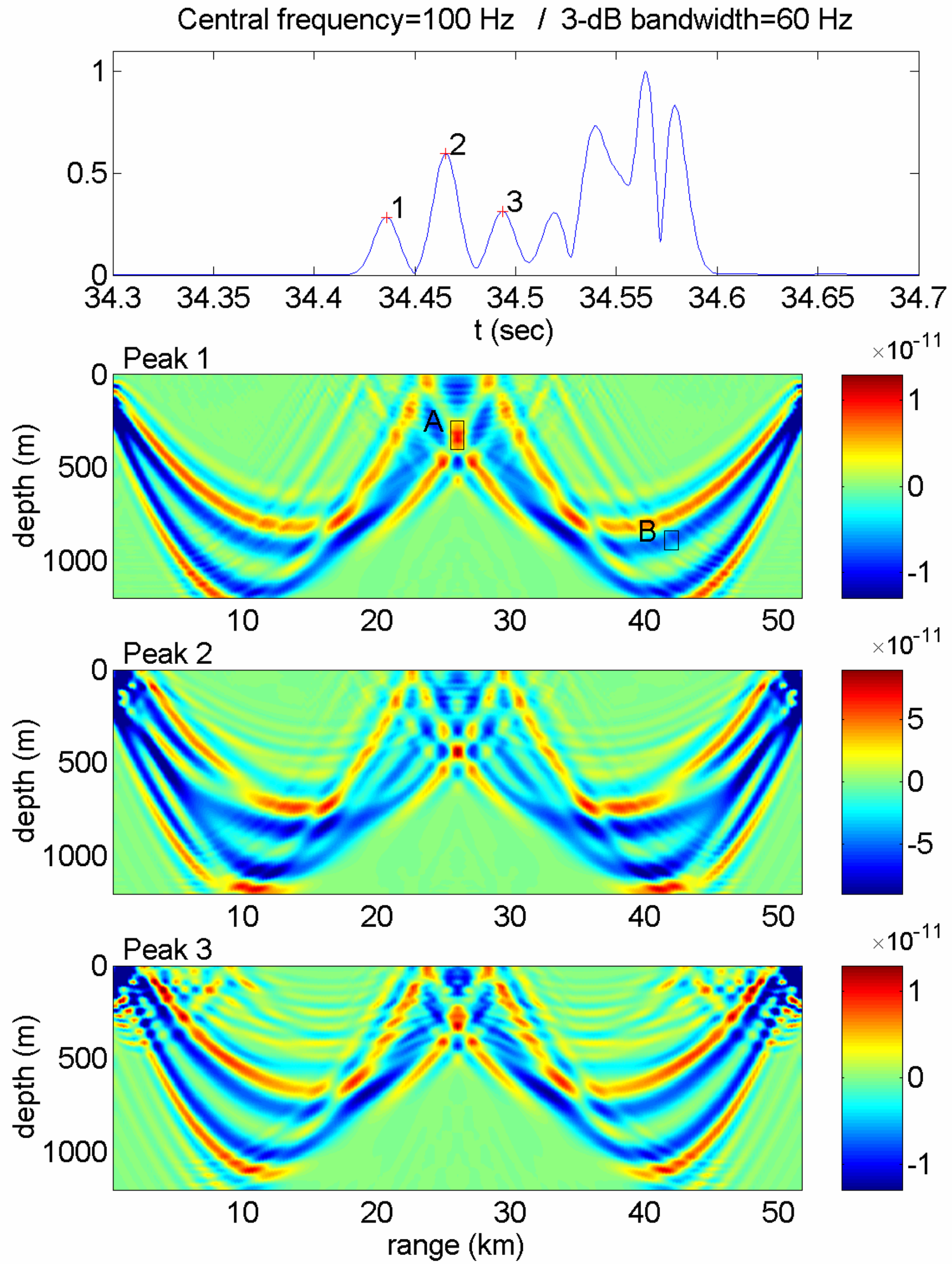




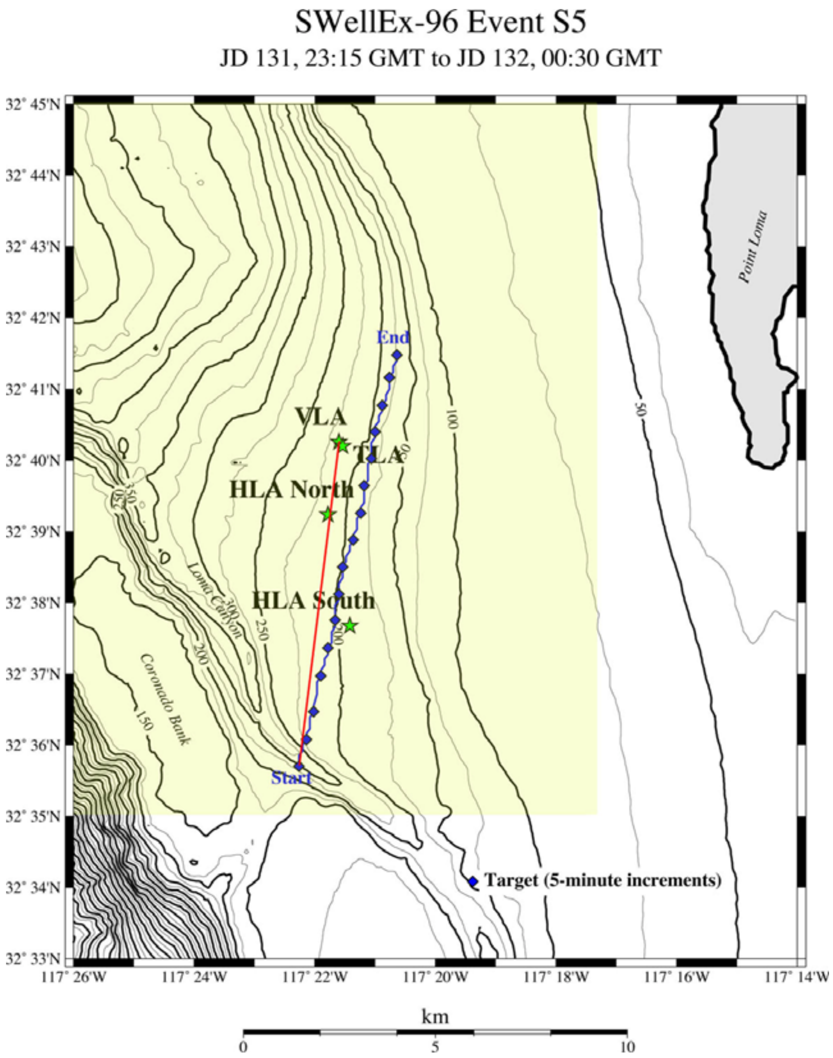
Numerical Example

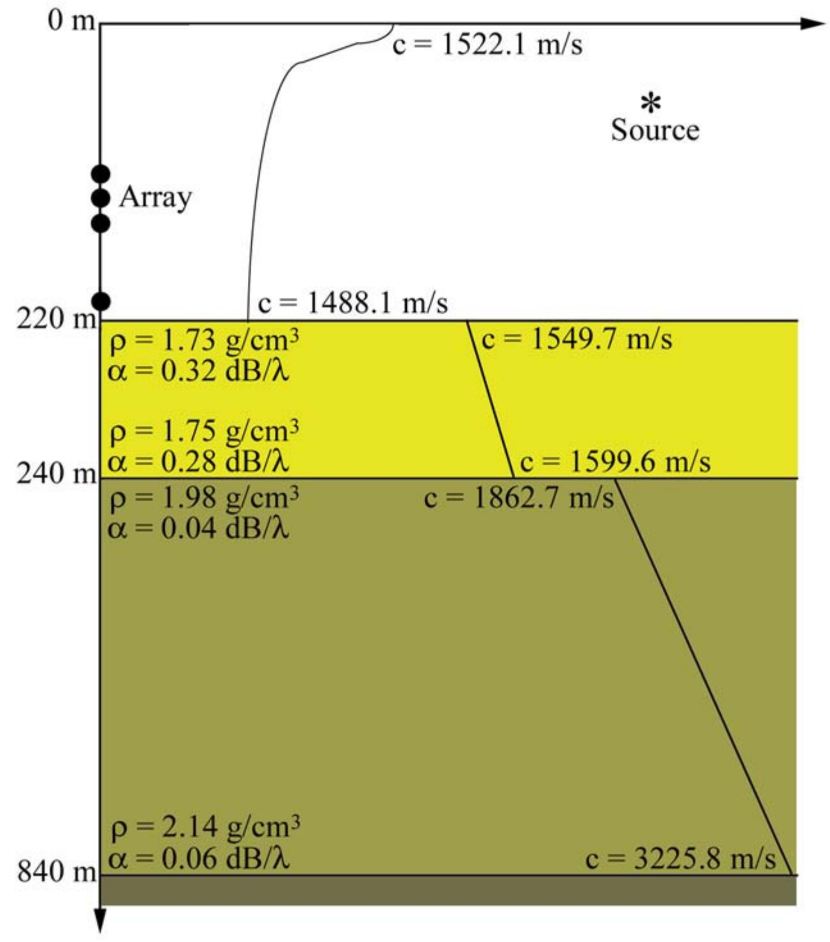
Wave Theory (100 Hz)



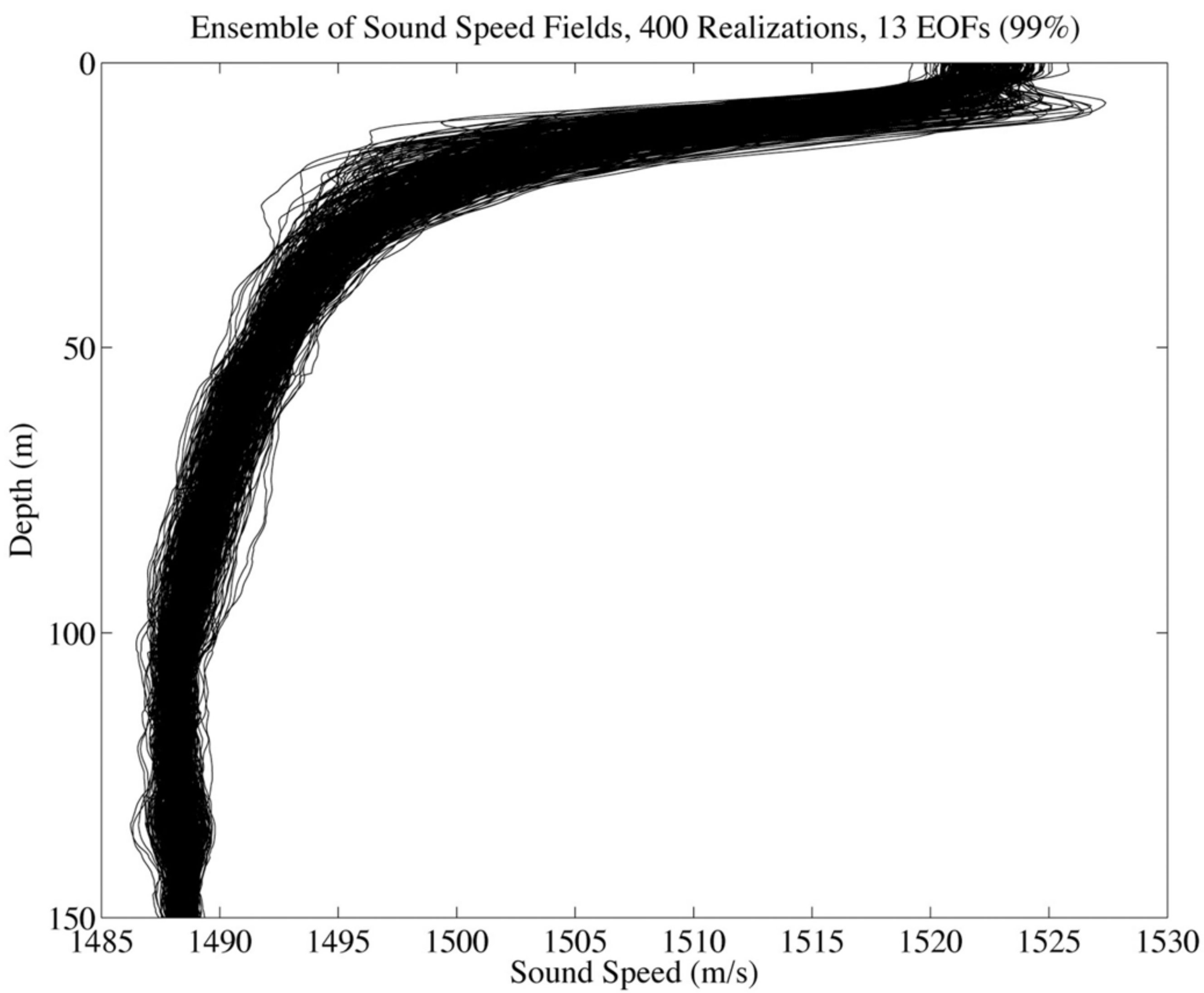


SWellEx-96 Experiment

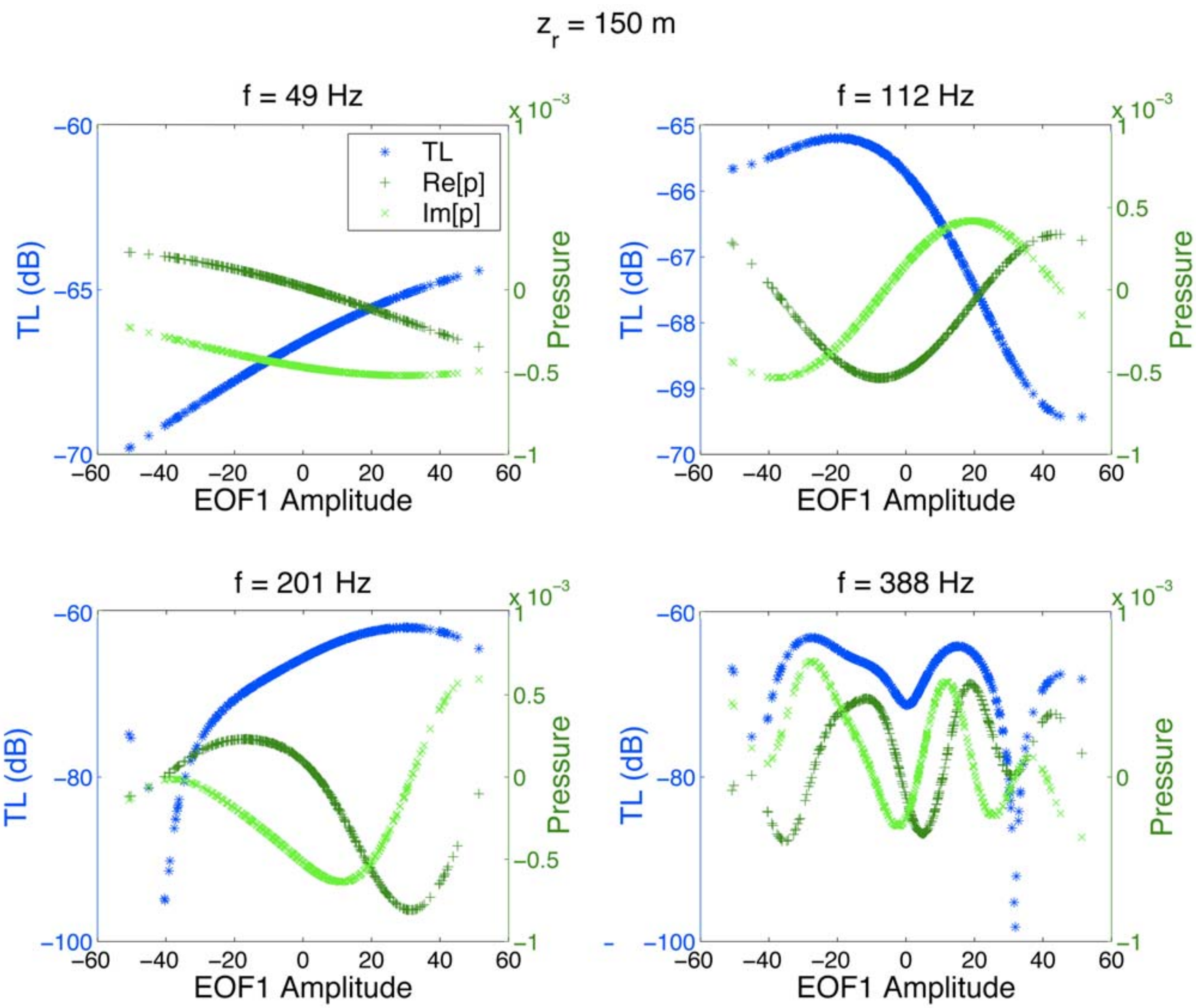




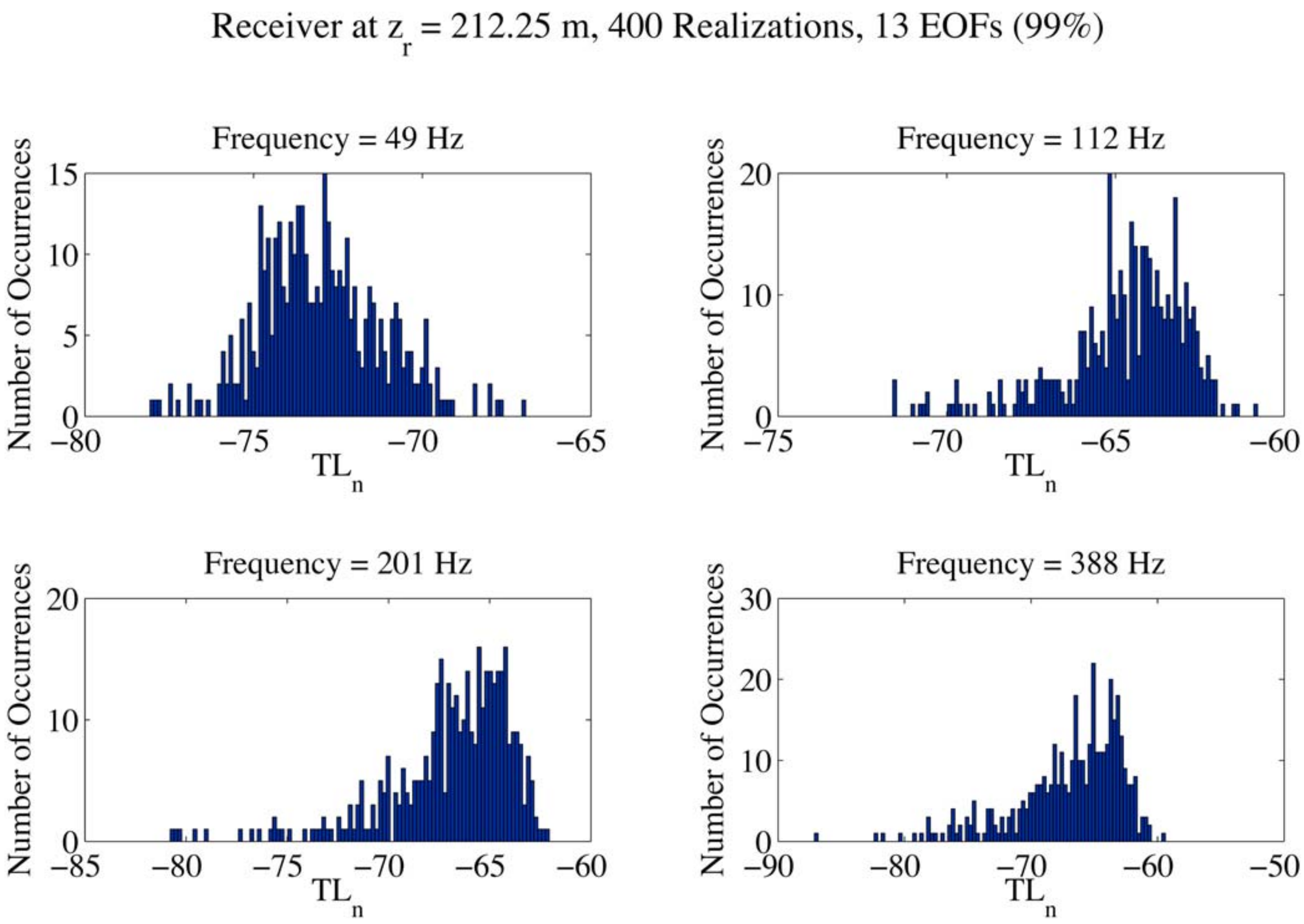
SSP Ensemble



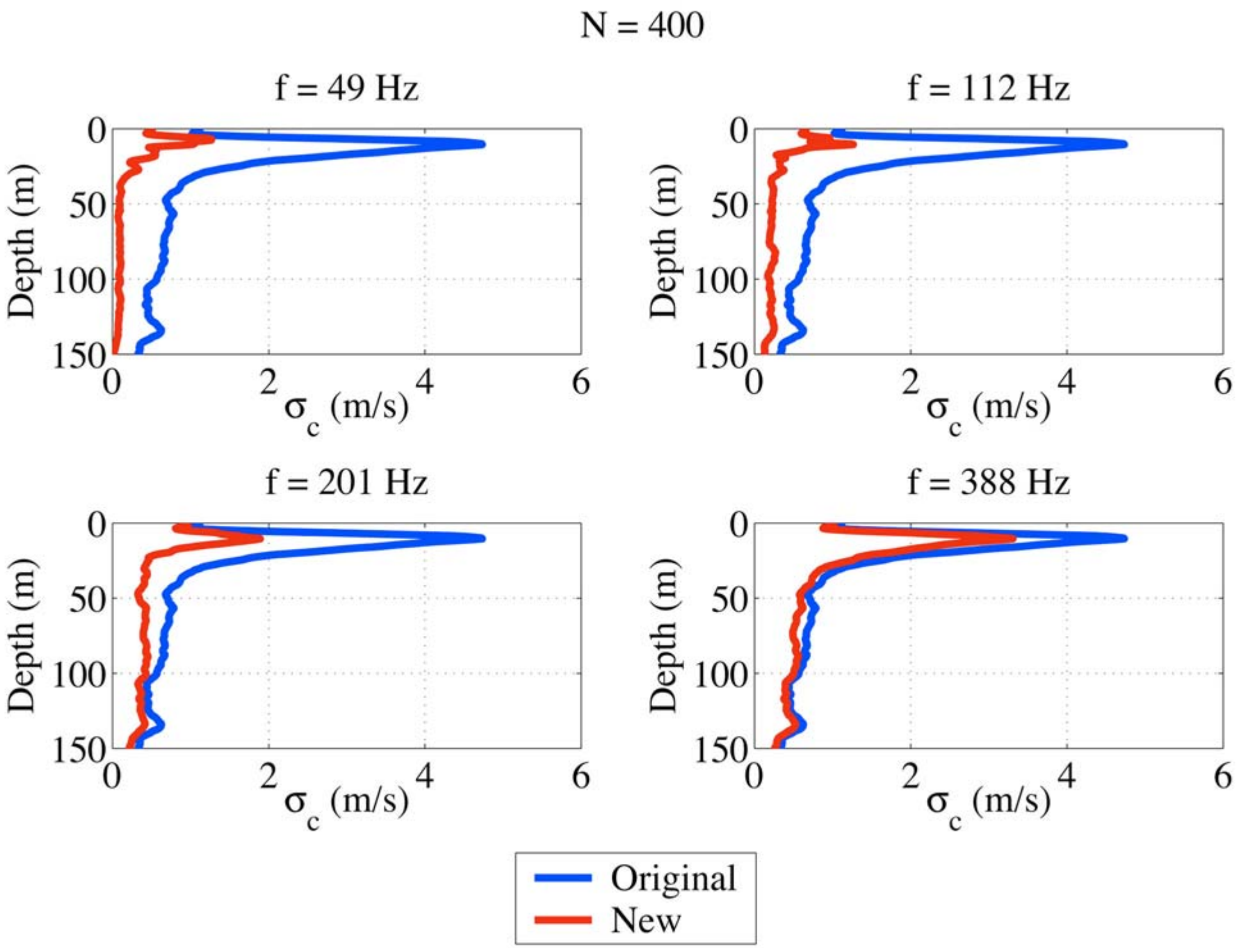
TL for SSP Ensemble



TL Distribution Functions



RMS Error for $\hat{c}(\underline{x})$ Given TL Data



ROMS/TOMS TL and AD Models: Tools for Generalized Stability Analysis and Data Assimilation

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(Scripps Institute of Oceanography, UCSD)

Linearization

- NL ROMS: $$\partial S_0 / \partial t = \yen\left(S_0\right)$$

- Perturbation: $$S = S_0 + s$$

$$\partial S_0 / \partial t + \partial s / \partial t = \square\left(S_0\right) + \partial \square / \partial s \,|_{S_0}\; s$$

Linearization, Continued

- NL ROMS: $$\partial S_0 / \partial t = \text{¥}\left(S_0\right)$$
- RP ROMS: $$\partial s / \partial t = \left(\partial \square / \partial S\right)\big|_{S_0} s \equiv A s$$

$$s(t) = R(0,t)s(0)$$ (RPM)

- AD ROMS: $$-\partial s / \partial t = A^T s$$ (ADM)

$$s^\dagger(0) = R^T(t,0)s^\dagger(t)$$

Fast-growing Modes for Ensembles

- Singular vectors: $R^T(t,0)XR(0,t)$
- Eigenmodes of $R(0,t)$ and $R^T(t,0)$

Extend to (Acoustic) Observables

- Transform from state S to data d(t):

$$d(t) = H(t)s(t) \qquad d(t) = H(t)R(0,t)s(0)$$

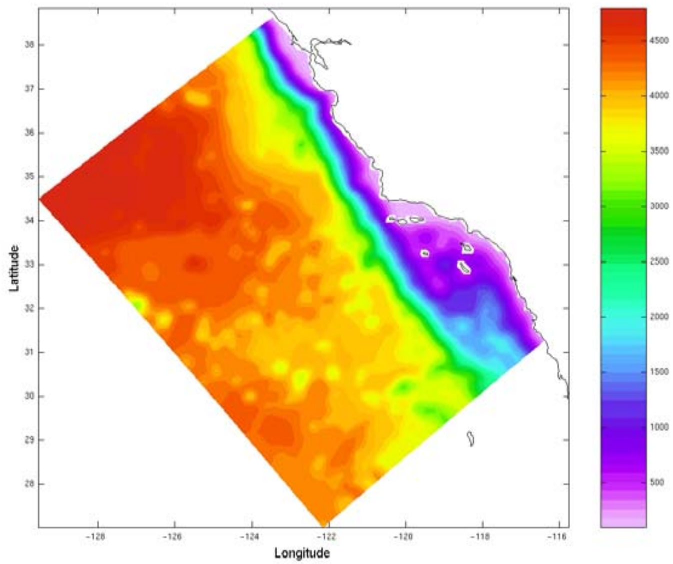
- Singular vectors: $R^T(t,0)H^T(t)XH(t)R(0,t)$

- Eigenmodes of $H(t)R(0,t)$ and

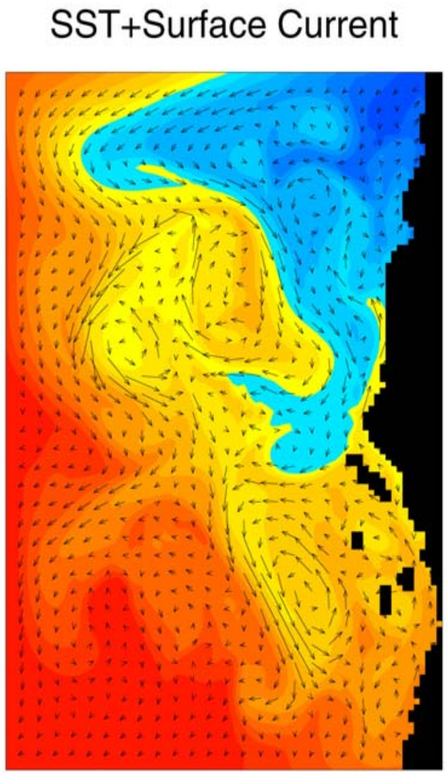
$$R^T(t,0)H^T(t)$$

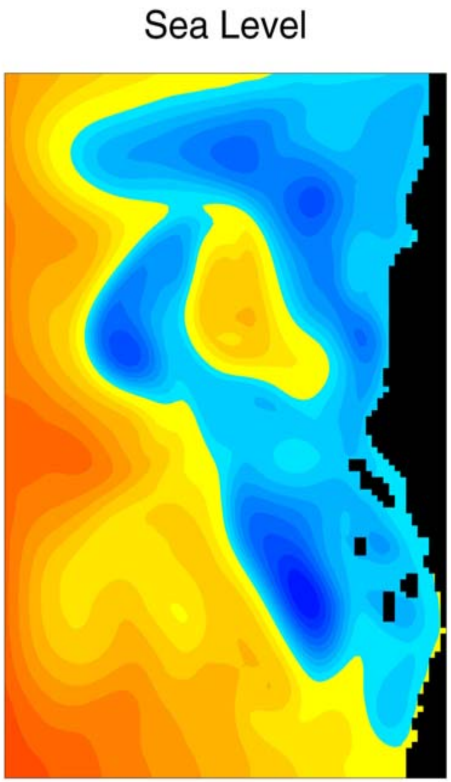
Southern California Bight (SCB)

- 1200km x 1000km
- 10km resolution
- 20 levels
- Di Lorenzo et al. (2003)

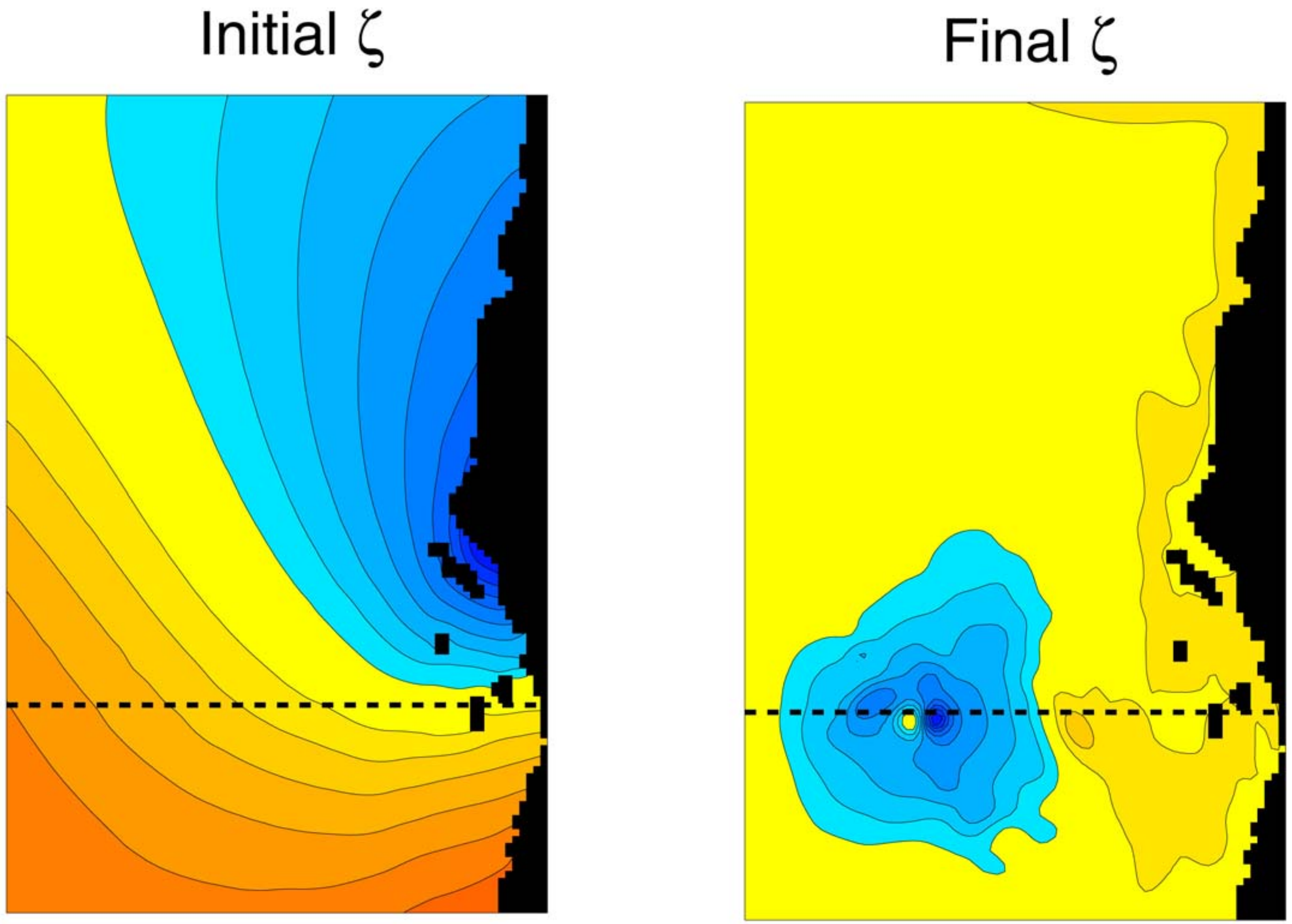


SCB Examples

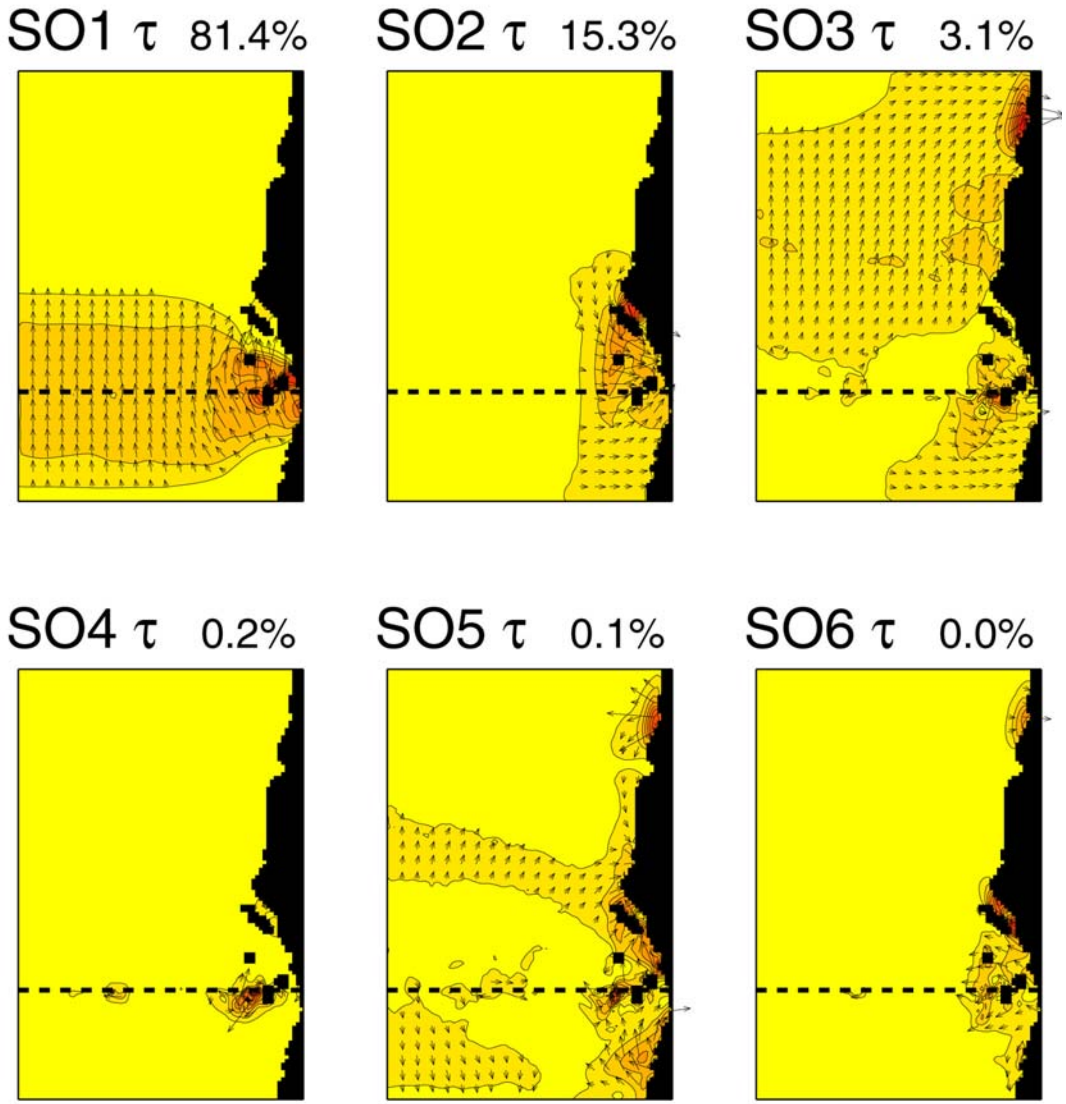




Transport Sensitivity – Initial Value



SCB Transport Sensitivity - Forcing



Generalized Stability Analysis

- Explore growth of perturbations in the ocean circulation
- Dynamics/sensitivity/stability of flow to naturally occurring perturbations
- Dynamics/sensitivity/stability due to error or uncertainties in forecast system
- Practical applications: ensemble prediction, adaptive observations, array design

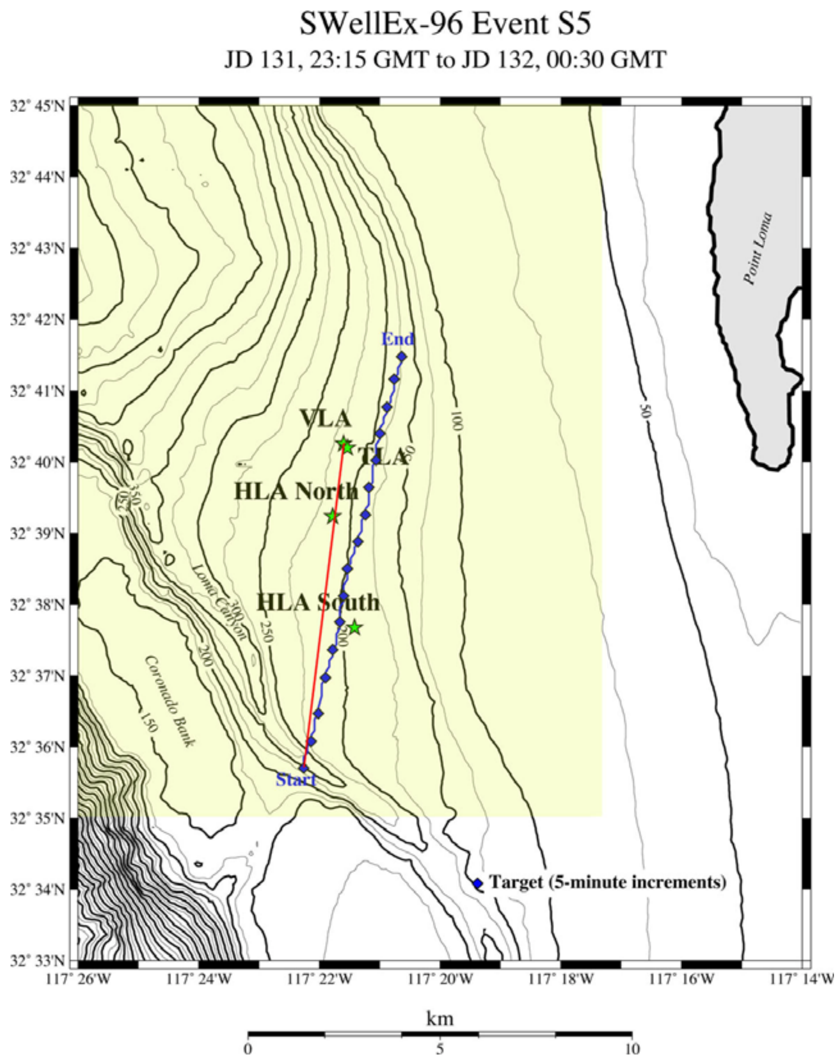
Oceanography Test Bed

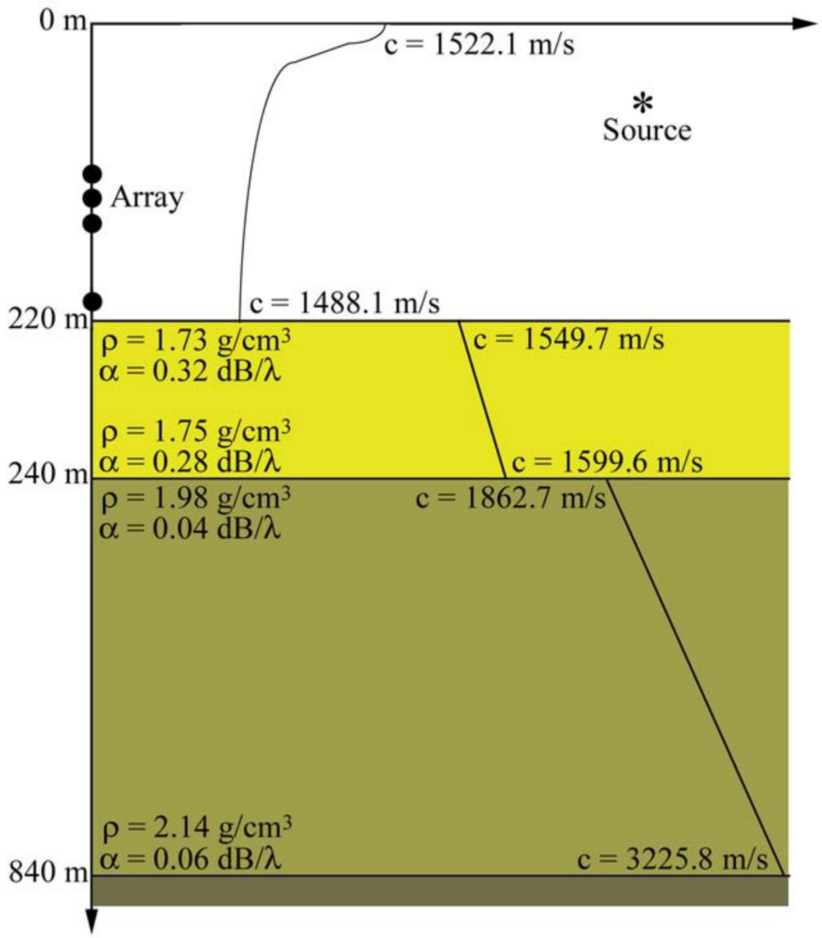
1) **High-resolution modeling**
of San Diego coastal flows

2) **Internal wave generation**
in SWellEX-96 area

E. Di Lorenzo, K. Winters, and B. Cornuelle

SWellEx-96 Experiment



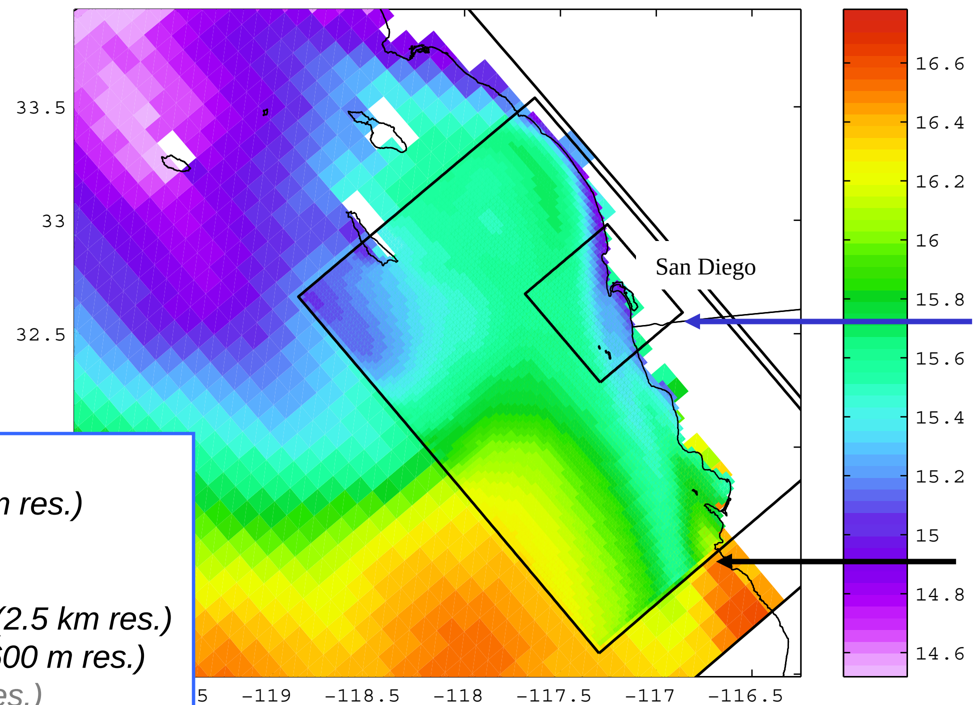


San Diego Coastal Ocean Observing System

Ocean Model Investigation

(ROMS,nested grids, daily winds, tides,climatological TS)

Model mean temperature *(Nov. 1996 to Dec. 1999)*



Nested Grid SDB2
600 m

Nested Grid SDB
2.5 km

Parent grid:
Southern California (9 km res.)

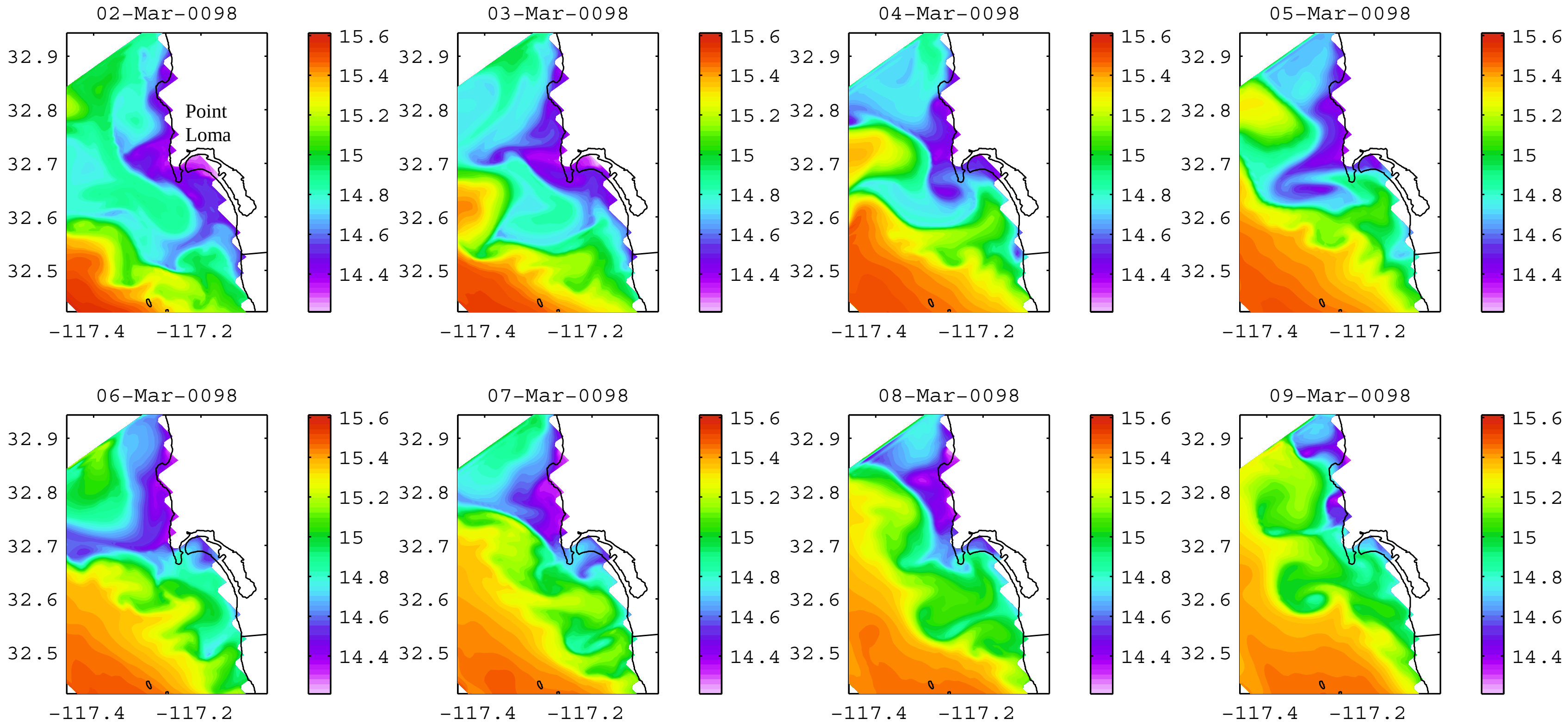
Nested grids:
San Diego Intermediate (2.5 km res.)
San Diego Bay Parent (600 m res.)
San Diego Bay (250 m res.)

The model simulation for the SCCOOS nested system goes from Nov. 1996 to Dec. 1999. It uses daily winds from a regional atmospheric model and CalCOFI data at the open boundaries of the Parent Grid. Heat Fluxes are set to climatological values.

High-resolution modeling

SST

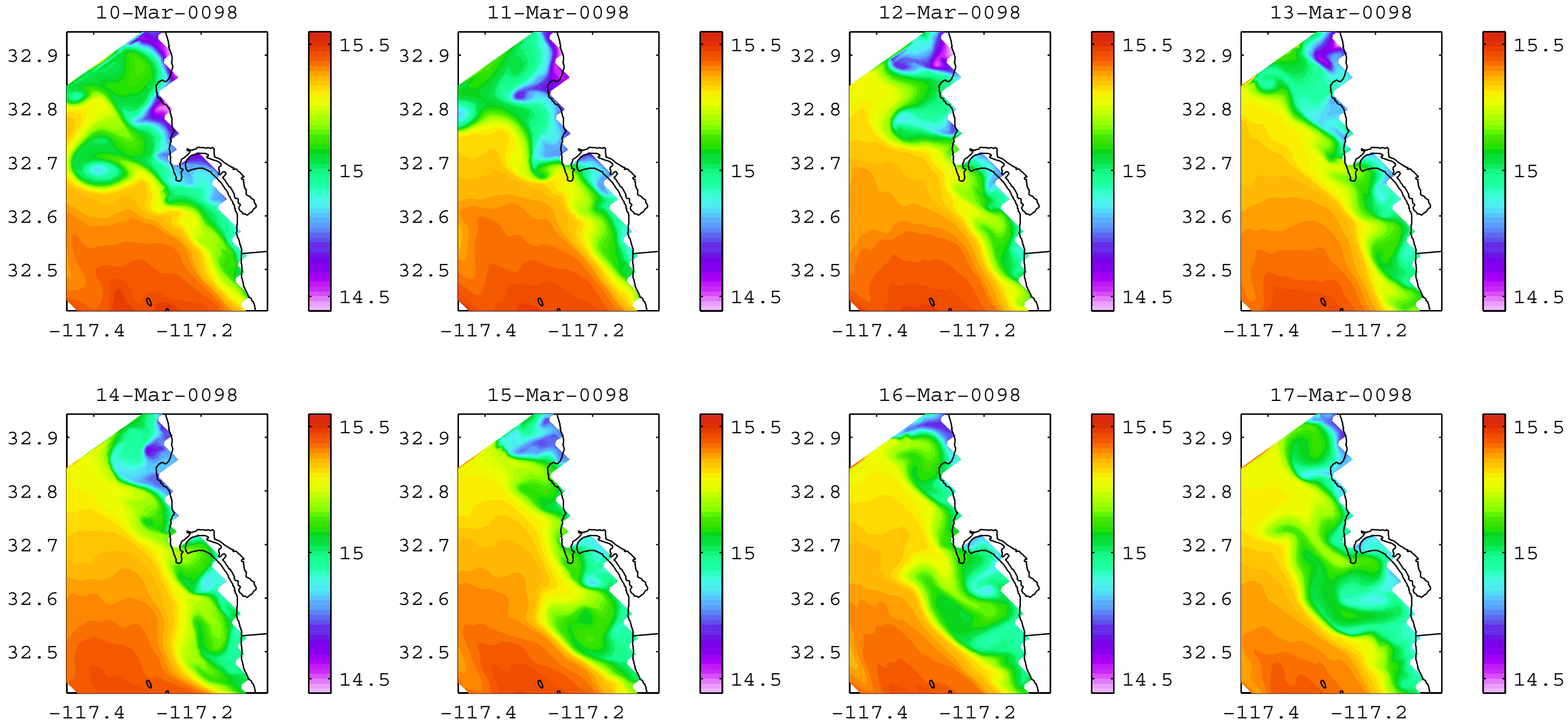
from ***March 2, 1998*** *to* ***March 17, 1998***



panel 1

High-resolution modeling

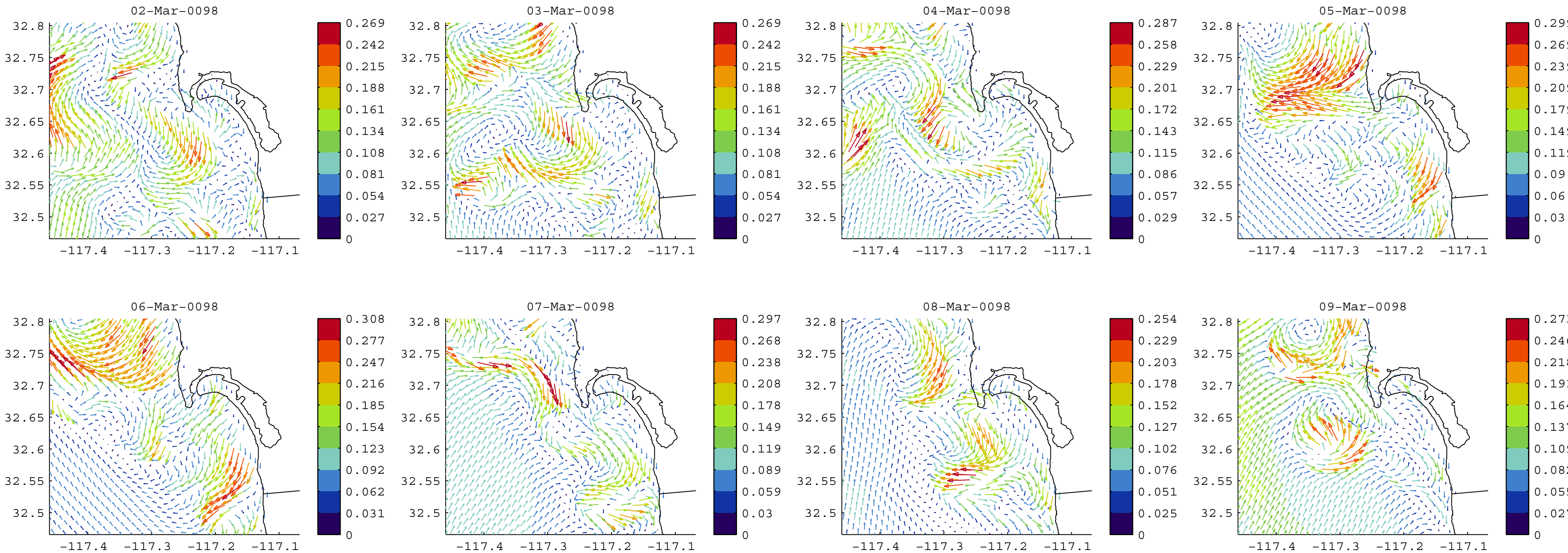
SST *from **March 2, 1998** to **March 17, 1998***



panel 2

High-resolution modeling

Currents *from* ***March 2, 1998*** *to* ***March 17, 1998***



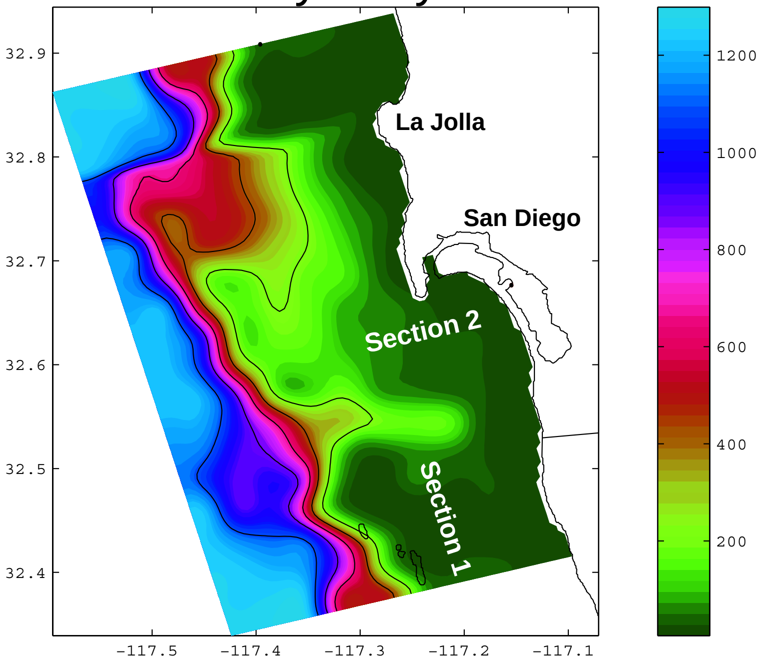
Done: 50 yr climatology run (2.5 km)

In progress: compare to HF coastal radar

simulate particle dispersion

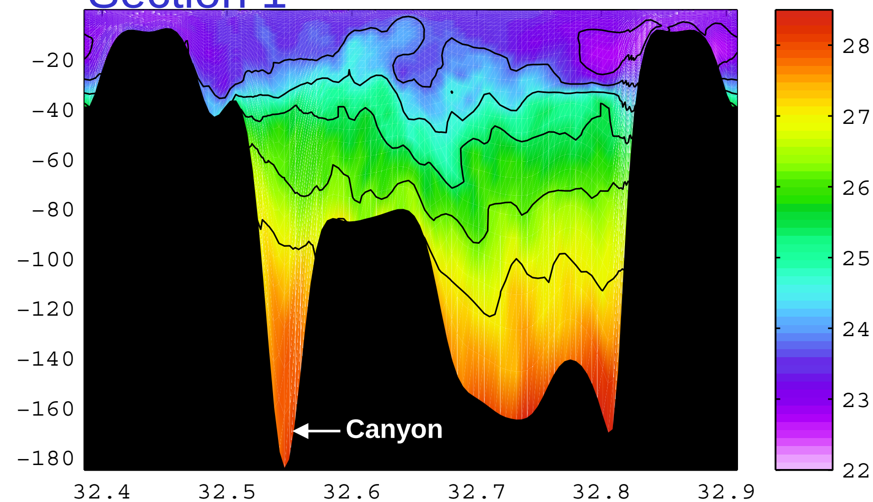
Internal Wave generation

Model Bathymetry

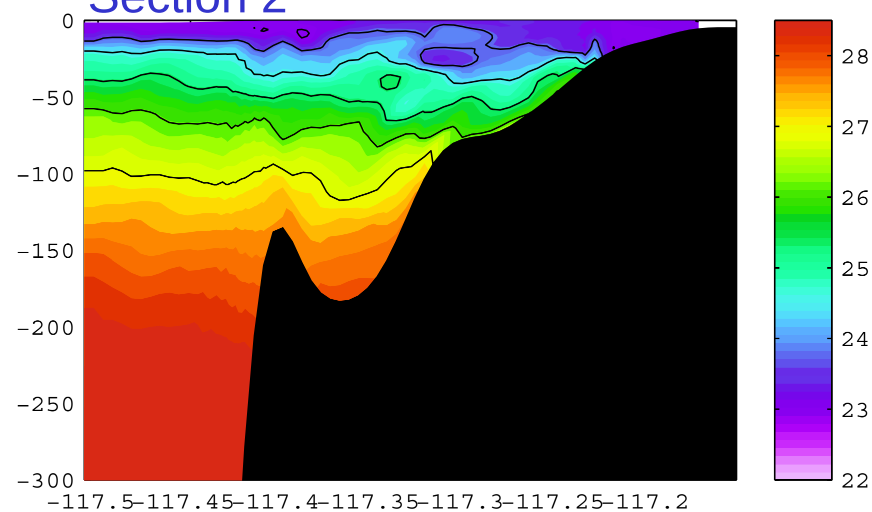


Isopycnals

Section 1



Section 2

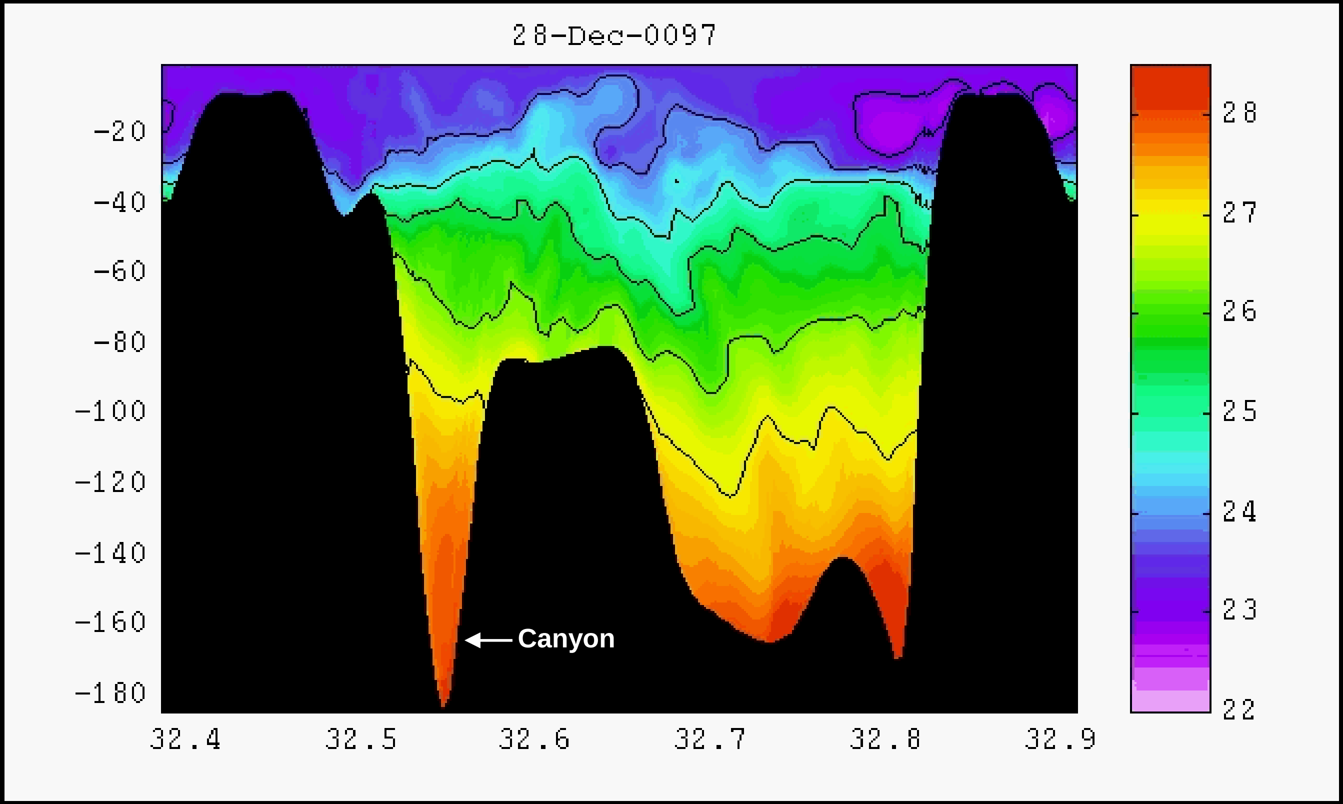


<u>Model setup:</u>

ROMS, 500 m resolution, 30 s-levels
6 hourly driving
Daily wind breeze + mean component

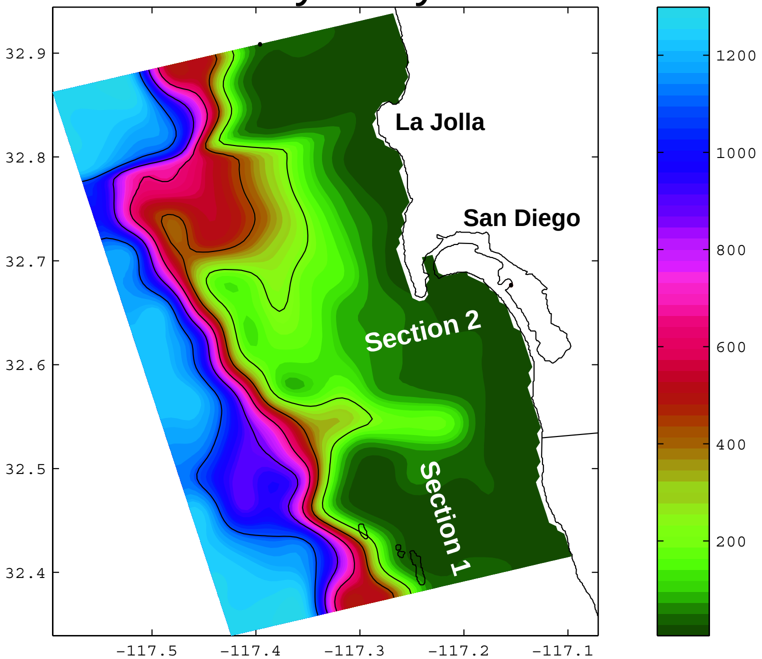
Internal Wave generation

Section 1



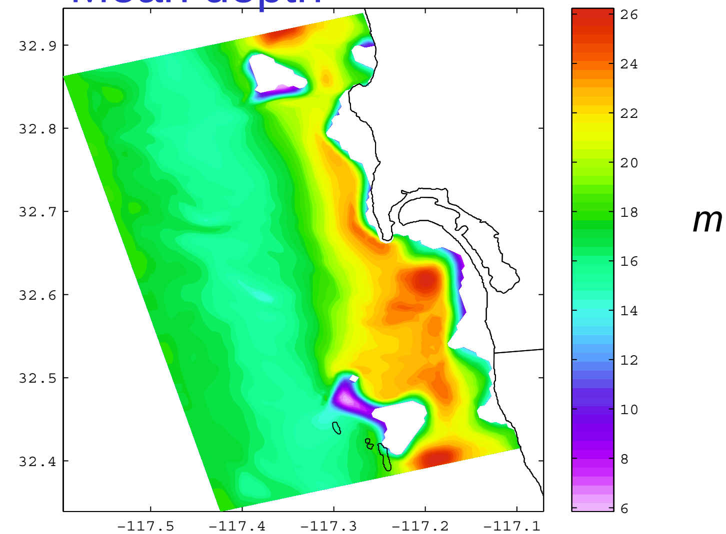
Internal Wave generation

Model Bathymetry

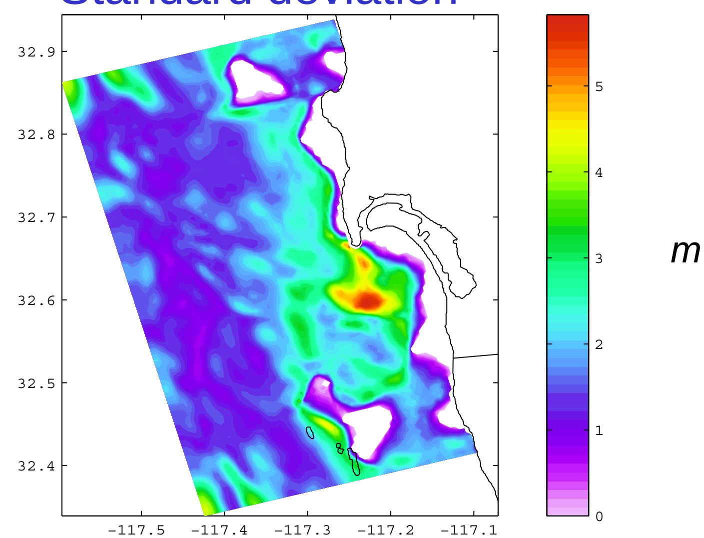


Isopycnal 23.8 statistics

Mean depth



Standard deviation



<u>Model setup:</u>

ROMS, 500 m resolution, 30 s-levels
6 hourly driving
Daily wind breeze + mean component

Internal Wave generation

horizontal view