Application of Bayes’ Theorem to the Estimation of Bottom Scattering Parameters and Reverberation Uncertainty

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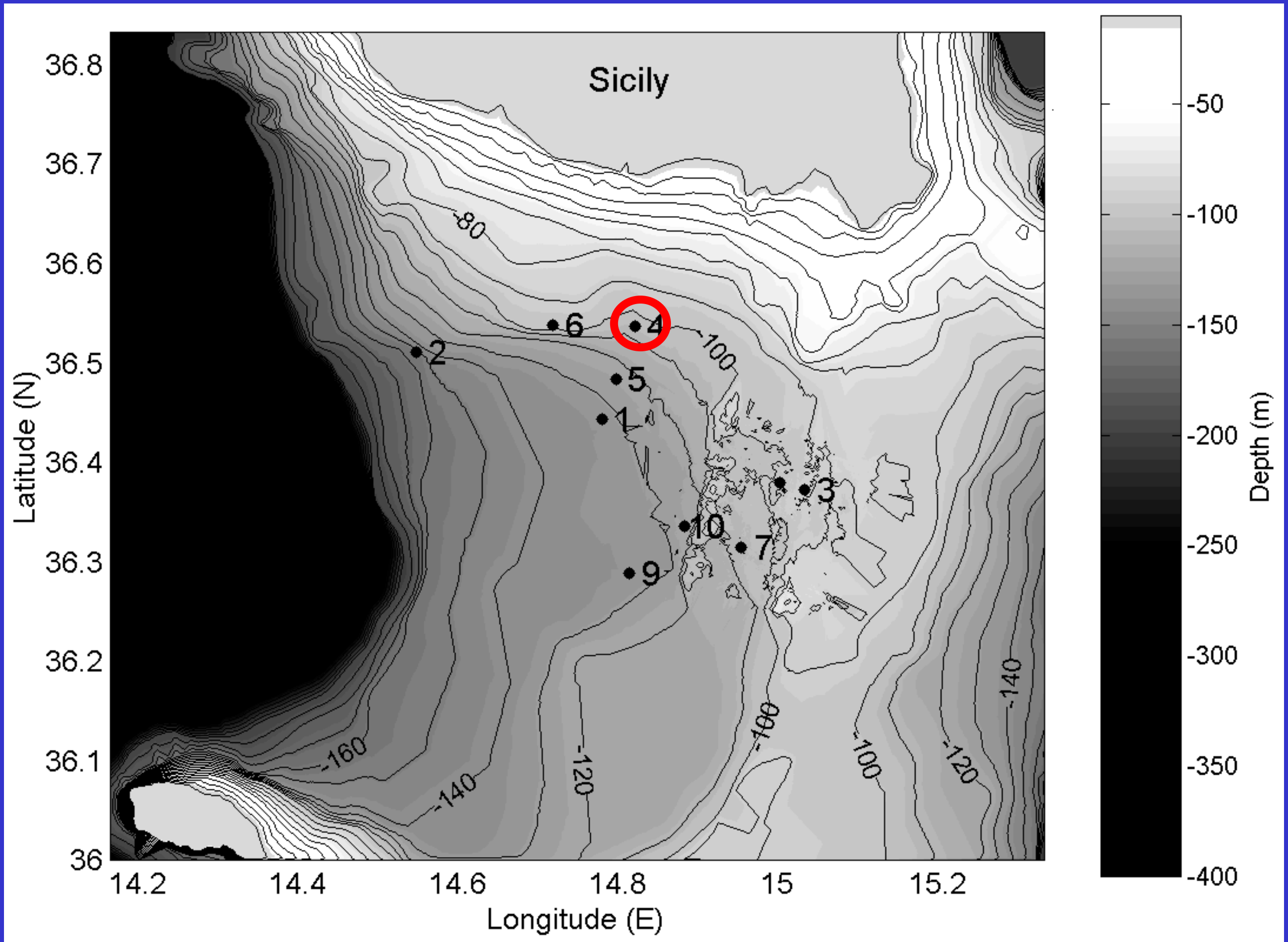
NRL

Objective

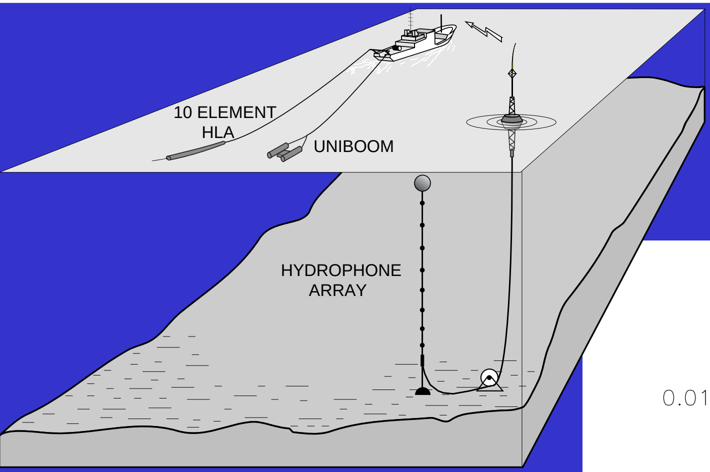
- Apply Bayes' theorem to the estimation of bottom scattering parameters using the latest generation bottom scattering models for complicated layered media
- Reduce the model parameter space by utilizing bottom forward model obtained through broadband reflection analysis
- Using a posteriori model probabilities, obtain estimates of scattering strength and corresponding reverberation uncertainty at off-experiment frequencies

Example: Reflection and Scattering Data Collected on Malta Plateau

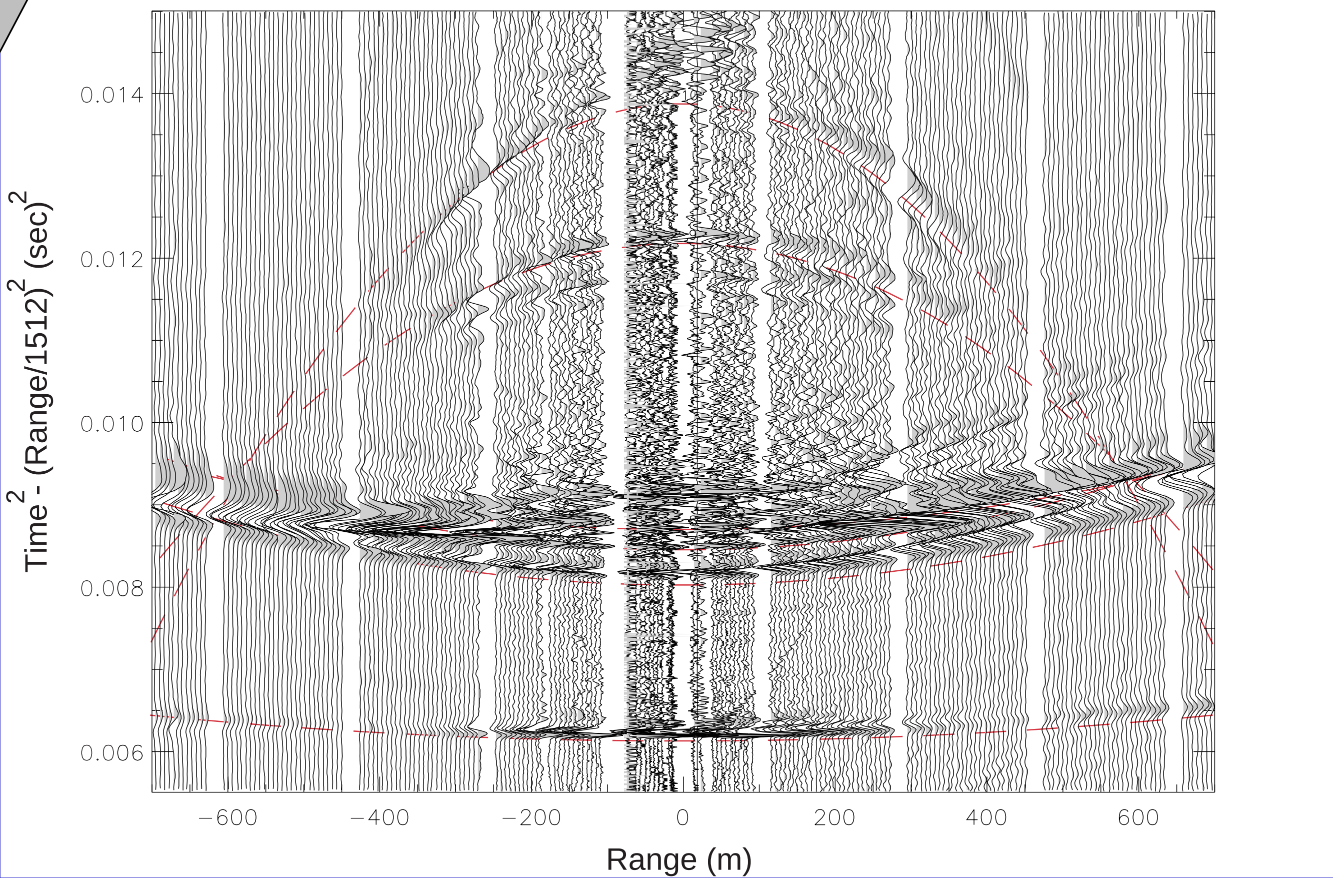
- "Site 4" Malta Plateau



Broadband Reflection Analysis Site 4

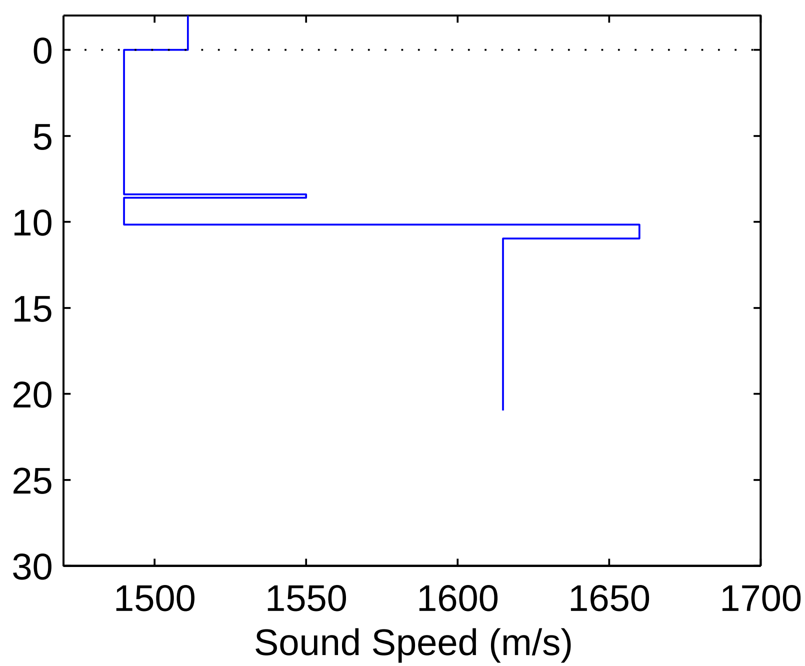


– Holland, IEEE J. of Oceanic Eng., 2002

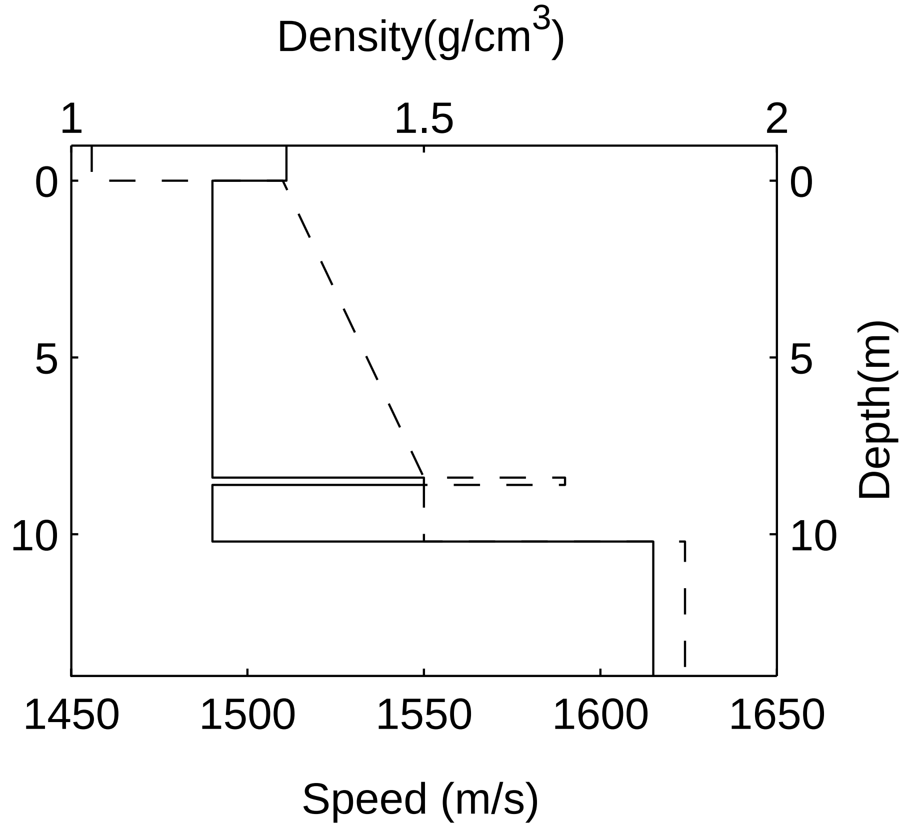


Refection Analysis for Site 4 Sediment SS and Density Profile

- Full

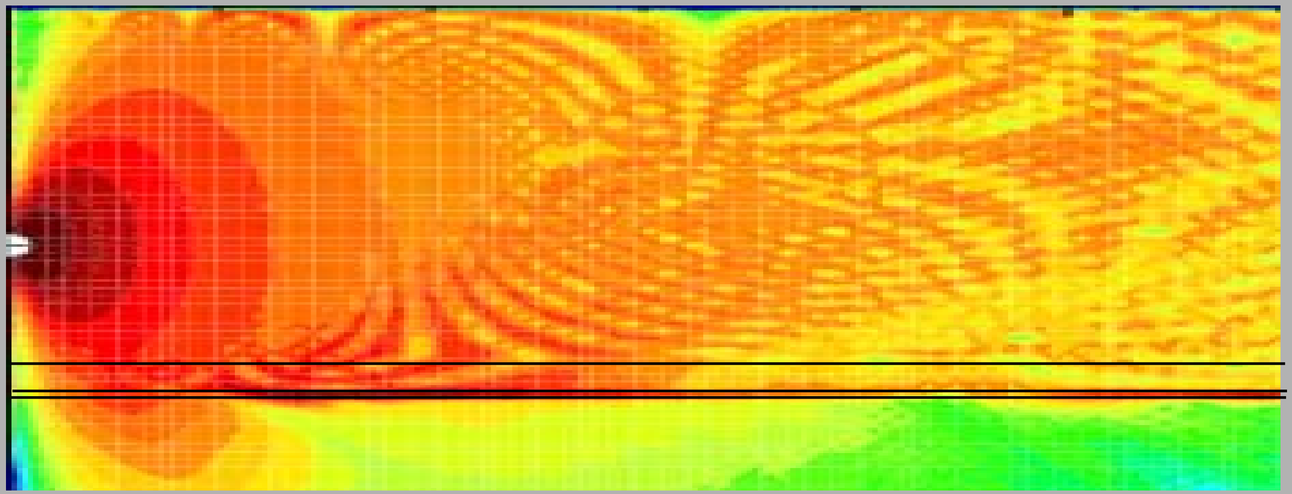


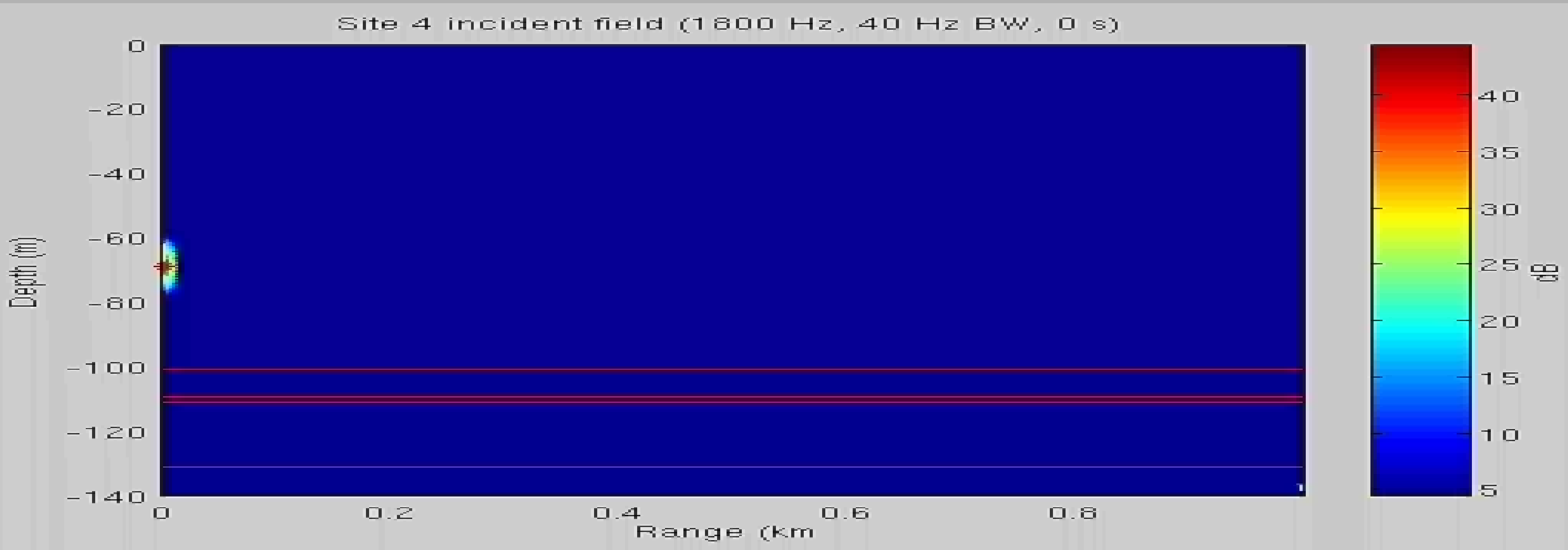
- Simplified



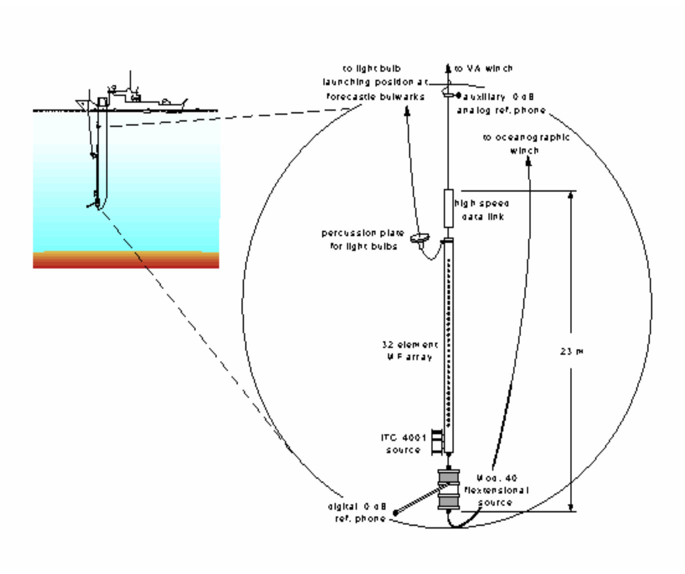
– Holland, IEEE J. of Oceanic Eng 2002.

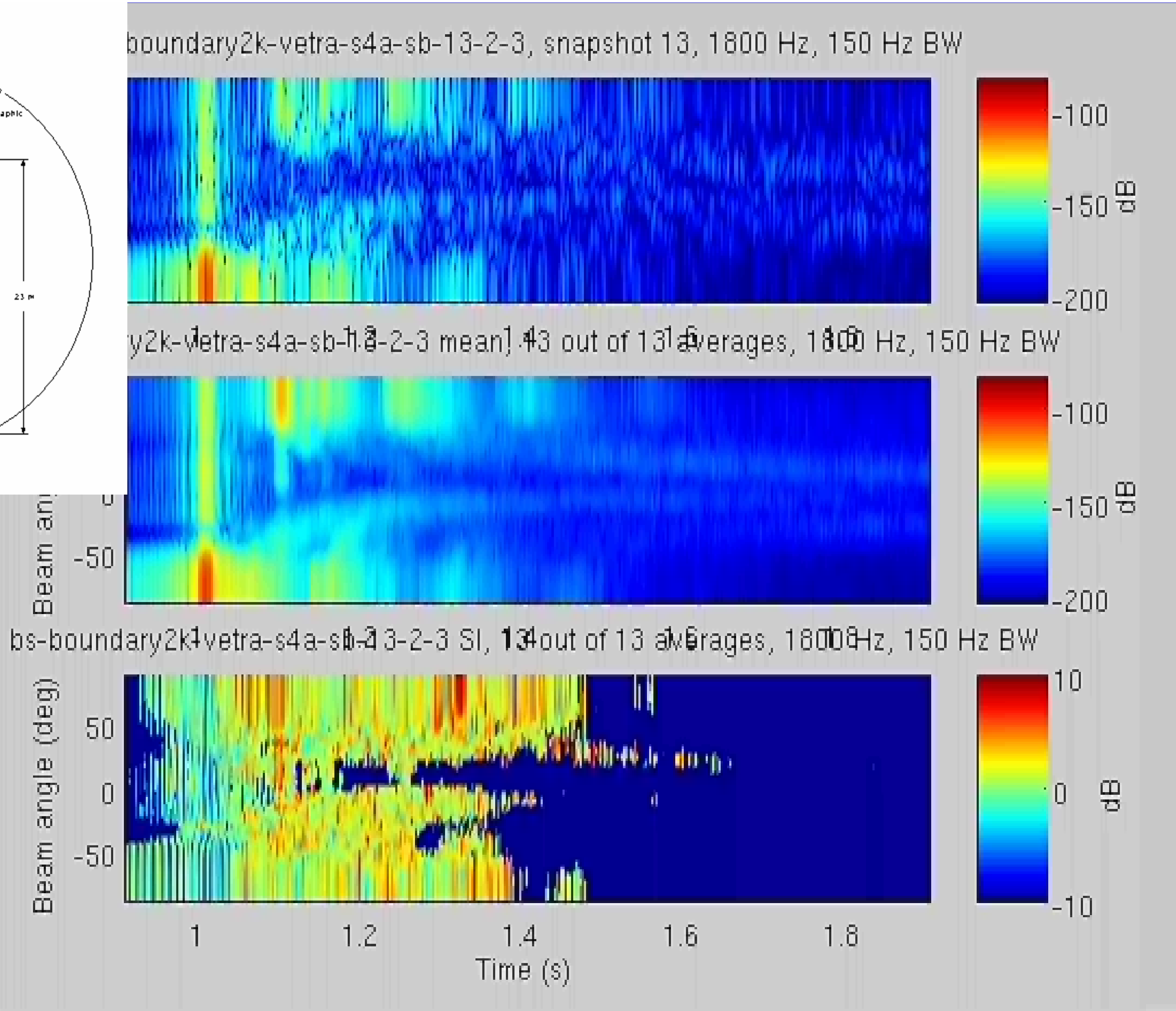
Forward Propagation Modeling to Scatterers 1800 Hz



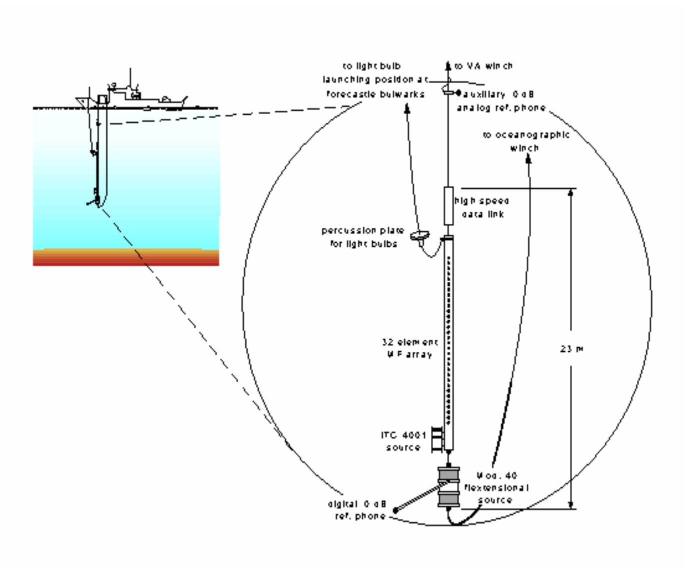


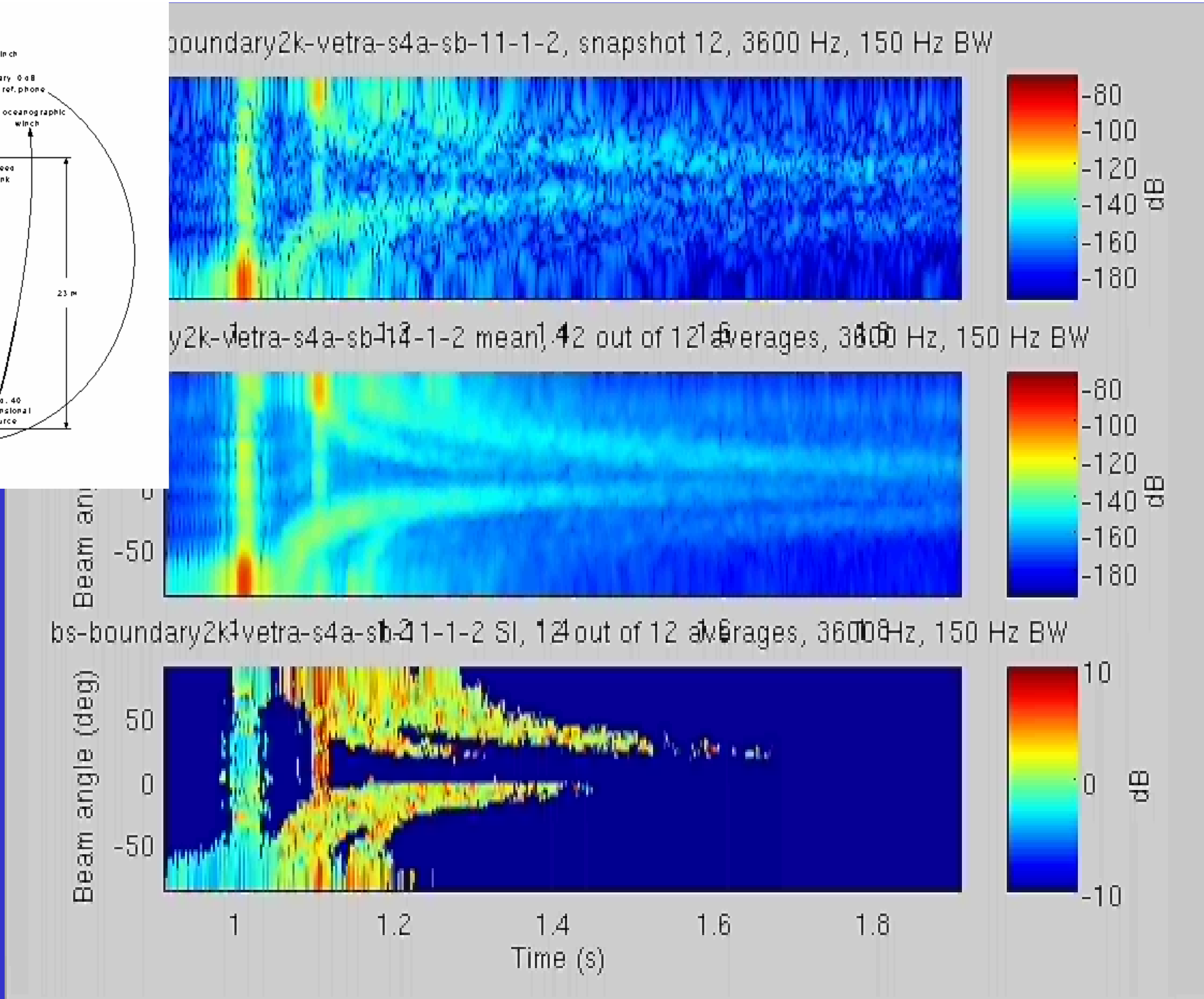
Scattering Analysis
Data Site 4 1800 Hz





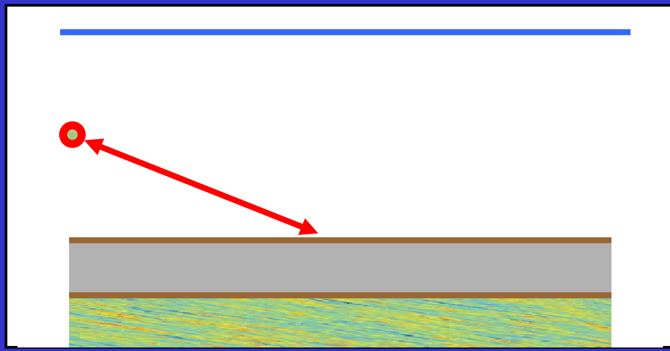
Data Site 4 3600 Hz

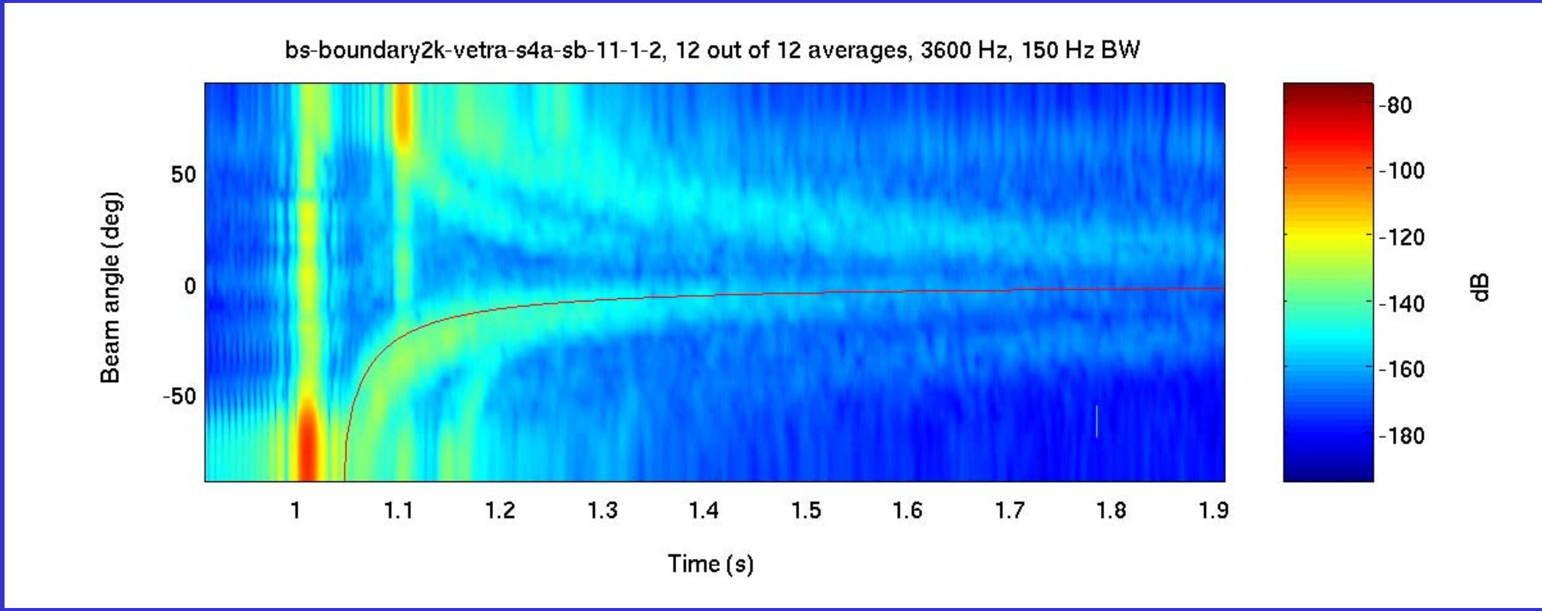




Possible Scattering Horizons

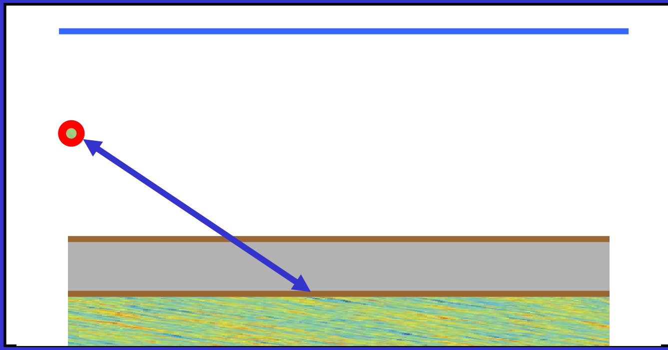
- Sediment-water interface

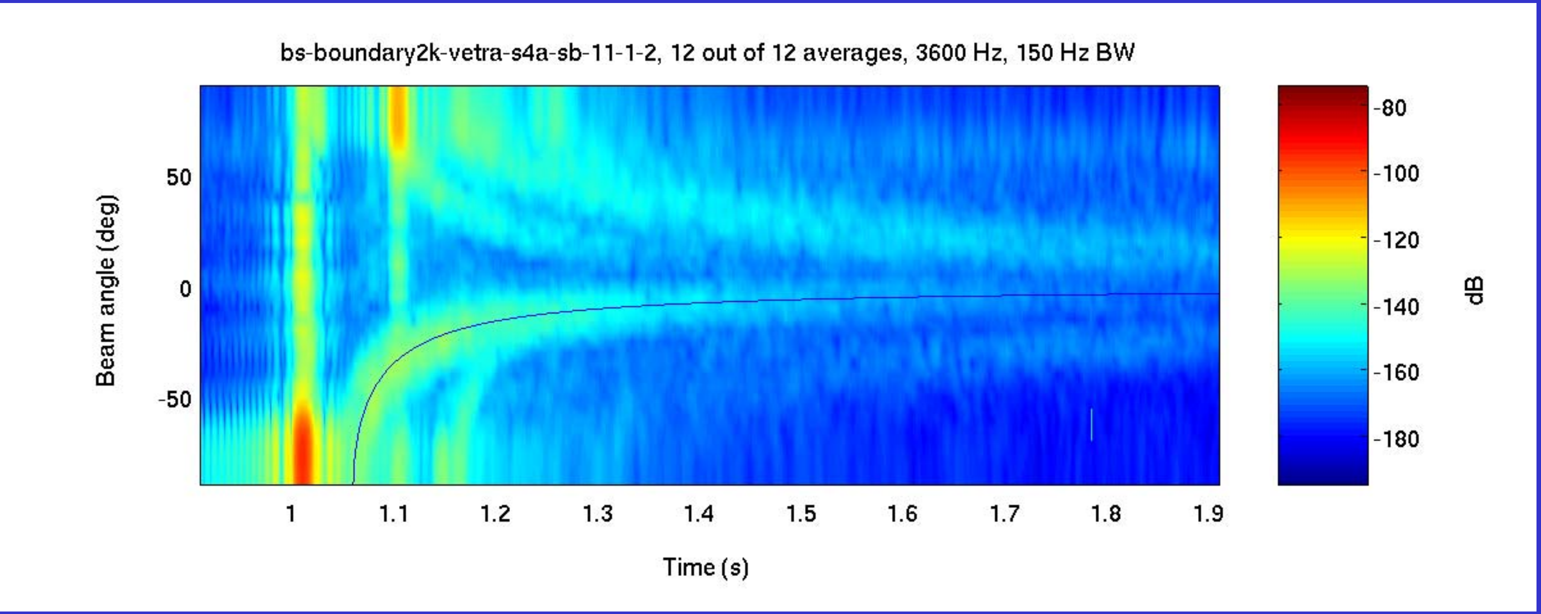




Possible Scattering Horizons

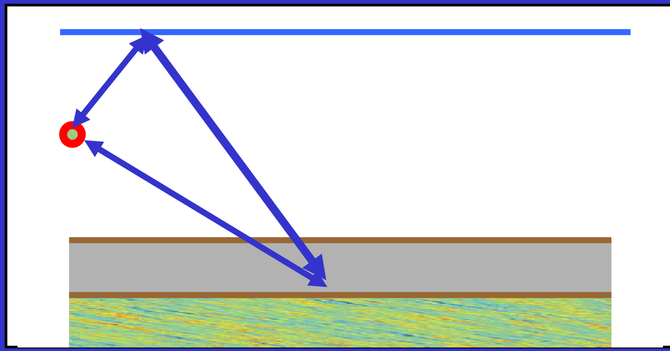
- Basement-sediment interface

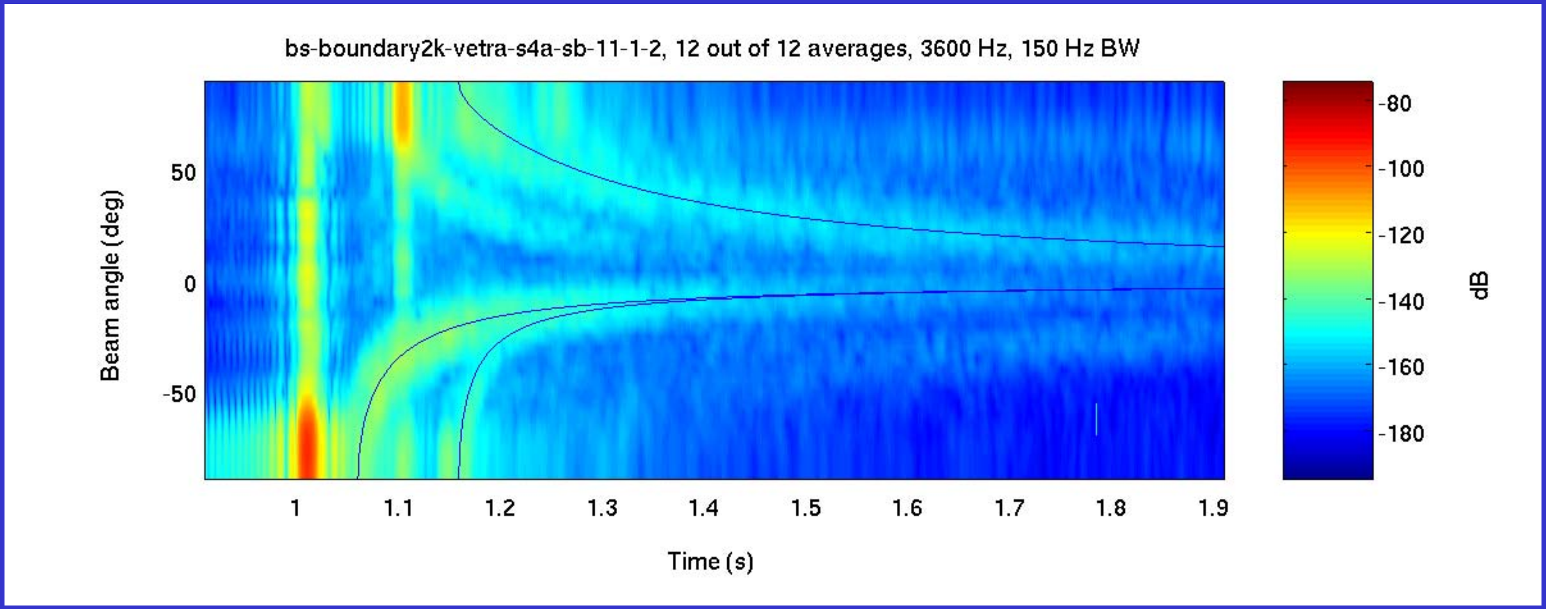




Possible Scattering Horizons

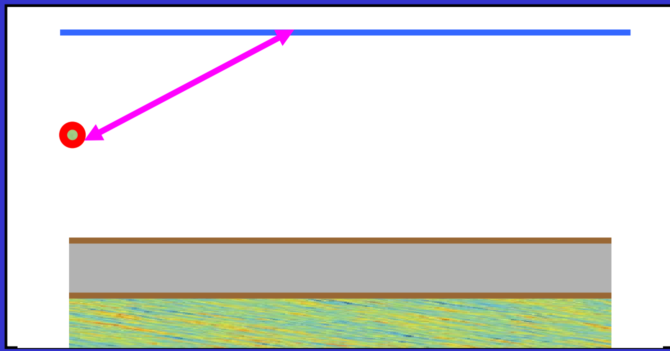
- Basement-sediment interface
  - higher multiples

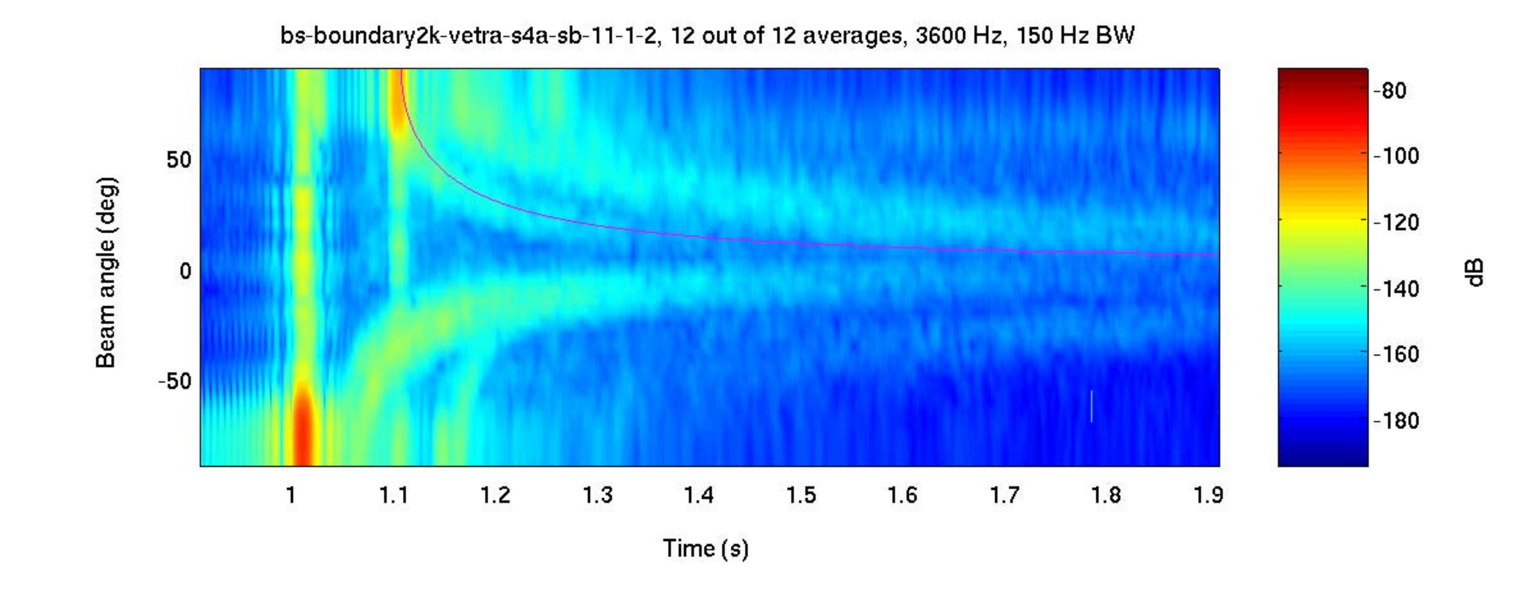




Free Surface Too!

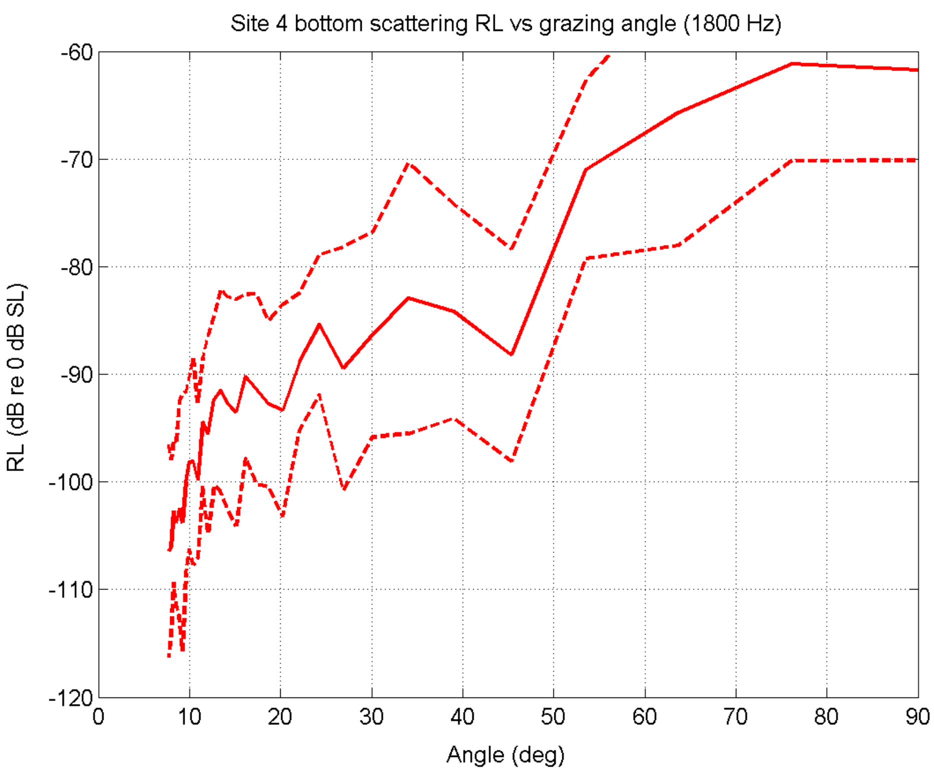
- Sea surface can be a contributor in shallow water

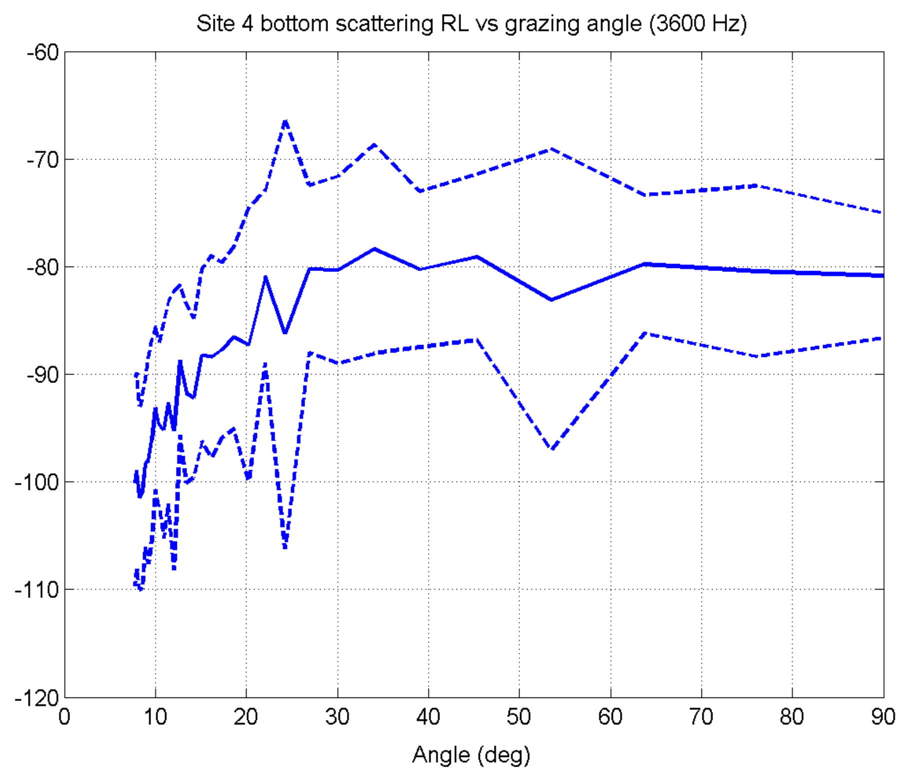




RL vs. Frequency and Grazing Angle

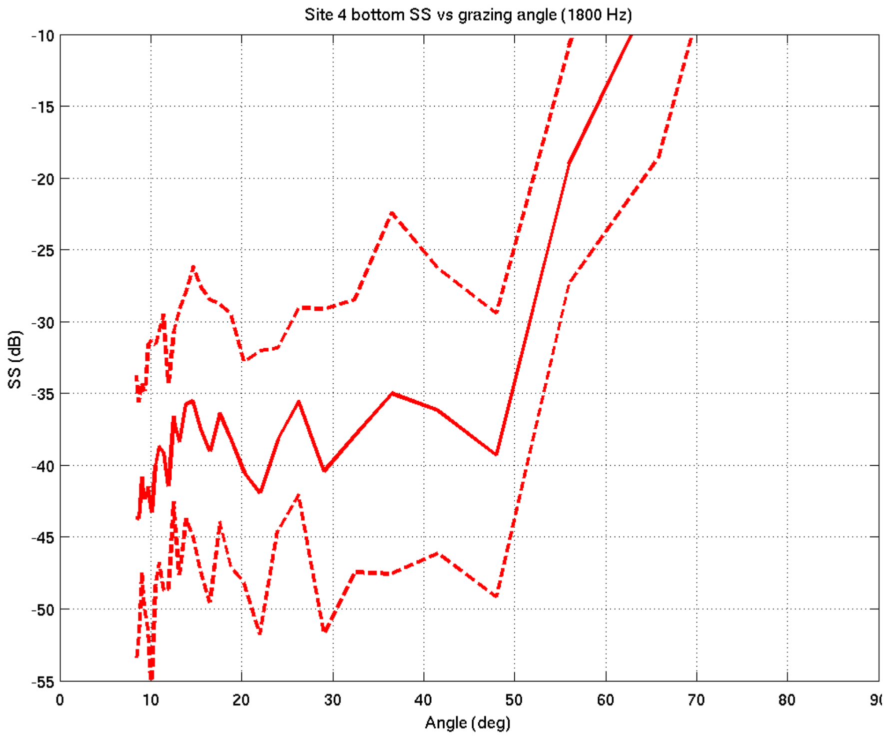
- 1800 Hz
- 3600 Hz

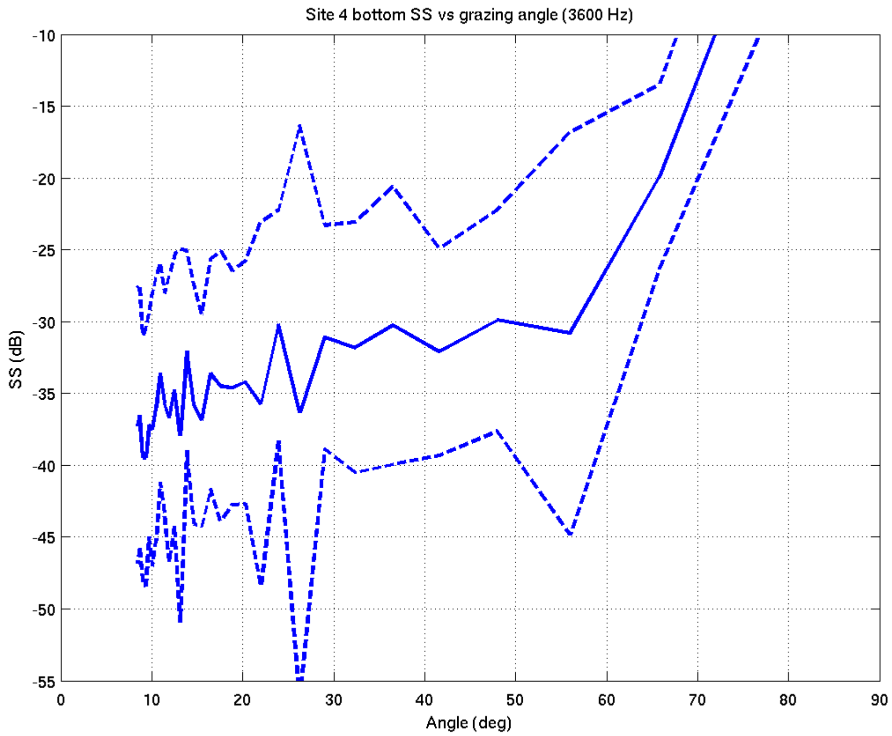




SS vs. Frequency and Grazing Angle

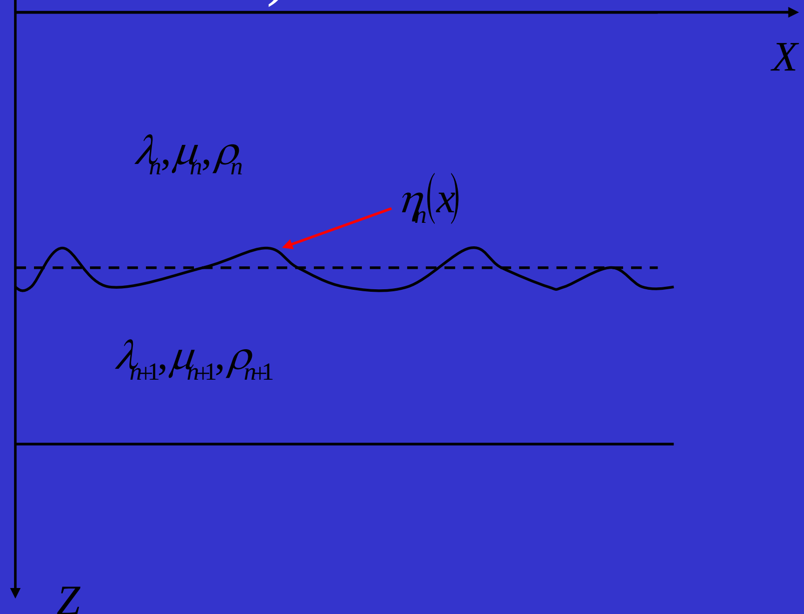
- 1800 Hz
- 3600 Hz





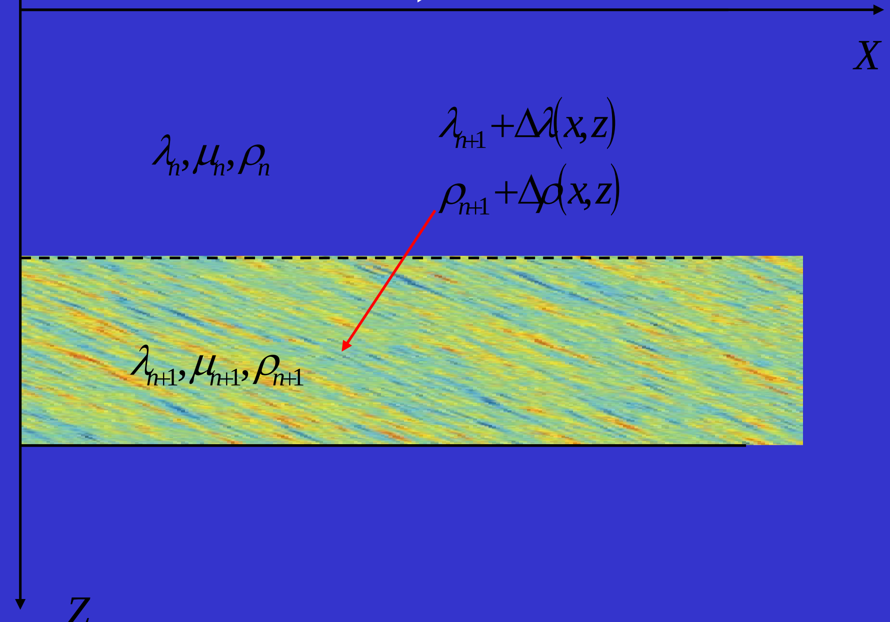
Models for Surface and Volume Scattering in OASES

- Rough surface scattering (Schmidt and Kuperman 1995)



  - 3-D
  - fluid-elastic

- Volume scattering (LePage and Schmidt 2000, 2003)



  - 2-D (3-D monostatic)
  - fluid

Volume Scattering Approach: Single Scattering MSP (Born approx)

- Inhomogeneous Helmholtz equation
  - sound speed inhomogeneity

$$\left\{\nabla^2 + k_b^2\right\} p_s(x', z') = 2\frac{\Delta c}{c}(x', z') k_b^2\, p(x', z')$$

  - density inhomogeneity

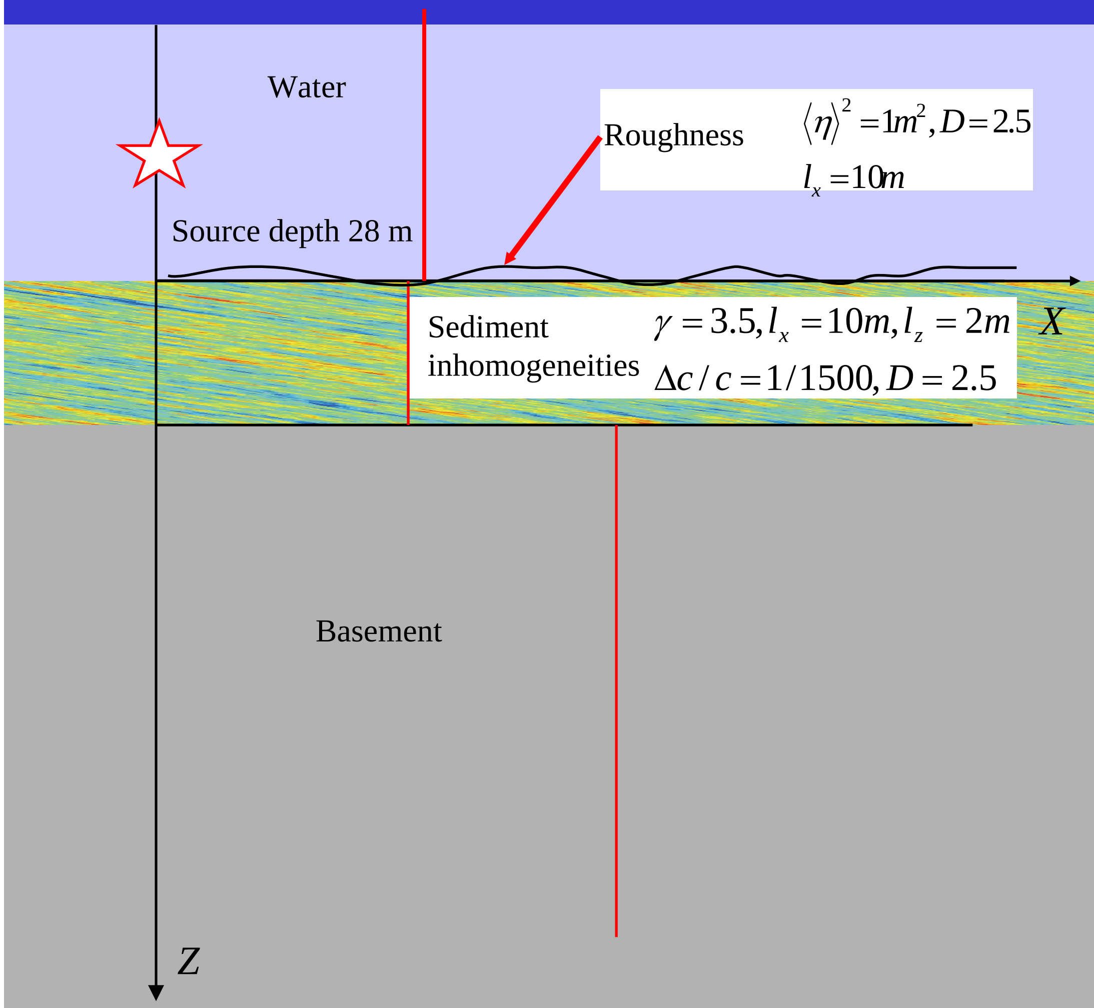
$$\left\{\nabla^2 + k_b^2\right\} p_s(x', z') = \nabla \frac{\Delta \rho}{\rho} \bullet \nabla p(x', z')$$

  - total, 100% correlated density and sound speed

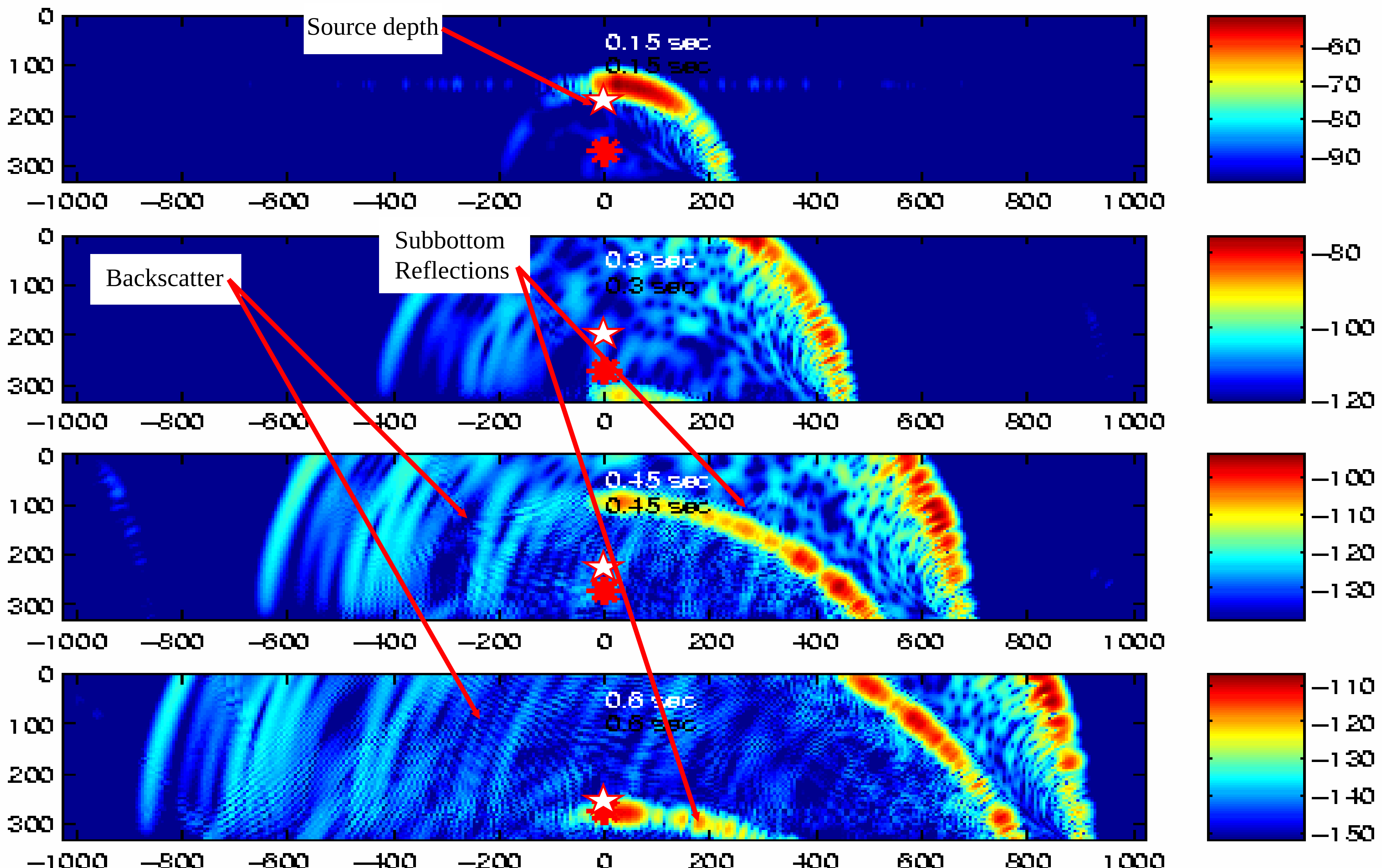
$$\left\{\nabla^2 + k_b^2\right\} p_s(x', z') = 2\frac{\Delta c}{c}(x', z') k_b^2\, p(x', z') + 2\gamma \nabla \frac{\Delta c}{c}(x', z') \bullet \nabla p(x', z')$$

Illustrative Example: Slow Sediment

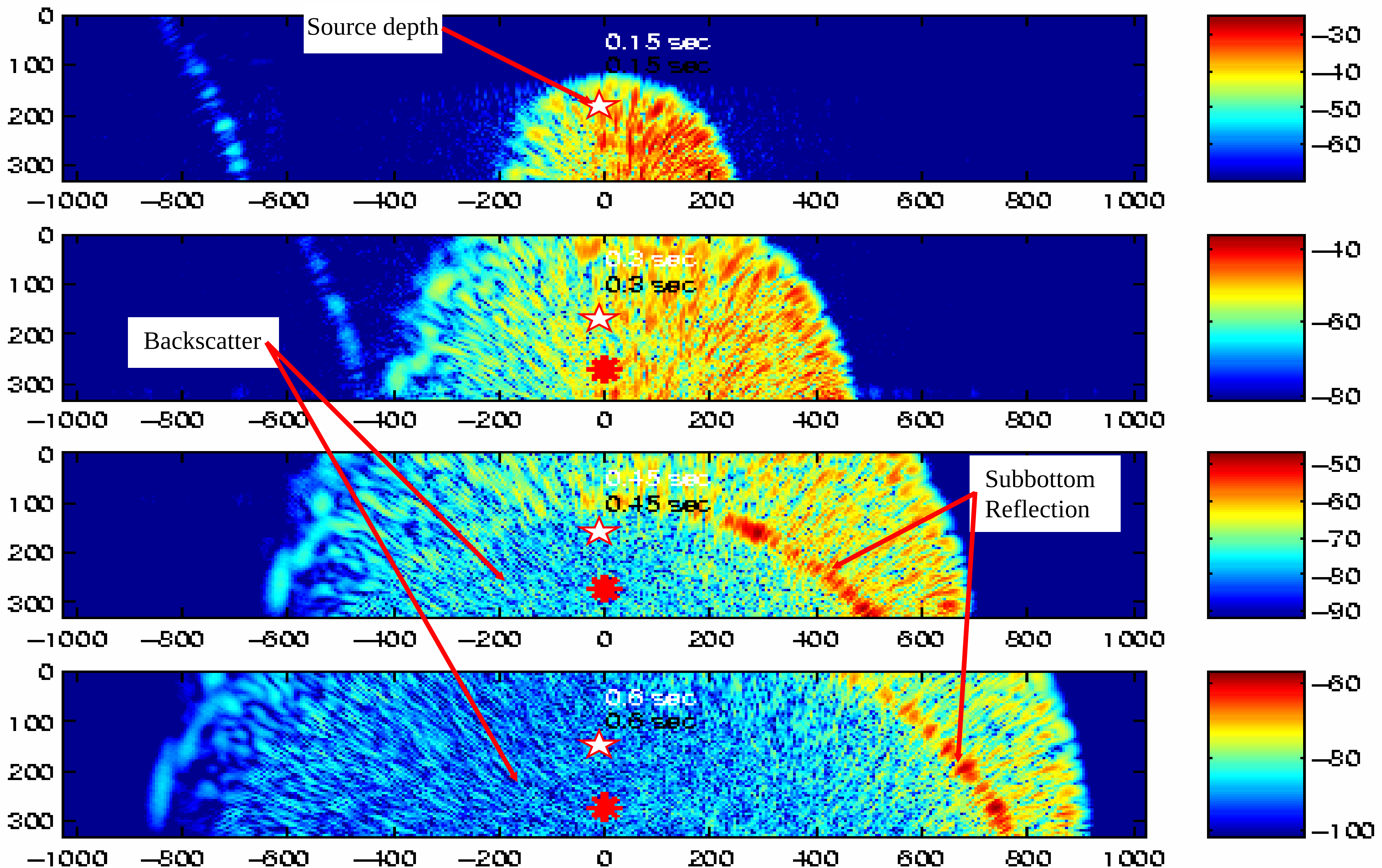
- Fluid halfspace
  - 1500 m/s
- Sediment layer
  - 1470 m/s
  - 200 m thick
  - ρ=1.65 g/cm^3
  - α=.01 dB/λ
- Basement
  - 2300 m/s
  - ρ=2.65 g/cm^3
  - α=.5 dB/λ



Slow Sediment Layer: Rough Surface Scattering 500 Hz

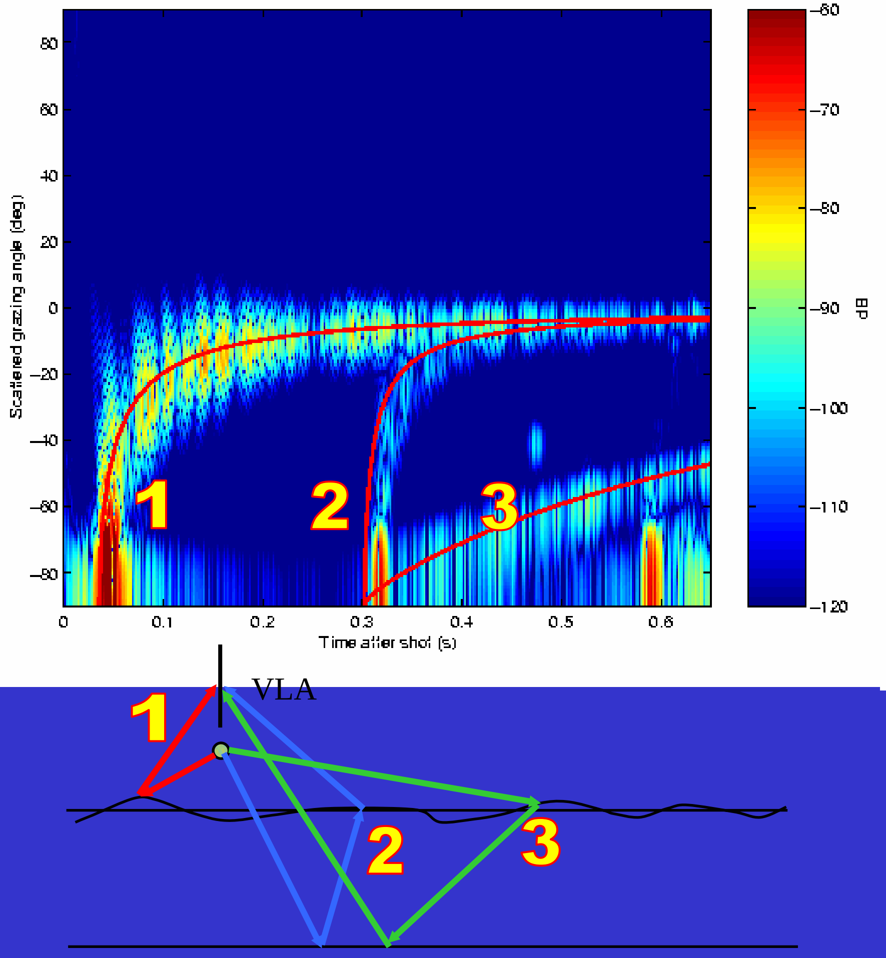


Slow Sediment Layer: Volume Scattering 500 Hz

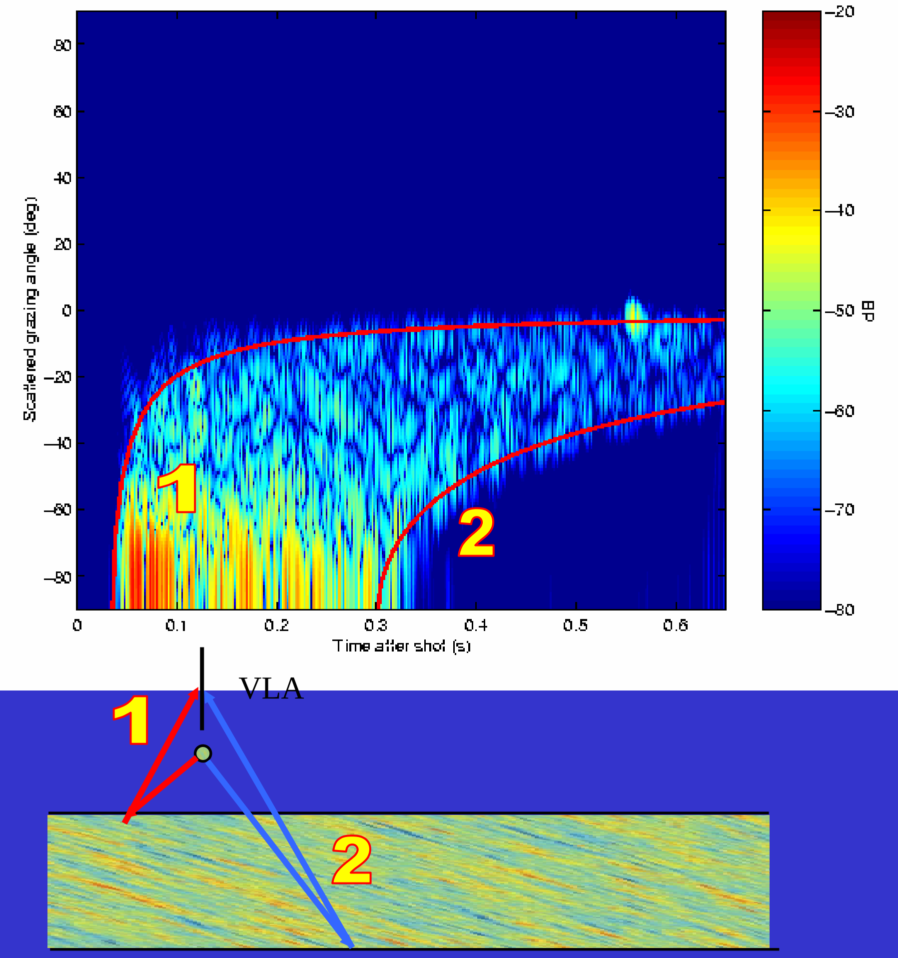


Scattered Field Measurements on a VLA from Slow Sediment Layer

- Surface scattering

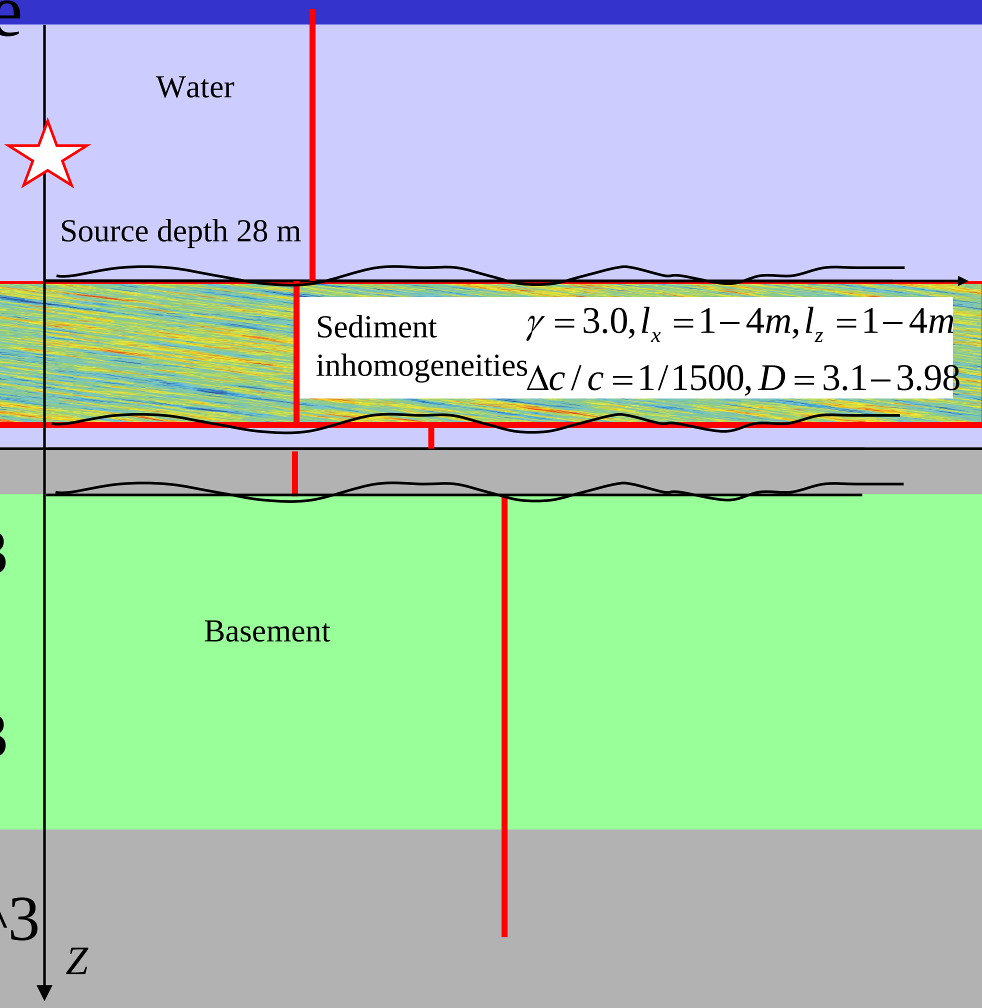


- Volume scattering

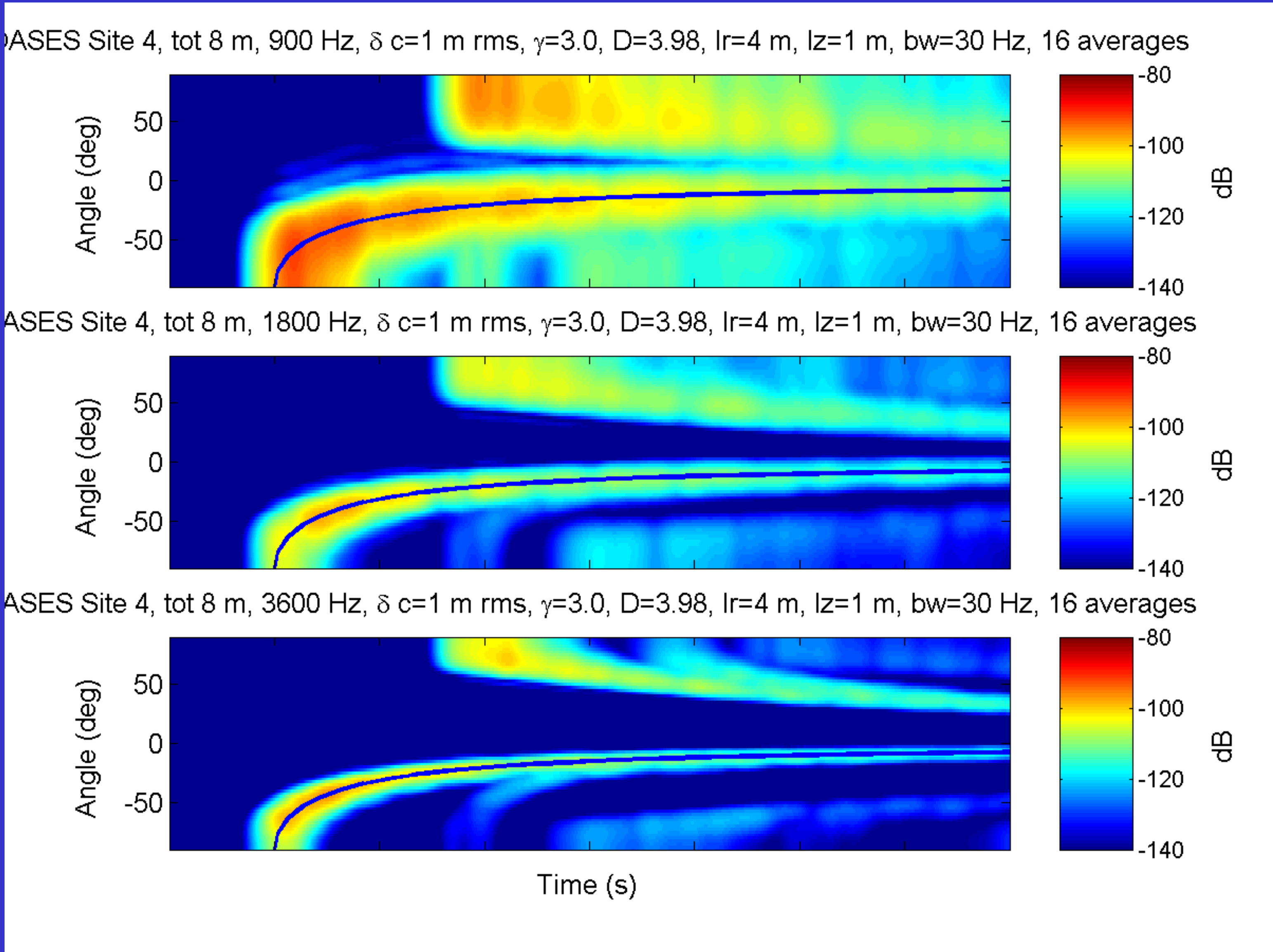


Malta Site 4 Hypothesis 1: Sediment Volume Scattering

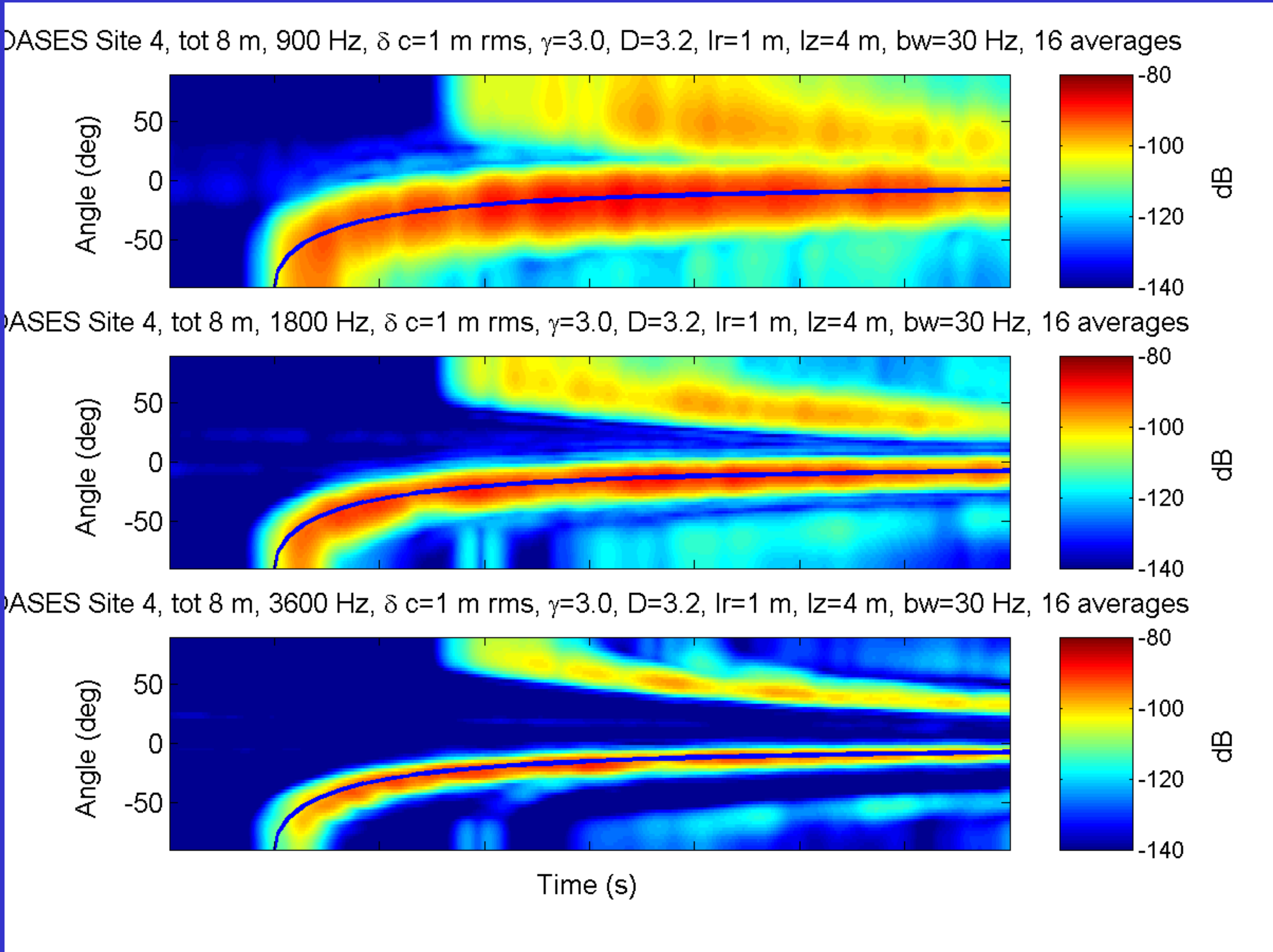
- 101 m Deep Waveguide
  - 1511 m/s
- 8.4 m Slow Sediment
  - 1480 m/s
  - ρ=1.32 g/cm^3
- 0.2 m Fast Interstices
  - 1550 m/s, ρ=1.7 g/cm^3
- 1.6 m Slow Sediment
  - 1490 m/s, ρ=1.6 g/cm^3
- Fast Basement
  - 1615 m/s, ρ=2.65 g/cm^3



Time-Angle Evolution Sediment Inhomogeneity Scattering (Best)



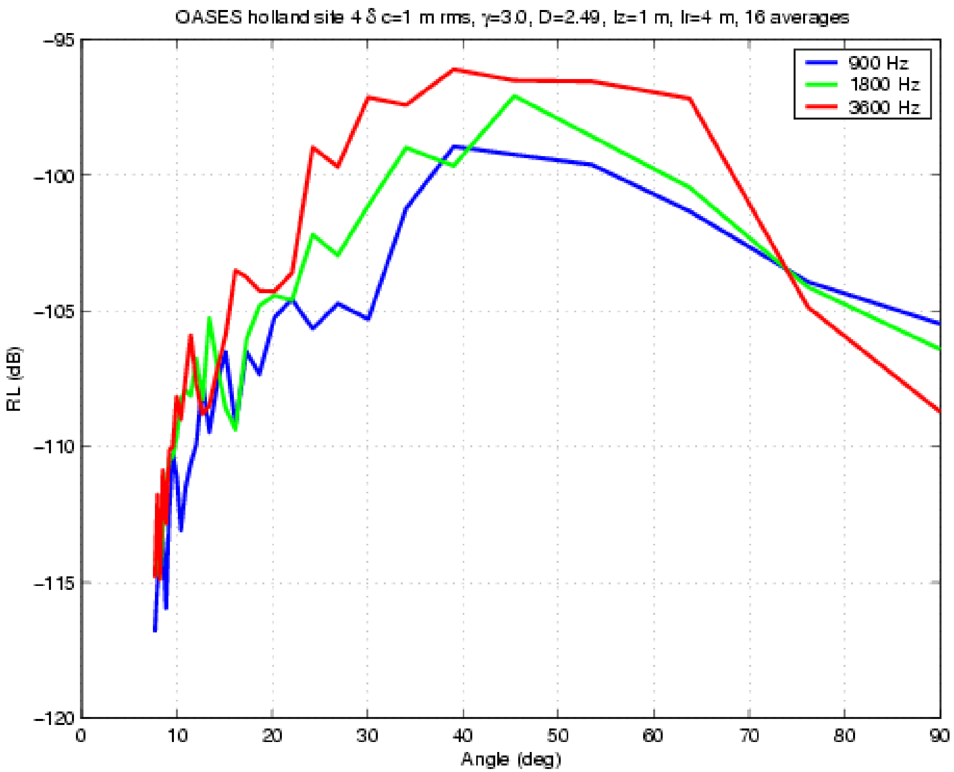
Time-Angle Evolution Sediment Inhomogeneity Scattering (Worst)

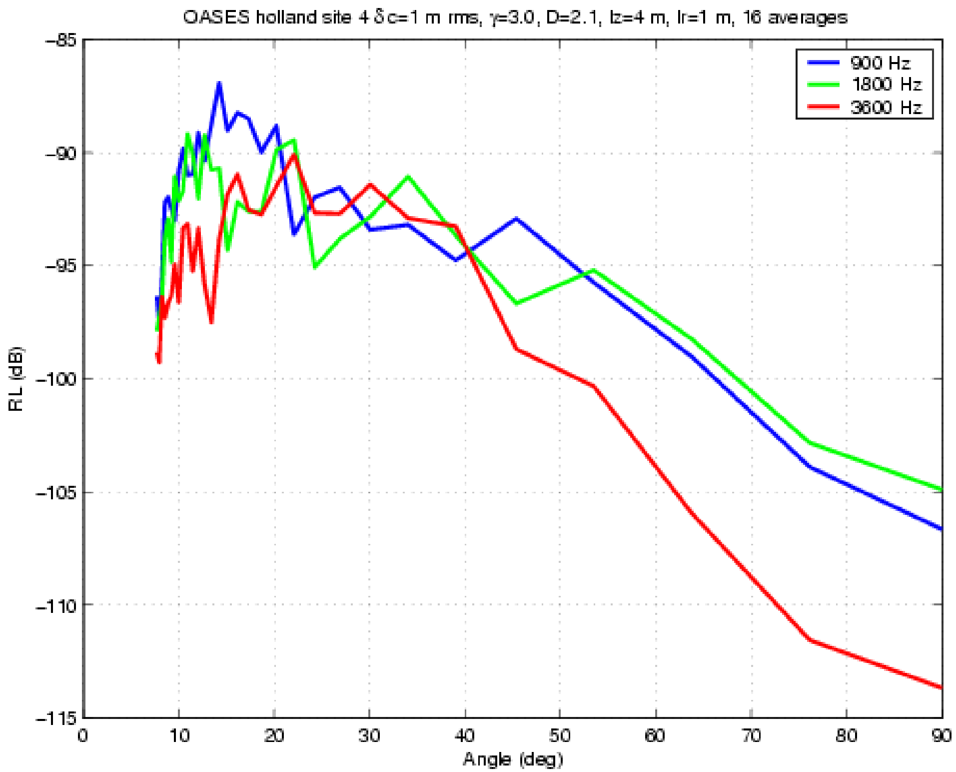


Best and Worst of Hypothesis 1

- l_z=1m, l_r=4m, D=3.98, δc_{rms}=6.5 m/s
- $P(O|m)/P(O|H_1)=1.00$

- l_z=4m, l_r=1m, D=3.1, δc_{rms}=1.4 m/s
- $P(O|m)/P(O|H_1)=0.48$

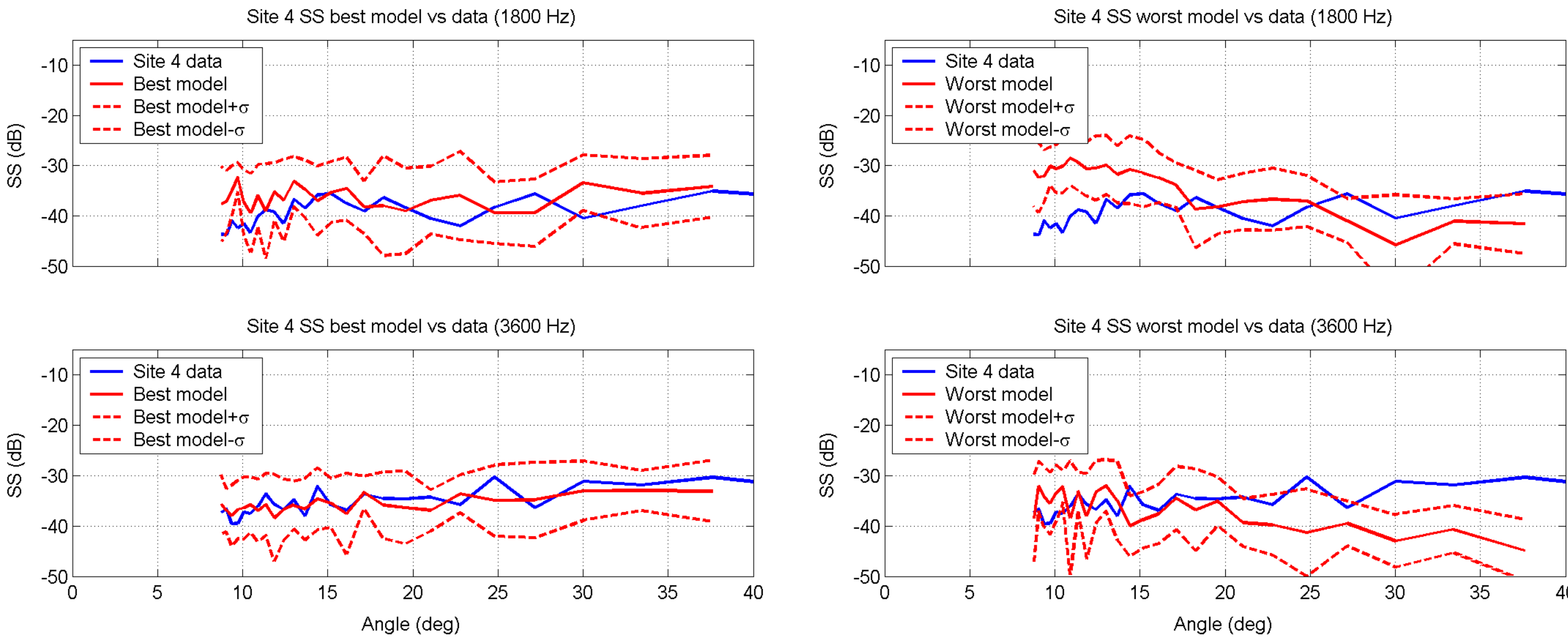




Best and Worst of Hypothesis 1 SS

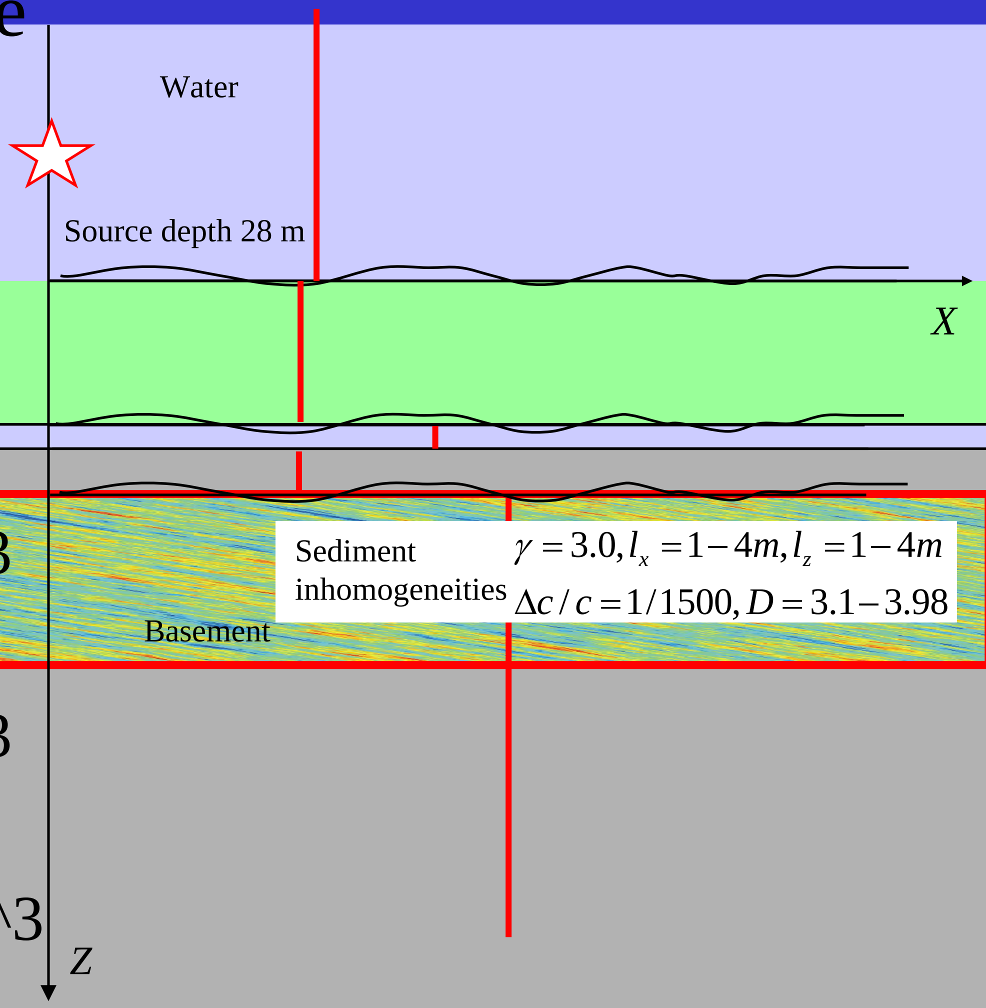
- l_z=1m, l_r=4m, D=3.98, δc_{rms}=6.5 m/s
- $P(O|m)/P(O|H_1)$=1.00

- l_z=4m, l_r=1m, D=3.1, δc_{rms}=1.4 m/s
- $P(O|m)/P(O|H_1)$=0.48



Malta Site 4 Hypothesis 2: Basement Volume Scattering

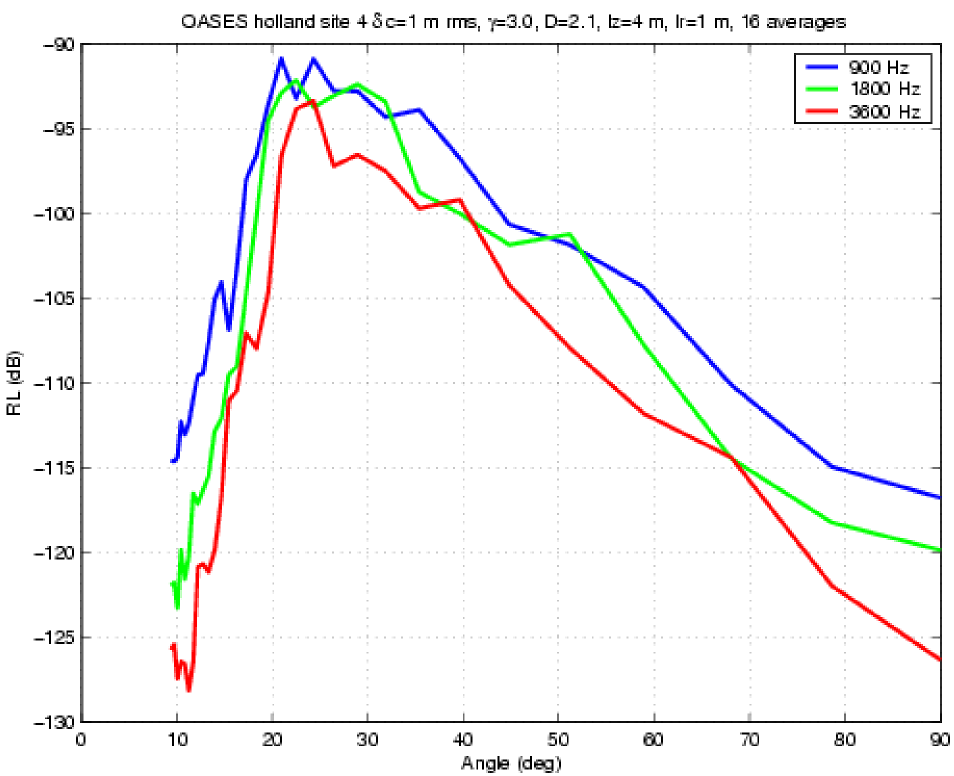
- 101 m Deep Waveguide
  - 1511 m/s
- 8.4 m Slow Sediment
  - 1480 m/s
  - ρ=1.32–1.5 g/cm^3
- 0.2 m Fast Interstices
  - 1550 m/s, ρ=1.7 g/cm^3
- 1.6 m Slow Sediment
  - 1490 m/s, ρ=1.6 g/cm^3
- Fast Basement
  - 1615 m/s, ρ=2.65 g/cm^3

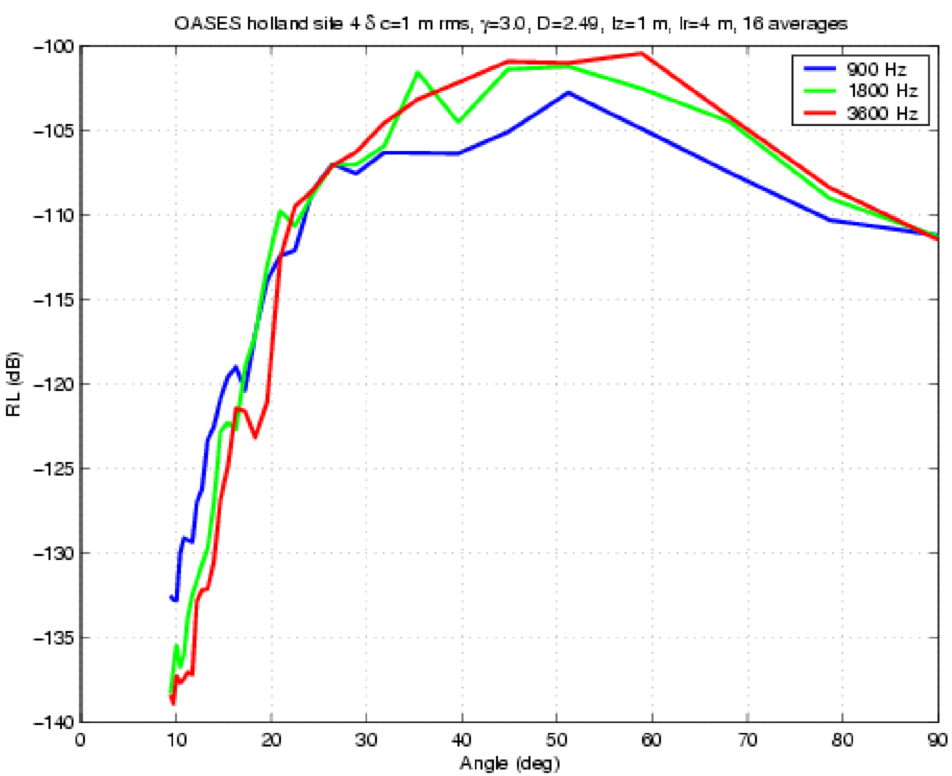


Best and Worst of Hypothesis 2

- l_z=4m, l_r=1m, D=3.1, δc_{rms}=8.9 m/s
- $P(O|m)/P(O|H_1)$=0.62

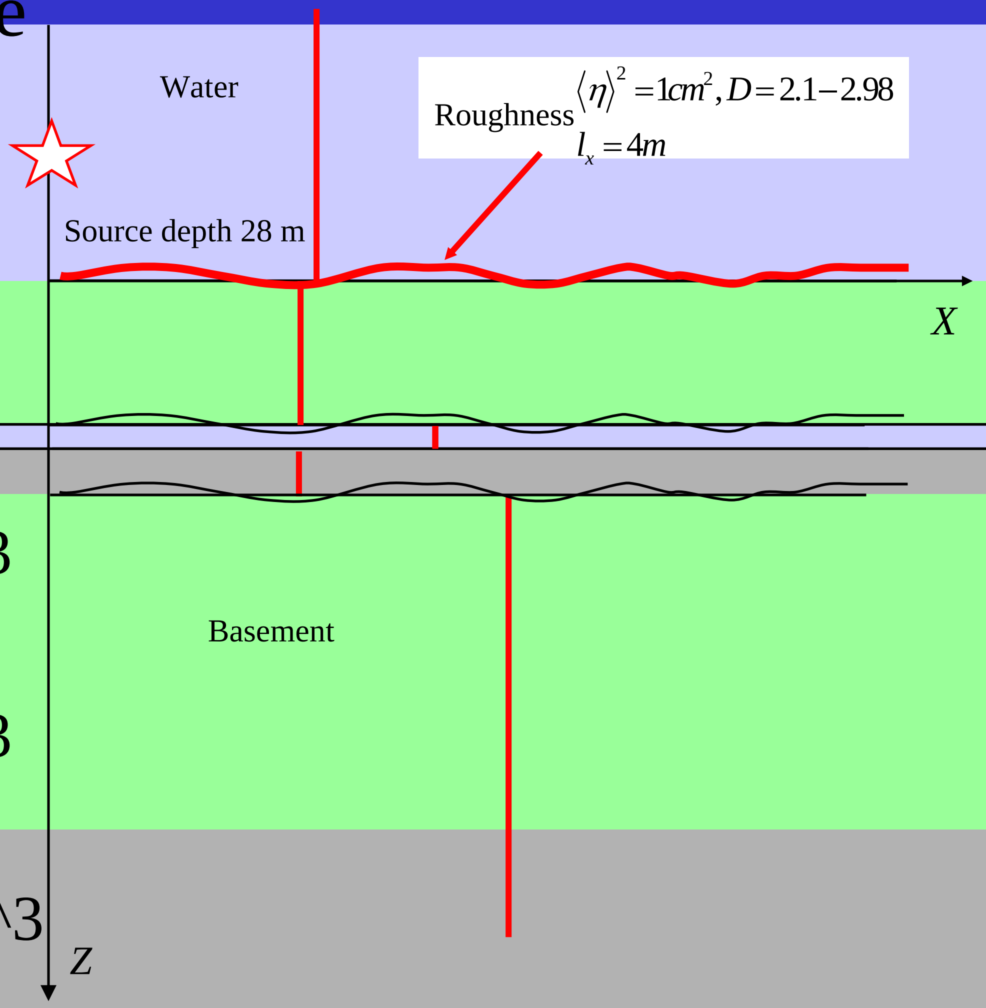
- l_z=1m, l_r=4m, D=3.98, δc_{rms}=38 m/s
- $P(O|m)/P(O|H_1)$=0.40





Malta Site 4 Hypothesis 3: Sediment Roughness Scattering

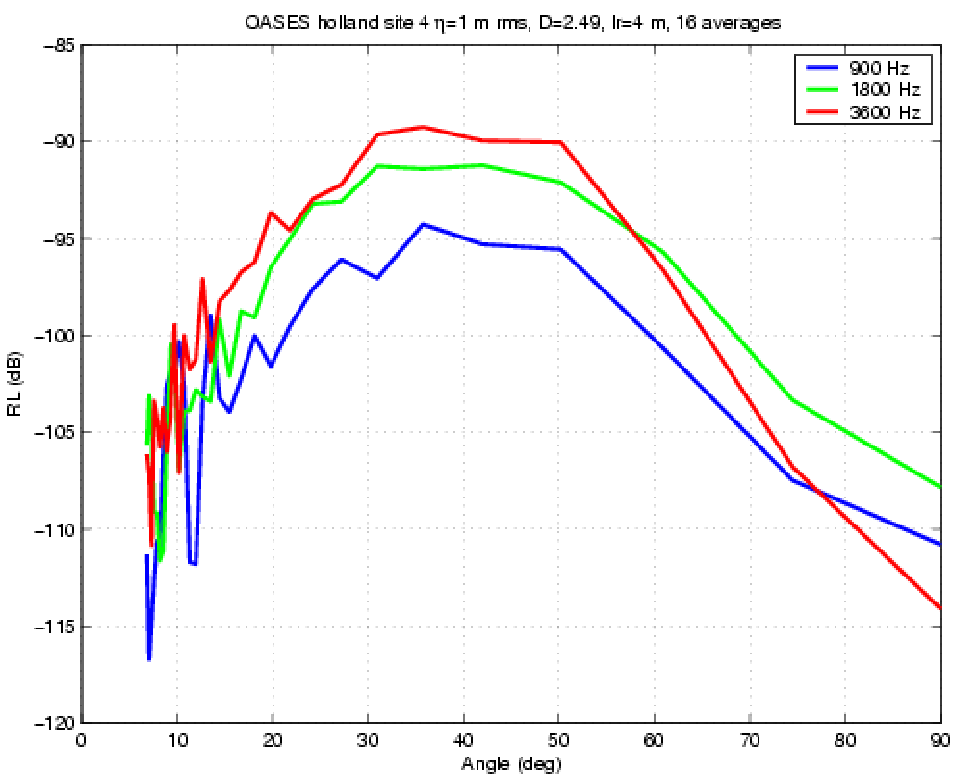
- 101 m Deep Waveguide
  - 1511 m/s
- 8.4 m Slow Sediment
  - 1480 m/s
  - ρ=1.32–1.5 g/cm^3
- 0.2 m Fast Interstices
  - 1550 m/s, ρ=1.7 g/cm^3
- 1.6 m Slow Sediment
  - 1490 m/s, ρ=1.6 g/cm^3
- Fast Basement
  - 1615 m/s, ρ=2.65 g/cm^3

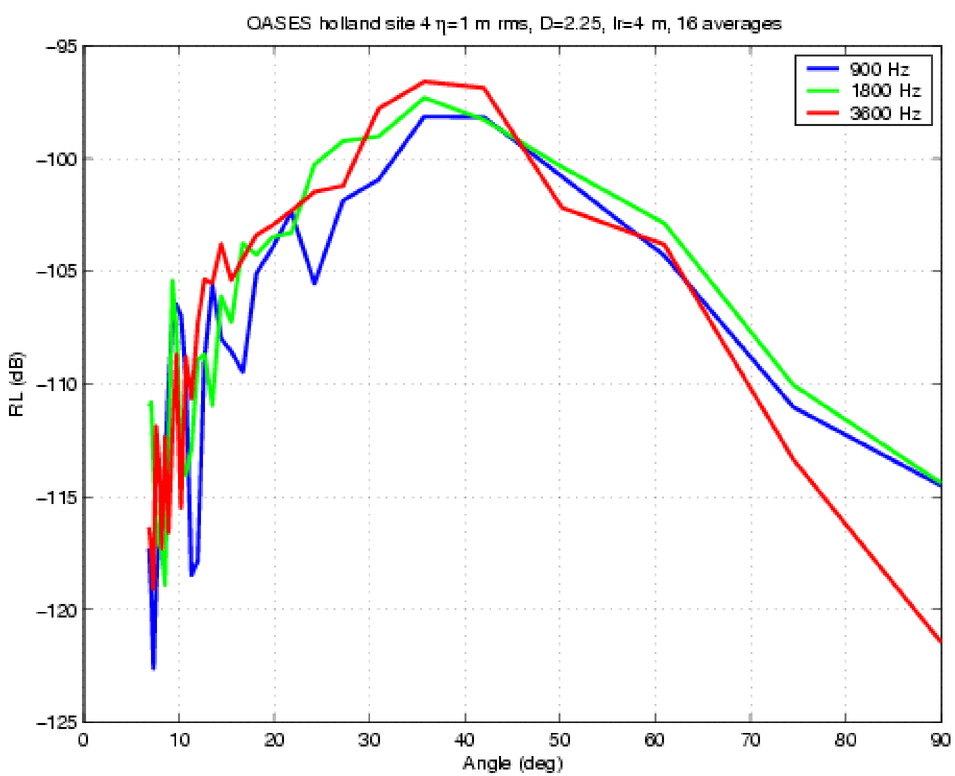


Best and Worst of Hypothesis 3

- $l_r=4m, \quad D=2.98,$ $\delta\eta_{rms}=2.1\ cm$
- $P(O|m)/P(O|H_1)=0.84$

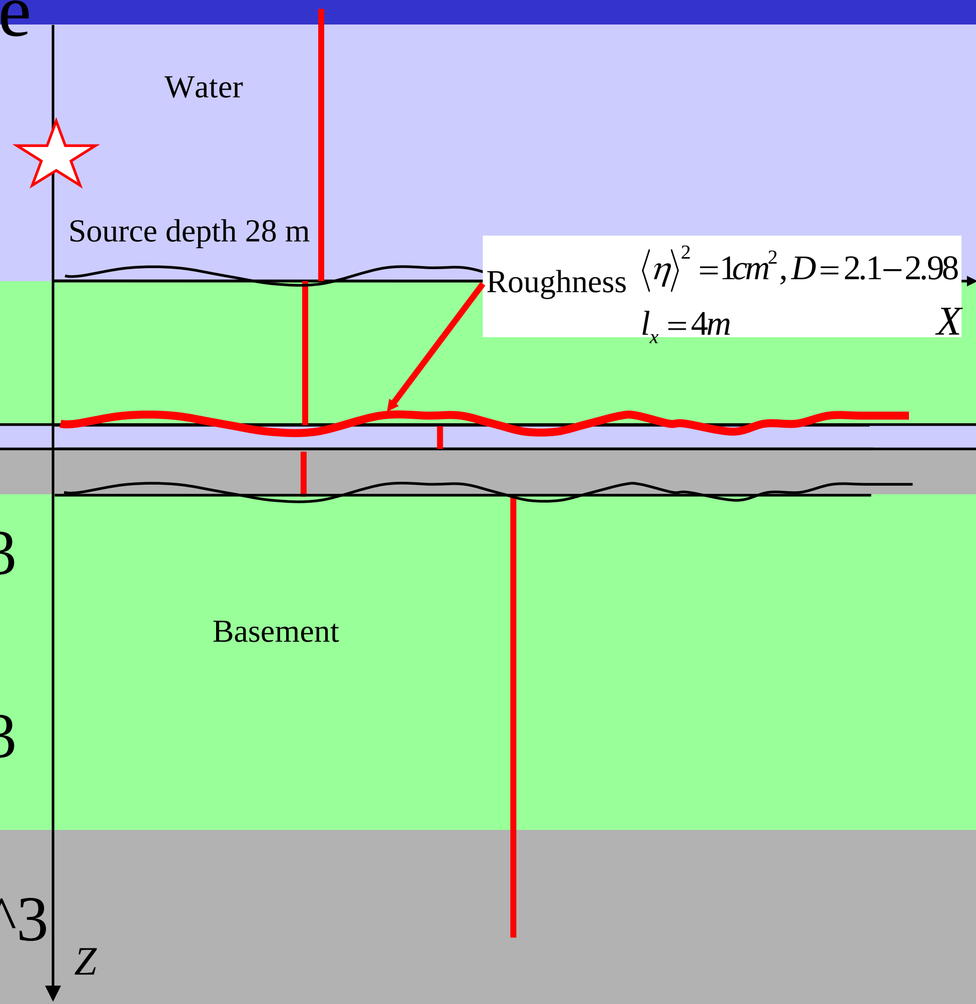
- $l_r=4m, D=2.5,$ $\delta\eta_{rms}=5.2\ cm$
- $P(O|m)/P(O|H_1)=0.78$





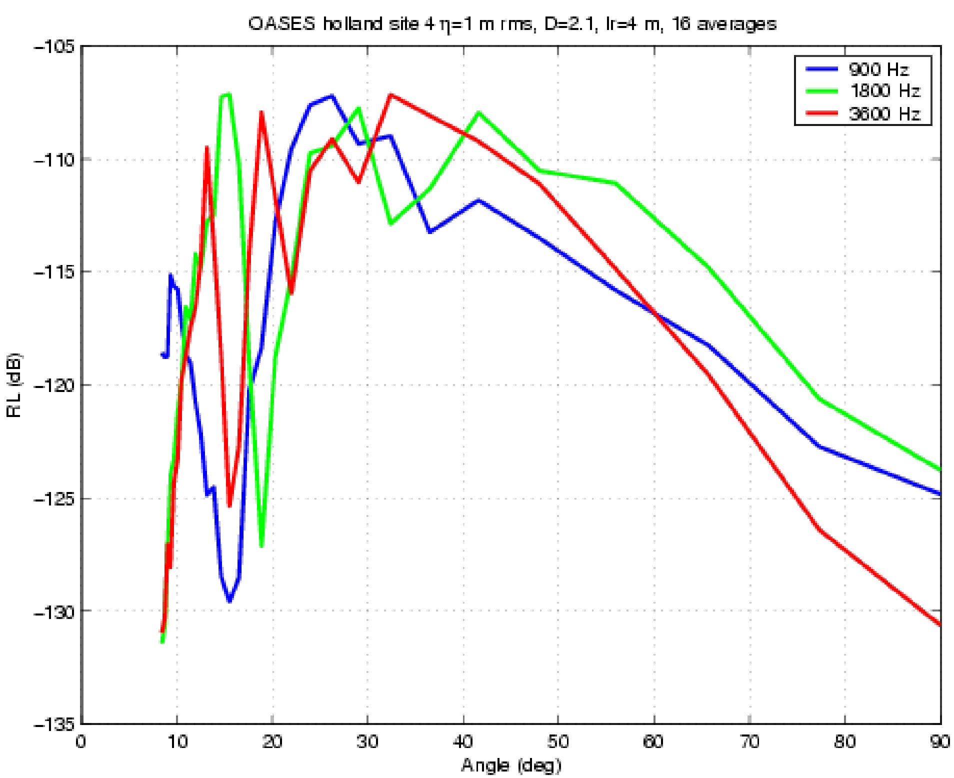
Malta Site 4 Hypothesis 4: Interstices Rough Surface Scatter

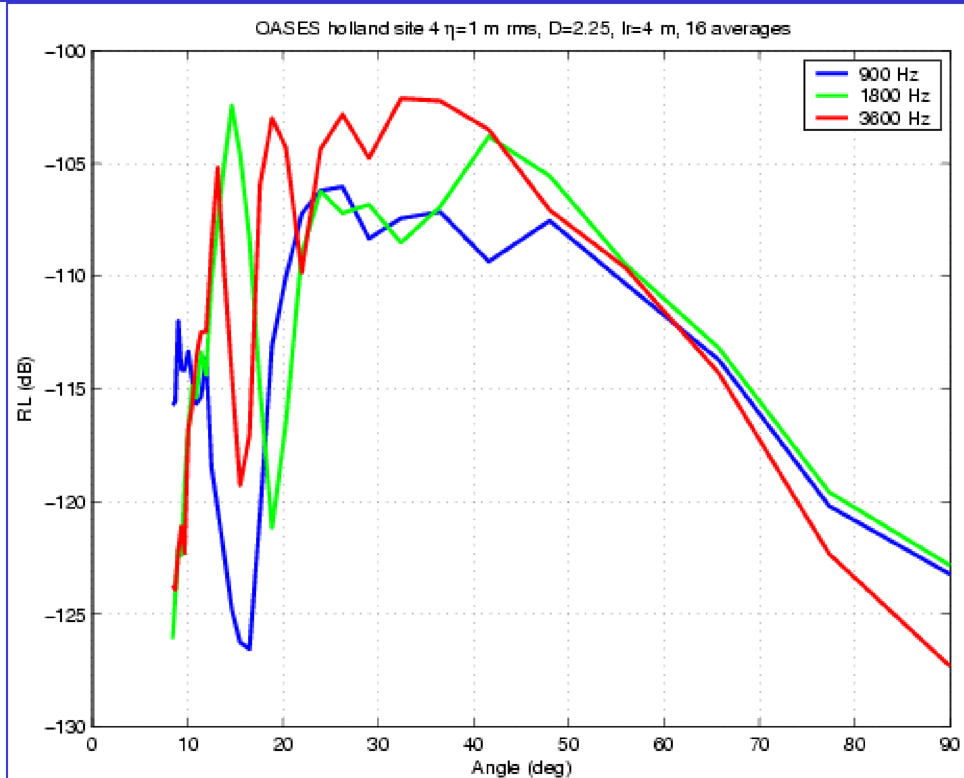
- 101 m Deep Waveguide
  - 1511 m/s
- 8.4 m Slow Sediment
  - 1480 m/s
  - ρ=1.32–1.5 g/cm^3
- 0.2 m Fast Interstices
  - 1550 m/s, ρ=1.7 g/cm^3
- 1.6 m Slow Sediment
  - 1490 m/s, ρ=1.6 g/cm^3
- Fast Basement
  - 1615 m/s, ρ=2.65 g/cm^3



Best and Worst of Hypothesis 4

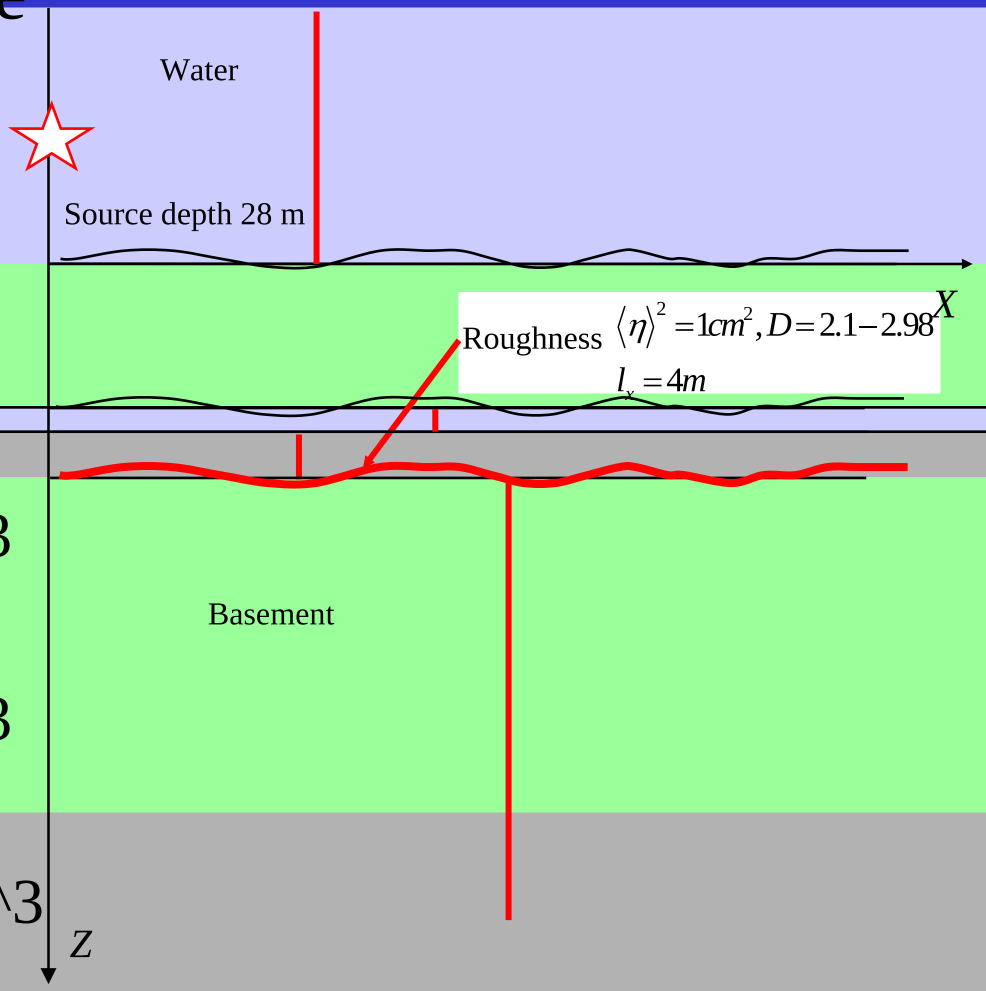
- l_r=4m, D=2.1, $\delta\eta_{rms}$=11.6 cm
- $P(O|m)/P(O|H_1)=0.69$
- l_r=4m, D=2.5, $\delta\eta_{rms}$=6.9 cm
- $P(O|m)/P(O|H_1)=0.61$





Malta Site 4 Hypothesis 5: Basement Rough Surface Scatter

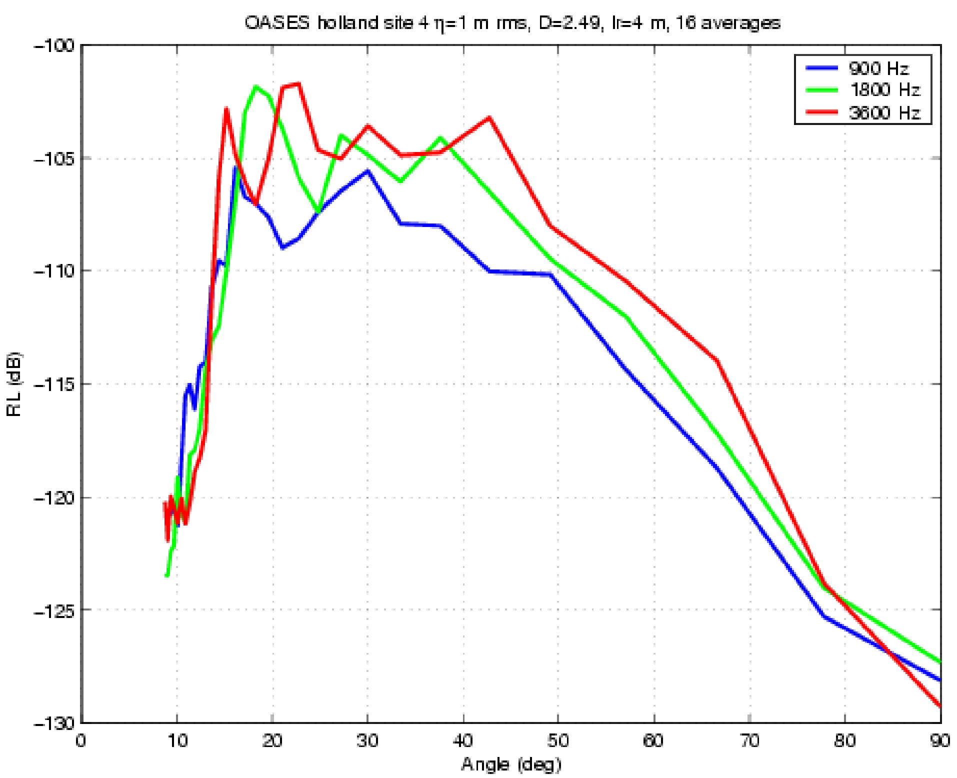
- 101 m Deep Waveguide
  - 1511 m/s
- 8.4 m Slow Sediment
  - 1480 m/s
  - ρ=1.32–1.5 g/cm^3
- 0.2 m Fast Interstices
  - 1550 m/s, ρ=1.7 g/cm^3
- 1.6 m Slow Sediment
  - 1490 m/s, ρ=1.6 g/cm^3
- Fast Basement
  - 1615 m/s, ρ=2.65 g/cm^3

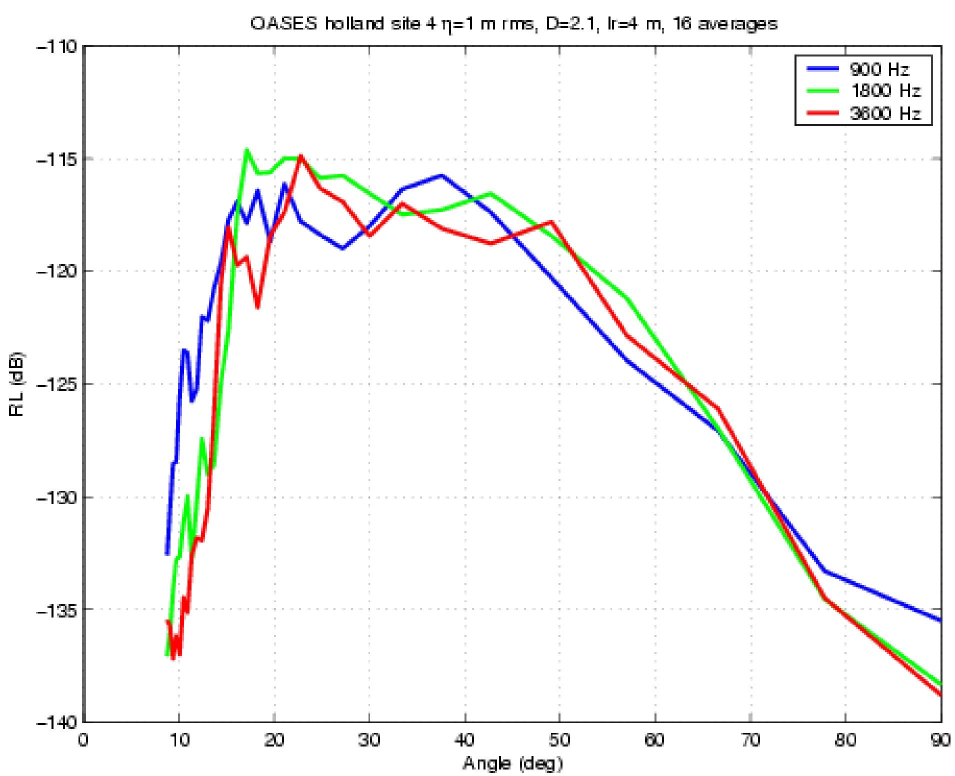


Best and Worst of Hypothesis 5

- $l_r = 4m, \quad D = 2.98, \; \delta\eta_{rms} = 6.3\ cm$
- $P(O|m)/P(O|H_1) = 0.87$

- $l_r = 4m, D = 2.1, \; \delta\eta_{rms} = 28\ cm$
- $P(O|m)/P(O|H_1) = 0.65$





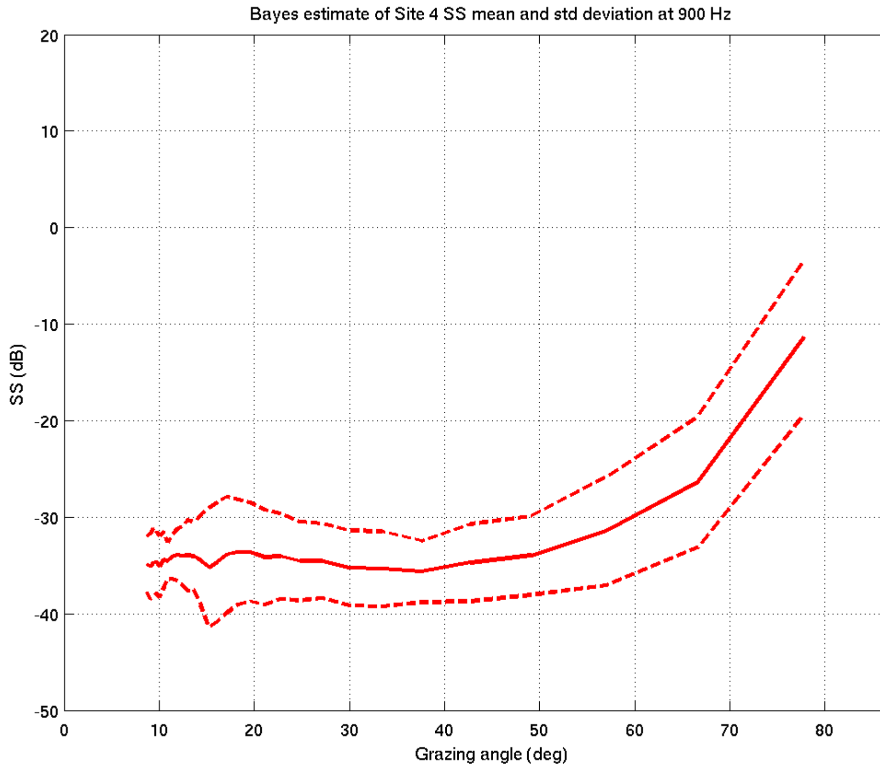
Bayesian Model Probabilities

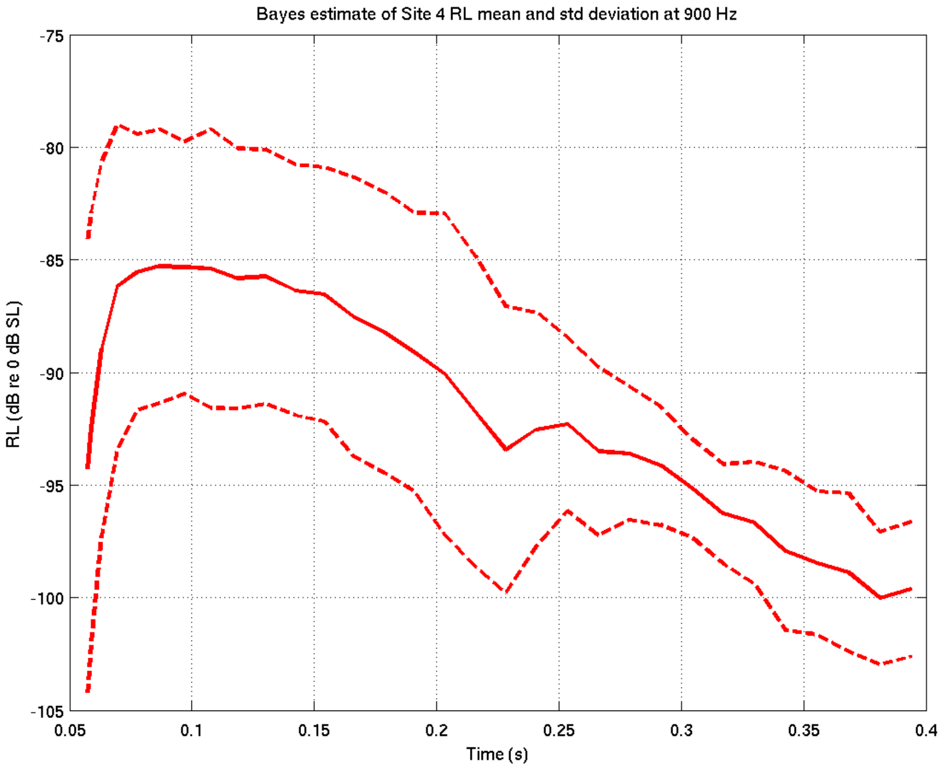
$$p(m|O) = \frac{p(O|m)\,p(m)}{\int_M p(O|m)\,p(m)\,dm}$$

$p(m\|O)$	Volume Inhomogeneities Slow layer			Roughness		
	$l_r = 4m$, $l_z = 1m$	$l_r = 1m$, $l_z = 1m$	$l_r = 1m$, $l_z = 4m$	Sediment-Basement Interface	Fast Layer	Sediment-Water Interface
Λ=1.9	0.0596	0.0593	0.0444	0.0542	0.0549	0.0547
Λ=1.75	0.0601	0.0586	0.0500	0.0539	0.0550	0.0548
Λ=1.51	0.0615	0.0588	0.0513	0.0578	0.0554	0.0559

Bayesian Model-Based Transfer of Uncertainty to 900 Hz

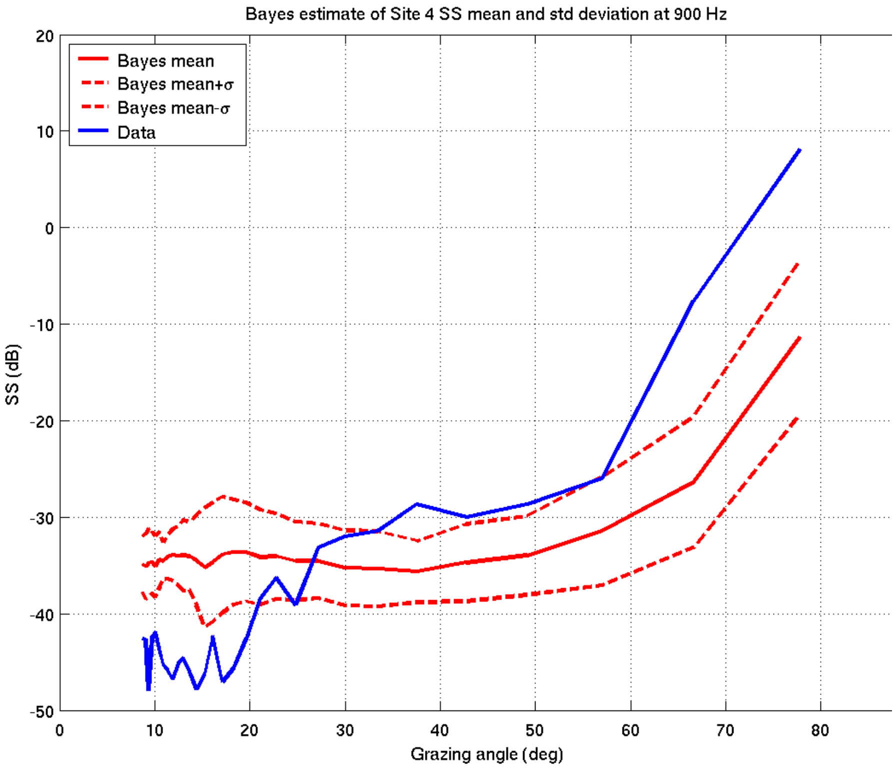
- SS
- RL

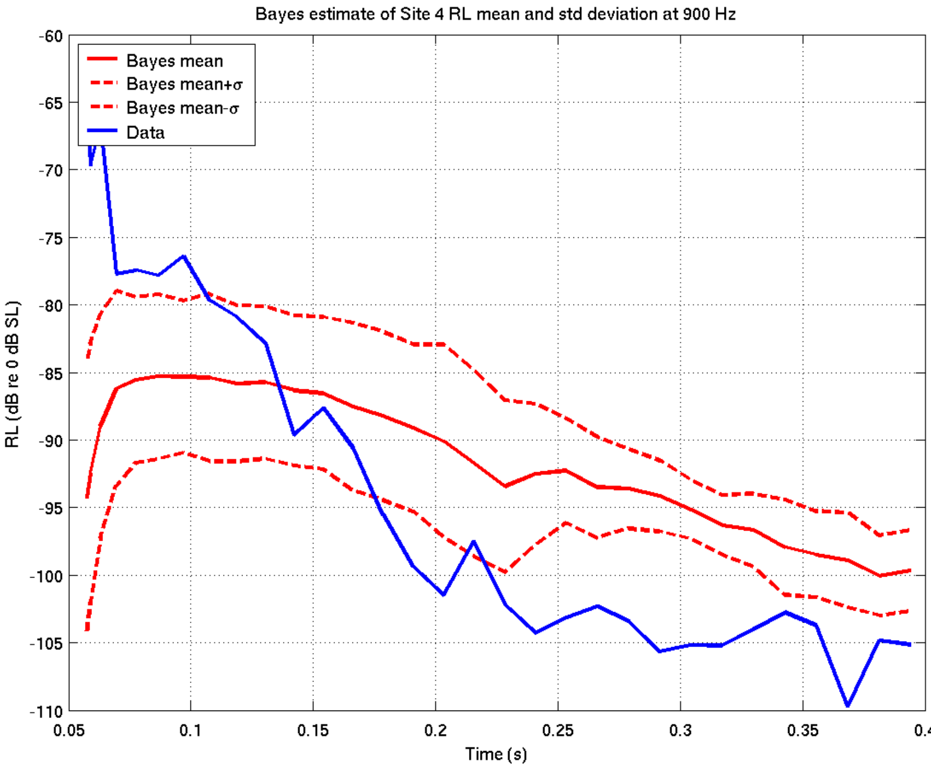




Performance of Bayesian Model-Based Transfer of Uncertainty

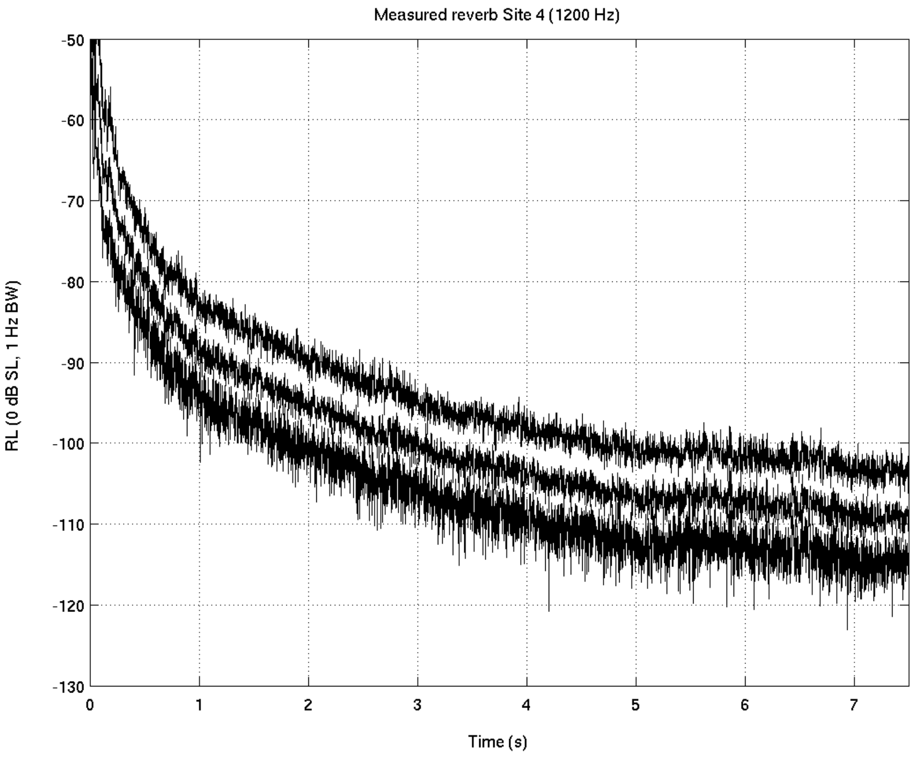
- SS
- RL

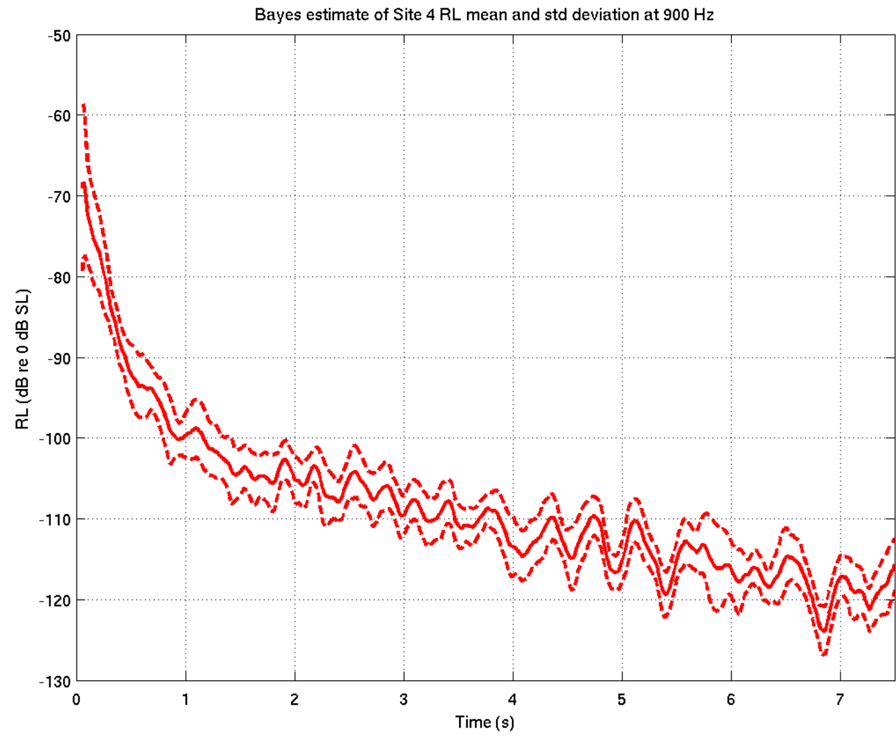




Bayesian Estimate of Reverberation vs Observation

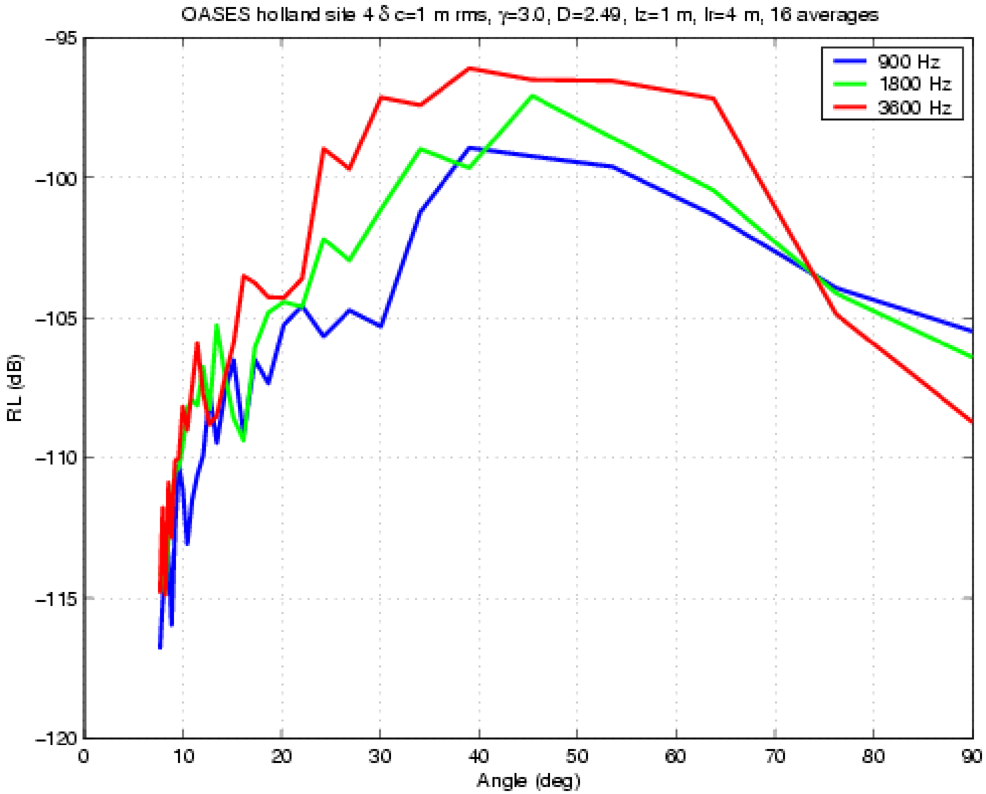
- Data Boundary 2004
- Estimate

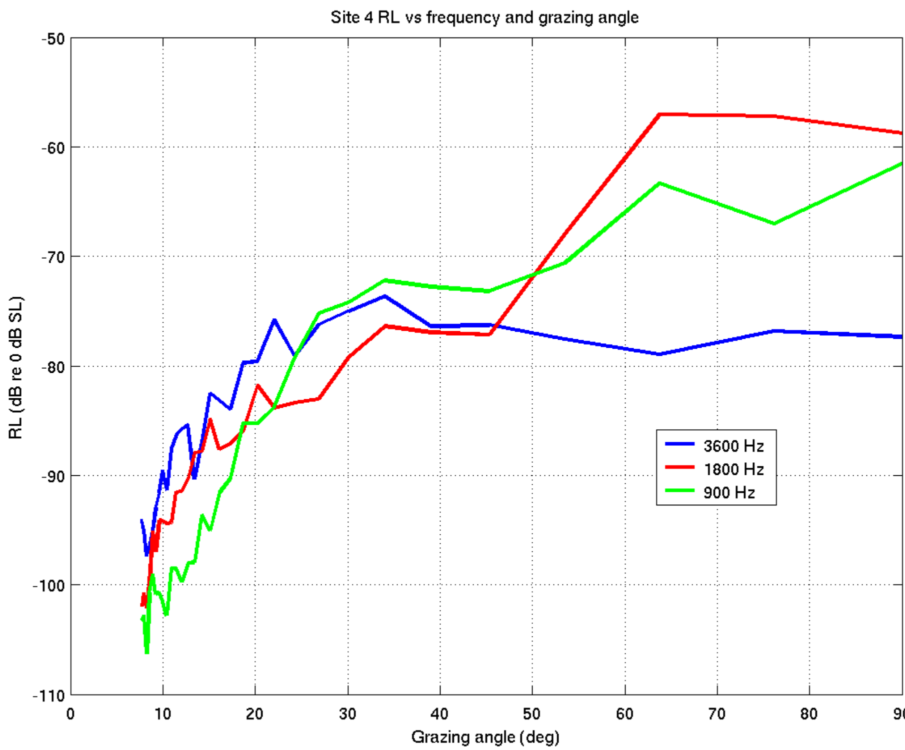




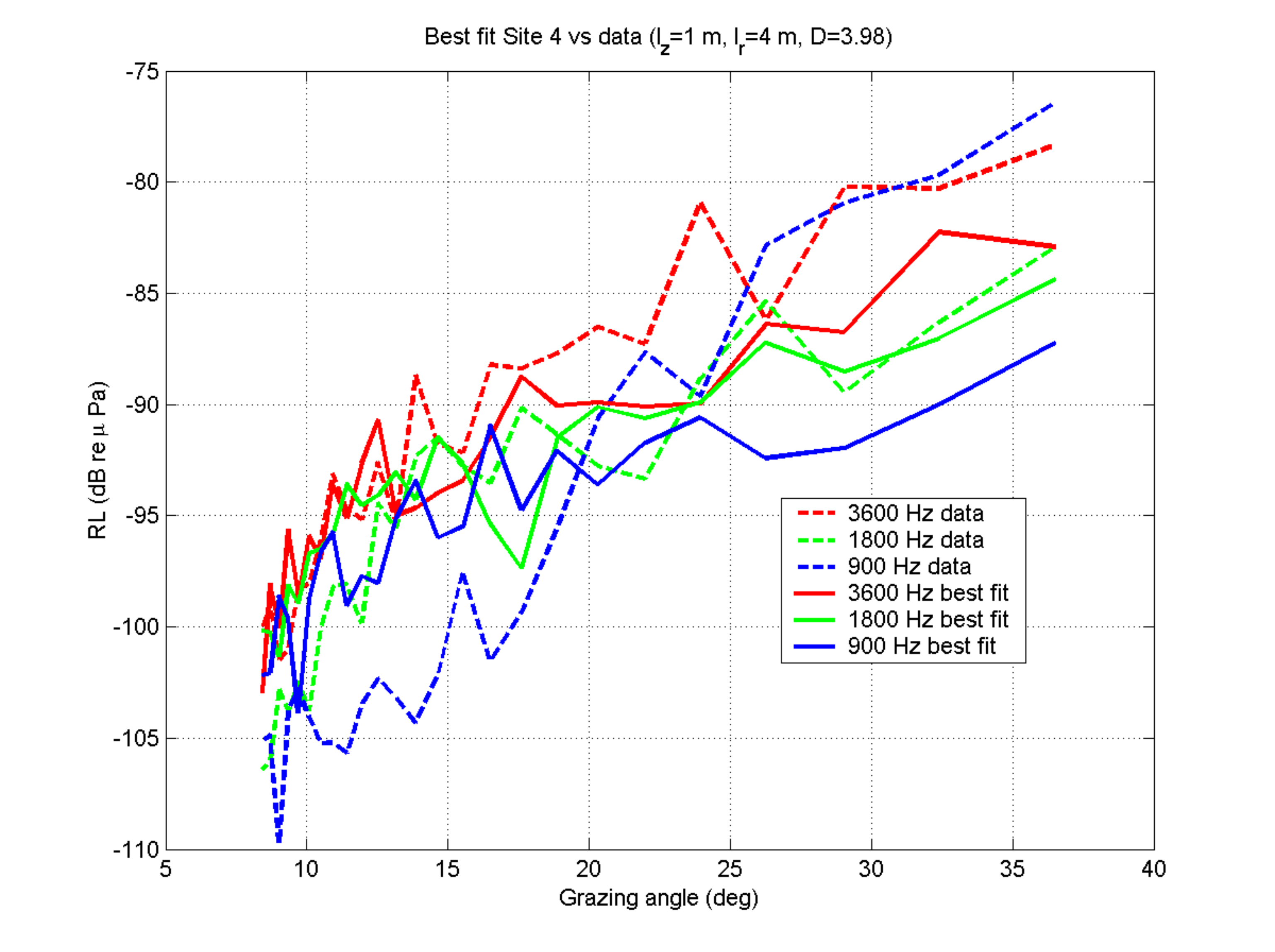
Culprit: Bias in Best Model

- Model
- Data



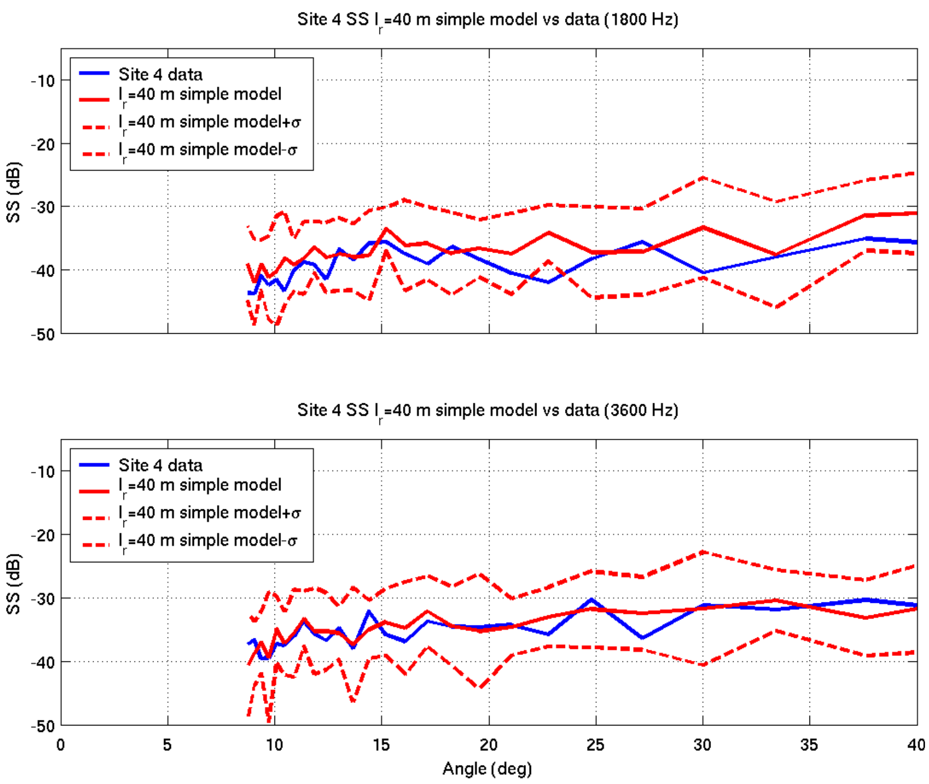


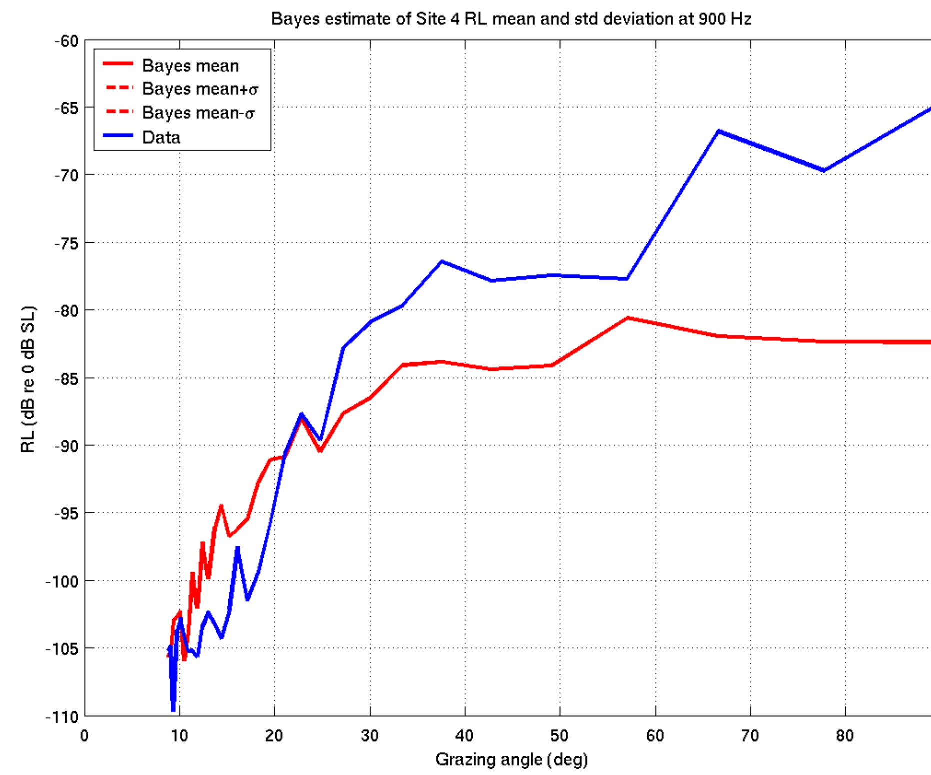
Culprit: Bias in Best Model



Performance of Modified Model

- SS vs 1800, 3600 Hz data
- Extrapolation performance 900 Hz





Conclusions

- Bayesian approach offers a statistically meaningful way of propagating experimental uncertainty to model predictions
  - Monte Carlo methods may be used to obtain p(O|m)
- Measurement certainty (low variance) required to resolve models
- Model mismatch due to finite model space integrals can introduce model probability biases
- Further work is required in the current example to obtain models for Site 4 with lower bias
  - Thin volume below interstices
  - Sensitivity to forward model estimate of attenuation