Resolution, Variance, and Seabed Variability

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Bottom Property Parameter Sensitivity

- Model-data sensitivities are required to construct an accurate picture of effect of variability on acoustic prediction uncertainty
  - Required quantities are the functional or Frechet derivatives

$$\left(\frac{\partial P}{\partial \kappa}, \frac{\partial P}{\partial \rho}\right)$$

  - Other quantities of interest are obtained from the derivative chain rule
- Linearize about a global model minimum

Linearization about Global Minimum

$$\mathbf{d} = \mathbf{g}(\mathbf{m})$$

$$= \mathbf{g}(\mathbf{m_0}) + \frac{\partial \mathbf{g}}{\partial \mathbf{m}}(\mathbf{m} - \mathbf{m_0}) + \ldots$$

$$\mathbf{d} - \mathbf{d_0} = \mathbf{G}(\mathbf{m} - \mathbf{m_0})$$

discrete $\boxed{\mathbf{d} = \mathbf{G}\mathbf{m}}$ continuous $\boxed{\mathbf{d} = \int \mathbf{G}(x)\mathbf{m}(x)dx}$

$\mathbf{G} = \frac{\partial \mathbf{g}}{\partial \mathbf{m}}$ is an N x M matrix of Frechet derivatives

N = number of data

M = number of model parameters

N > M $\Rightarrow$ overdetermined

M > N $\Rightarrow$ underdetermined

Data Resolution

- Assume we have found a generalized inverse ($\mathbf{G}^{-g}$) that solves our problem in some sense

$$\mathbf{m}^{\text{est}} = \mathbf{G}^{-g}\mathbf{d}^{\text{obs}}$$

- How well does our model estimate fit the data?

$$\mathbf{d}^{\text{pre}} = \mathbf{G}\,\mathbf{m}^{\text{est}} = \mathbf{G}\,\mathbf{G}^{-g}\,\mathbf{d}^{\text{obs}}$$

$$\text{data resolution matrix} = \mathbf{G}\,\mathbf{G}^{-g} = \mathbf{N}$$

$$\mathbf{N} = \mathbf{I} \Rightarrow \mathbf{d}^{\text{pre}} = \mathbf{d}^{\text{obs}} \qquad \text{zero prediction error}$$

$$\mathbf{N} \neq \mathbf{I} \Rightarrow \mathbf{d}^{\text{pre}} \neq \mathbf{d}^{\text{obs}} \qquad \text{non-zero prediction error}$$

- The N x N resolution matrix **N** characterizes whether the data can be independently predicted or resolved

Model Resolution

- True but unknown model $\mathbf{m}^{\text{true}}$ such that

$$\mathbf{G}\,\mathbf{m}^{\text{true}} = \mathbf{d}^{\text{obs}}$$

- How close is a particular estimate $\mathbf{m}^{\text{est}}$ to $\mathbf{m}^{\text{true}}$?

$$\mathbf{m}^{\text{est}} = \mathbf{G}^{-g}\mathbf{d}^{\text{obs}} = \mathbf{G}^{-g}\left[\mathbf{G}\,\mathbf{m}^{\text{true}}\right] = \mathbf{G}^{-g}\,\mathbf{G}\,\mathbf{m}^{\text{true}}$$

$$= \mathbf{R}\,\mathbf{m}^{\text{true}}$$

- $\mathbf{R} = \mathbf{G}^{-g}\mathbf{G}$ is the M x M model resolution matrix

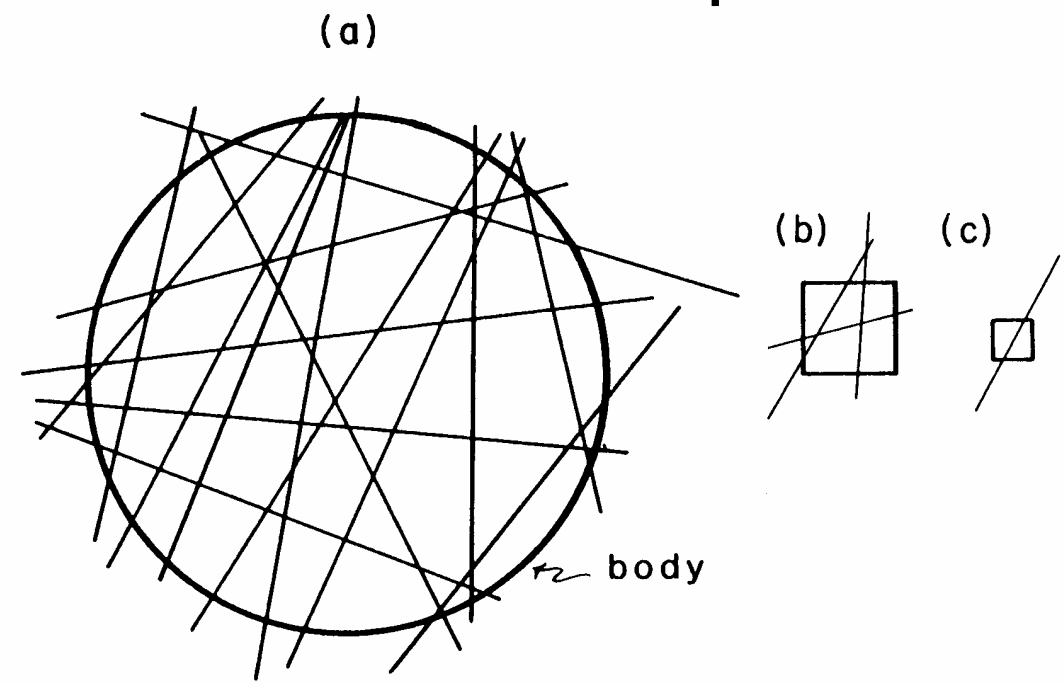
  R = I $\Rightarrow$ model is uniquely determined

  R ≠ I $\Rightarrow$ model is a weighted average of the true model

- Plots of the rows of the resolution matrix can be useful in determining to what scale features can actually be resolved

Trade-off of Resolution and Variance 1.

CT Scan Example



(b) Coarse discretization
- overdetermined problem
- low resolution (large box)
- low variance
  (large # of measurements/box)

(c) Fine discretization
- underdetermined problem
- high resolution (small box)
- large variance
  (small # of measurements/box)

Analytical Expressions for Partial Derivatives

$$\mathbf{d} = \mathbf{G}\mathbf{m} = \frac{\partial \mathbf{g}}{\partial \mathbf{m}}\mathbf{m}$$

$$\frac{\partial \mathbf{g}}{\partial \mathbf{m}} = \left(\frac{\partial P}{\partial \kappa}, \frac{\partial P}{\partial \rho}\right)$$

$$\frac{\partial P(z_r, t; z_s, 0)}{\partial \kappa(z)} = \frac{1}{\kappa^2(z)} \; P(z, t; z_r, 0) * P(z, t; z_s, 0) * \frac{\partial^2}{\partial t^2} s(t)$$

$$\frac{\partial P(z_r, t; z_s, 0)}{\partial \rho(z)} = \frac{1}{\rho^2(z)} \; \left\{ \left[\frac{\partial}{\partial z} P(z, t; z_r, 0) * \frac{\partial}{\partial z} P(z, t; z_s, 0) * s(t) \right] - \right.$$

$$\left. \left[p^2 P(z, t; z_r, 0) * P(z, t; z_s, 0) * \frac{\partial^2}{\partial t^2} s(t) \right] \right\}$$

- $\left(\frac{\partial P}{\partial \kappa}, \frac{\partial P}{\partial \rho}\right)$ can be computed from **one pass** of FFP or SAFARI/OASES or …

A Handful of Partial Derivatives

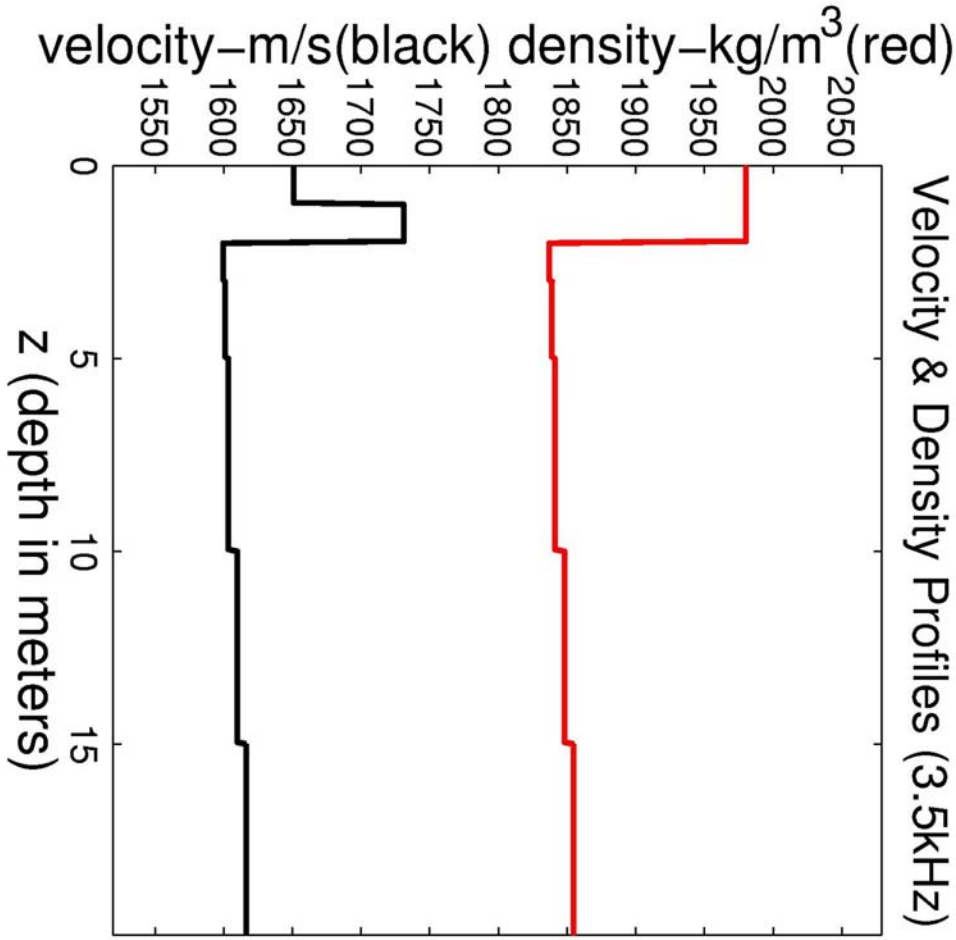
- If the source and receiver are assumed to be at the same level z, the partials are simple functions of the depth dependent Green's function *G*.

$$\frac{\partial P}{\partial \kappa} = -\frac{\omega^2}{\kappa^2}[G]^2 \qquad \frac{\partial P}{\partial \rho} = \frac{1}{\rho^2}\left\{\left[\frac{\partial G}{\partial z}\right]^2 - k_x^2 G^2\right\}$$

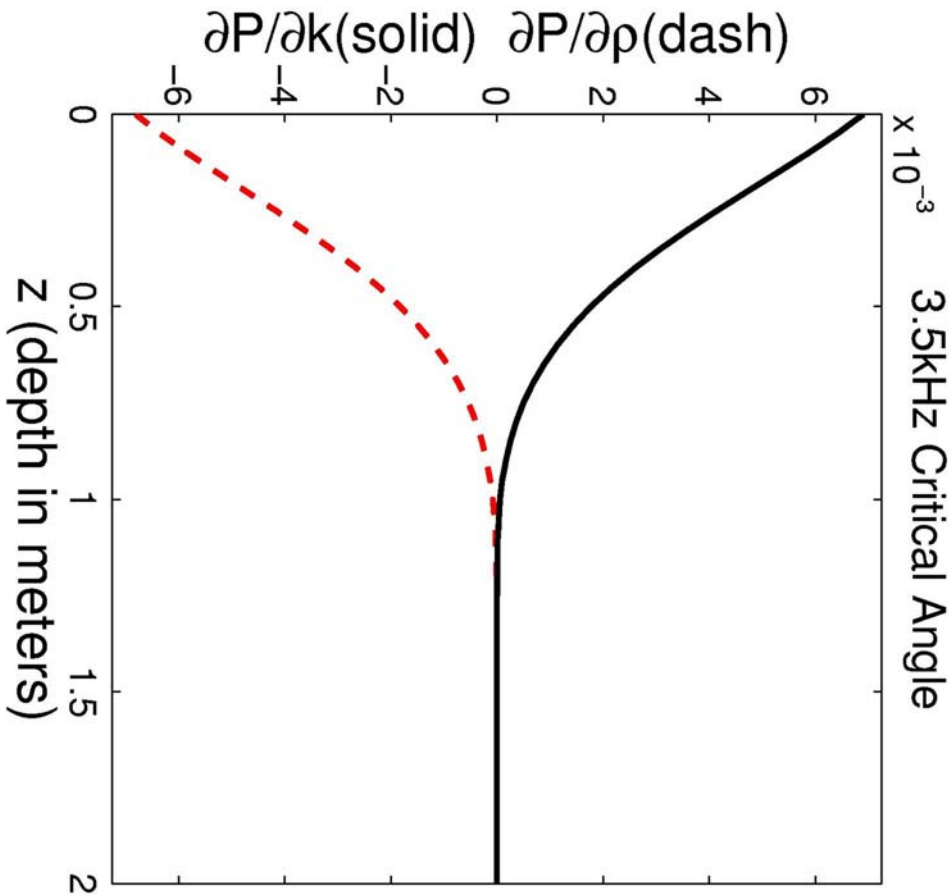
- Derivatives with respect to sound speed *c* and attenuation α are easily computed.

$$\frac{\partial P}{\partial c} = 2\rho c\frac{\partial P}{\partial \kappa} - 2\frac{\rho}{c}\frac{\partial P}{\partial \rho} \qquad \frac{\partial P}{\partial \alpha} = -2\left(\frac{ic\kappa}{\omega}\right)\frac{\partial P}{\partial \kappa}$$

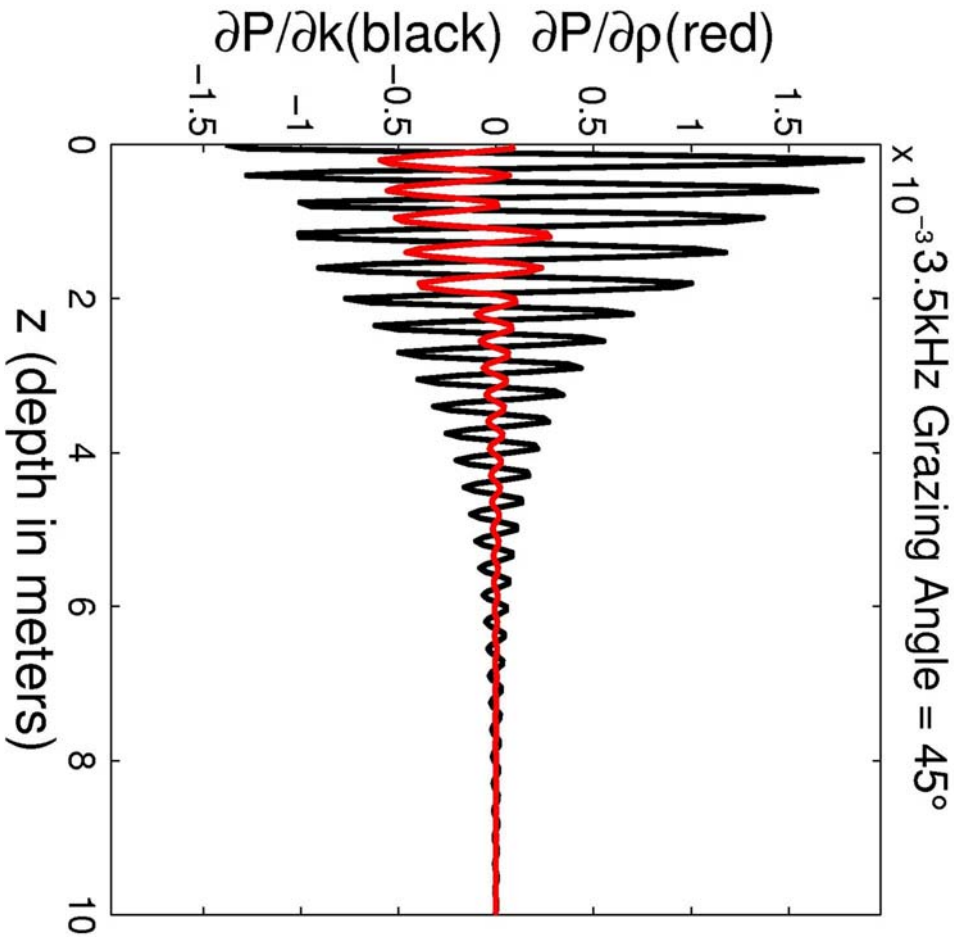
East China Sea - Continental Shelf



Frechet Derivative 1

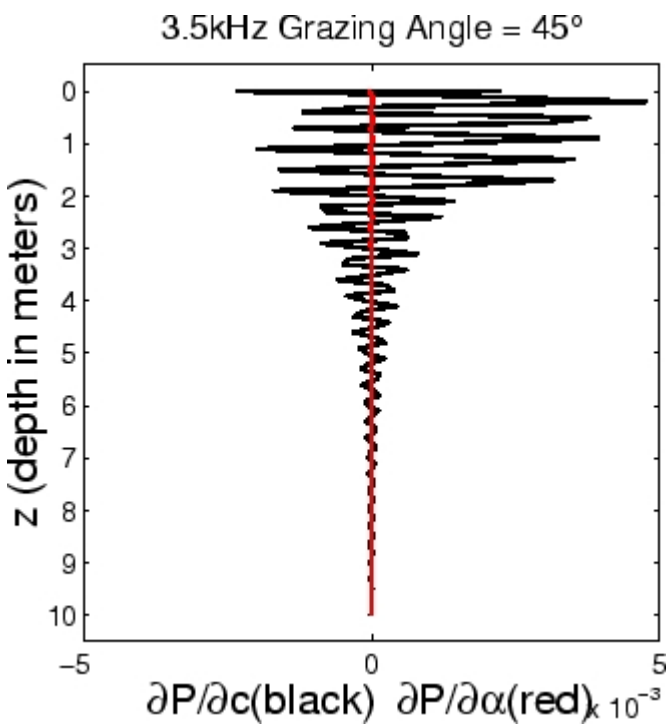


Frechet Derivative 2



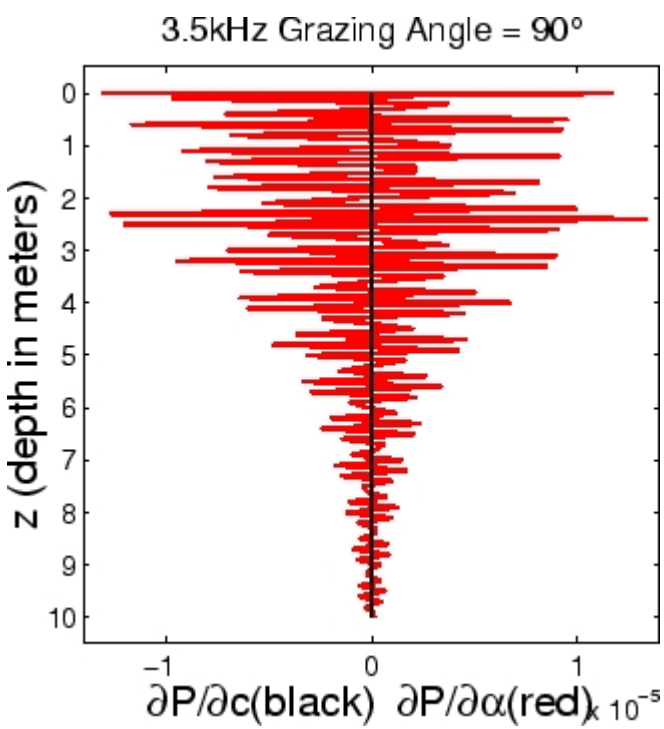
- At a grazing angle of 45 bulk modulus perturbations have a greater effect on the measured pressure than density, which will be poorly re resolved below 2m.
- Effects of density are out of phase with the effects of bulk modulus.

Frechet Derivative 3



- The derivative for the sound speed **c** is much larger than the derivative for the attenuation α. This isn't surprising considering the $\mathbf{1/\omega}$ dependence of $\partial \mathbf{P}/\partial\alpha$.

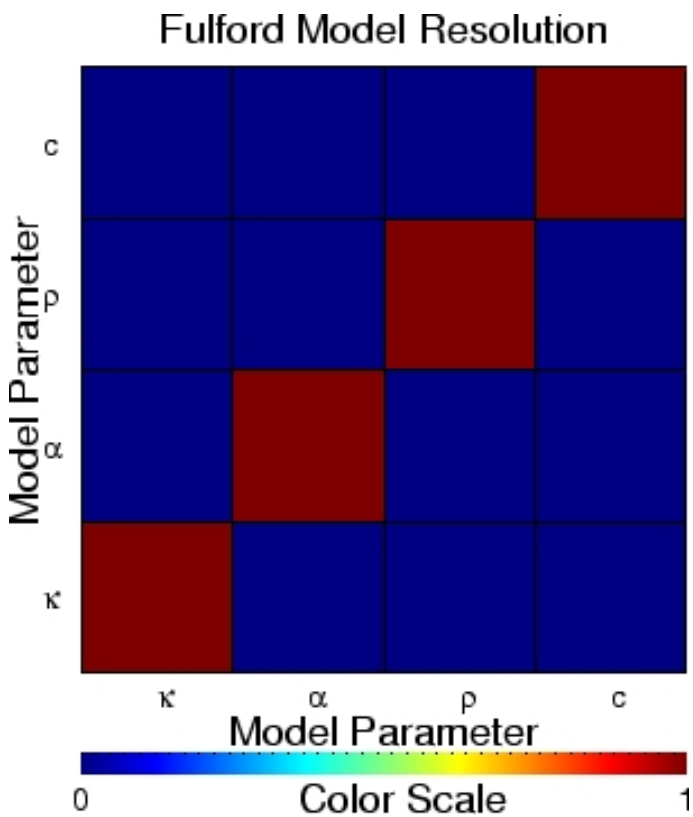
Frechet Derivative 4



- At normal incidence ∂P/∂c is identically 0. Pure sound speed perturbations are forward scattering. (This is why the parabolic equation works so well in ocean acoustics.)
- Note that the scale on the abcissa is amplified by a factor of 100 from the previous slide.

Model Resolution

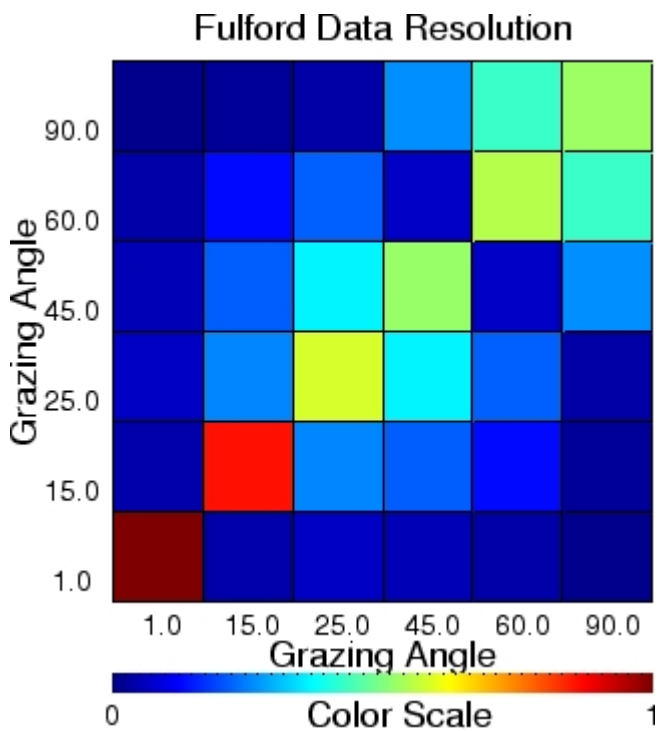
(4 Model Parameters, 6 Measurement Angles)



- Angles: 1˚, 15˚, 25˚, 45˚, 60˚, 90˚
- By the addition of more data, the model parameters at the water sediment interface can be resolved.
- The model resolution matrix for an over-determined problem is always diagonal.

Data Resolution

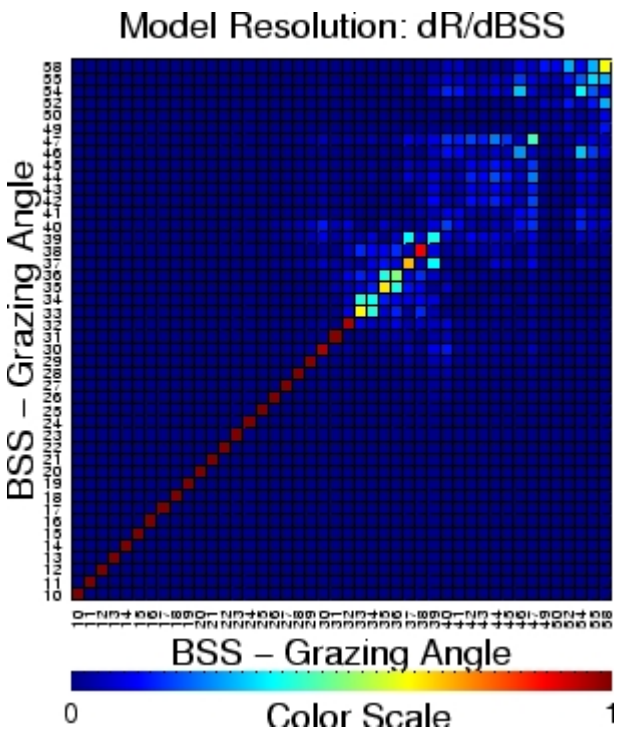
(4 Model Parameters, 6 Measurement Angles)



- At progressively higher angles the ability to predict the data degrades.
  - Data not uniquely predicted by model
- Conclusion: We have a model which fits the data. However, our model is not too good at predicting the high grazing angle data.

Model Resolution

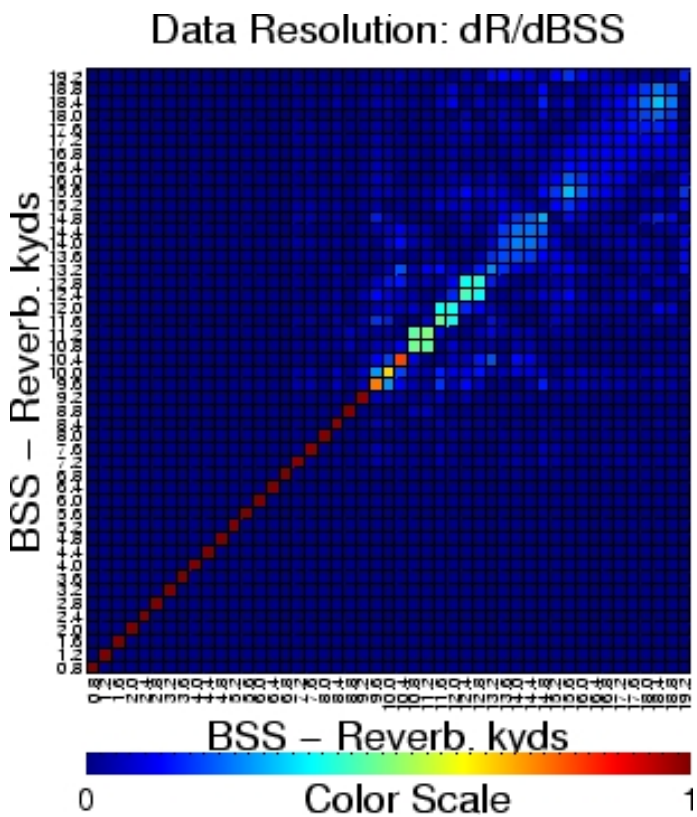
(SEPES: Greg Anderson, Darrell Jackson)
(Mixed over/under-determined)



- Model: 44 scattering angles
- Data: 47 reverberation samples
- Low scattering angles are well resolved
  - Lambert's law: strong angular dependence at low grazing angles
- Large grazing angles are poorly resolved
  - Lambert's law: weak angular dependence at high grazing angles

Data Resolution

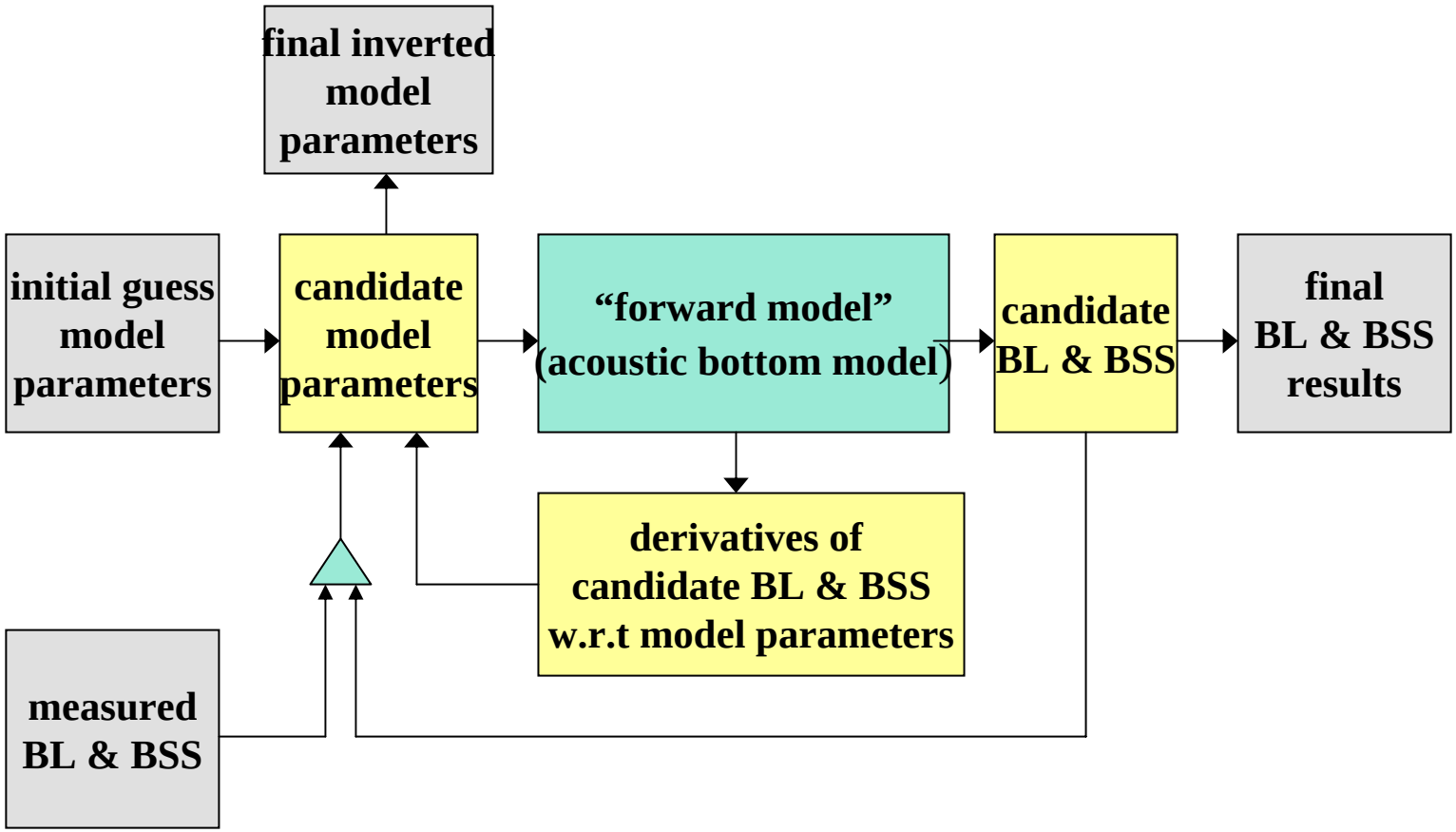
(SEPES: Greg Anderson, Darrell Jackson)



- How well can we predict the data from our model?
  - Reverb for low grazing angles is well predicted.
  - Reverb for high grazing angles is poorly predicted.
- Conclusion: At high grazing angles the data are not uniquely predicted by the model.

Optimization Process for Nonlinear Inversion Problem

(implemented in Fortran nonlinear optimization package SNOPT)

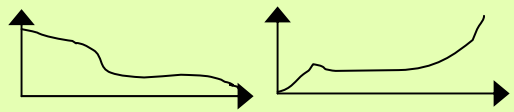


Feedback loop: minimize data misfit and meet constraints

Mathematical Description

objective function = data misfit:

$B(\phi) = \{ BL(\theta), BSS(\theta) \}$



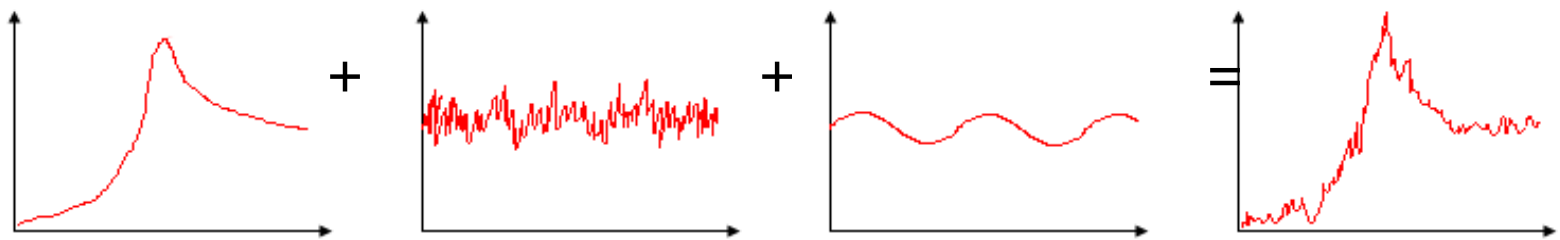
$$\underset{\underline{x}}{\text{minimize}} \quad f(\underline{x}) = \sum_{\phi} \left(\hat{B}(\phi) - B(\phi) \right)^2$$

constraint functions = BL & BSS bounds, param bounds, corr distance C:

$$\text{subject to} \quad \begin{bmatrix} lbnd_1 \\ lbnd_2 \\ lbnd_3 \end{bmatrix} \leq \begin{bmatrix} B(\underline{\phi}; \underline{x}) - \hat{B}(\underline{\phi}) \\ \underline{x} \\ C(\underline{x}) \end{bmatrix} \leq \begin{bmatrix} ubnd_1 \\ ubnd_2 \\ ubnd_3 \end{bmatrix}$$

Synthetic Input Data

(for both BL & BSS)



Output from bottom model for "target bottom"

Angle-independent Gaussian noise, equal standard deviations at all angles here

Additional (small amplitude) randomized sinusoidal trend

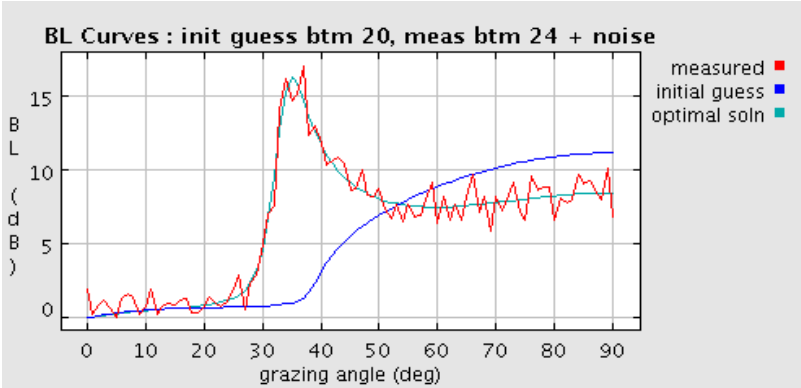
Synthetic data

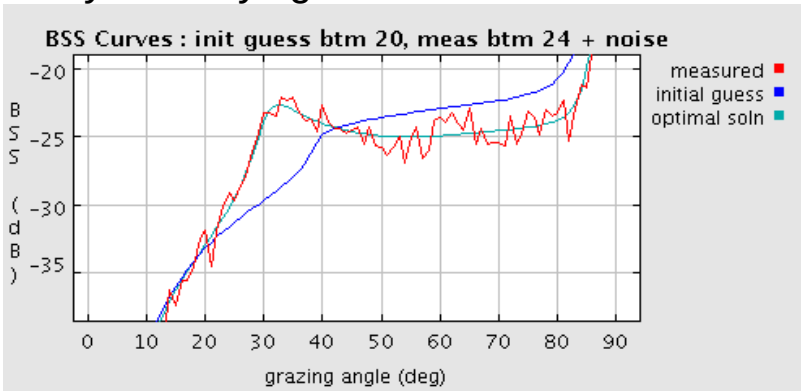
Real data only covers a small range of angles, but this synthetic data covers all 90º

Example Inversion Results

freq = 1500Hz
c0 = 1545m/s

measured data : 0.6m medium silt over muddy gravel
initial guess : 0.2m very fine sand over muddy sandy gravel

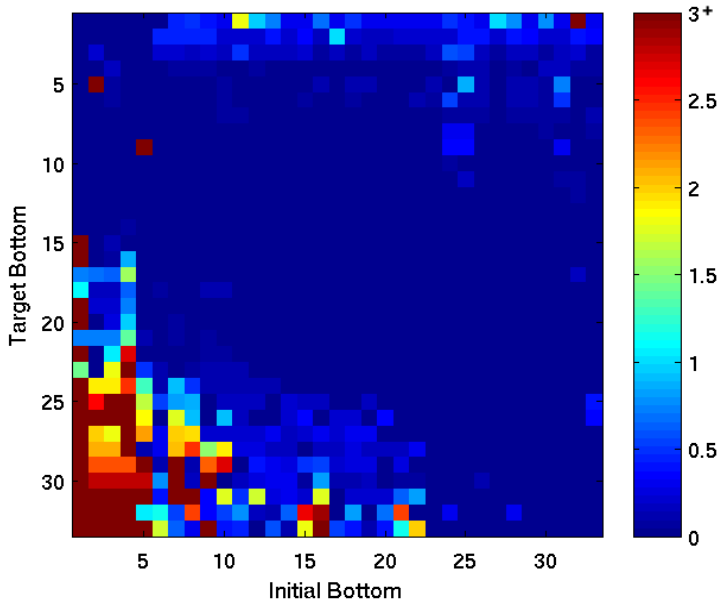




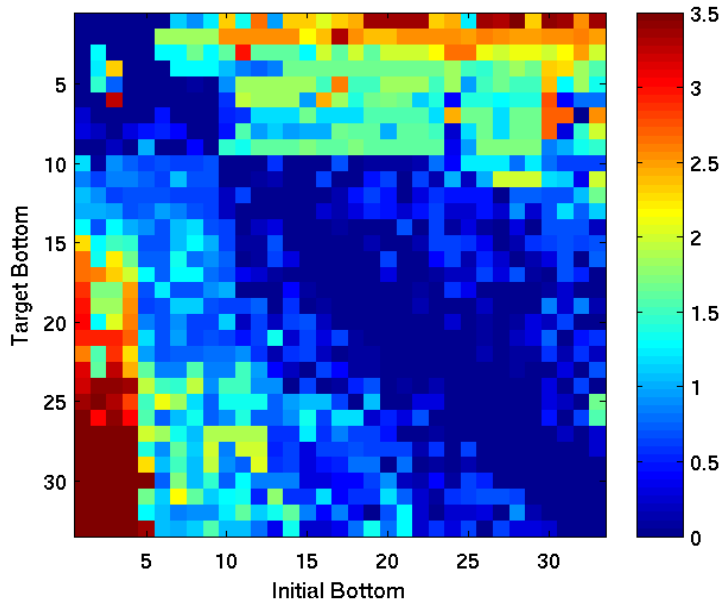
variable	mean		stdev	
rho2	1.68956801	+/-	0.06893113	
nu2	1.08795427	+/-	0.00834763	
delta2	0.00893704	+/-	0.00295667	
sigma2	0.00185622	+/-	0.00011848	
gamma2	3.92301159	+/-	2.11525432	*
w2	0.00001582	+/-	0.00062948	*
d2	0.63765071	+/-	0.01389444	
rho3	2.05942681	+/-	0.04665487	
nu3	1.16064270	+/-	0.00550594	
nu3t	0.01000000	+/-	0.01216358	*
delta3	0.02120026	+/-	0.00698733	
delta3t	0.01000000	+/-	24.29685238	*
sigma3	0.00179095	+/-	0.00010520	

Exploring Uniqueness Issues

Final cost function values



Sum-of-sqrs-of-normed-diffs between final and target parameter vectors



DEFINITIONS:

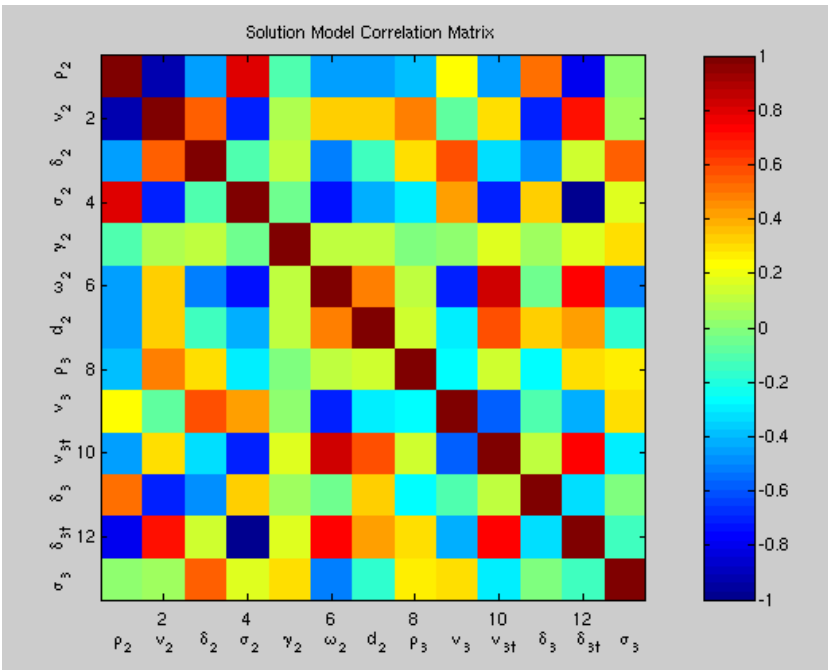
"Target bottom" – the one used to create synthetic data.

"Initial bottom" – the one used for the initial guess model parameters.

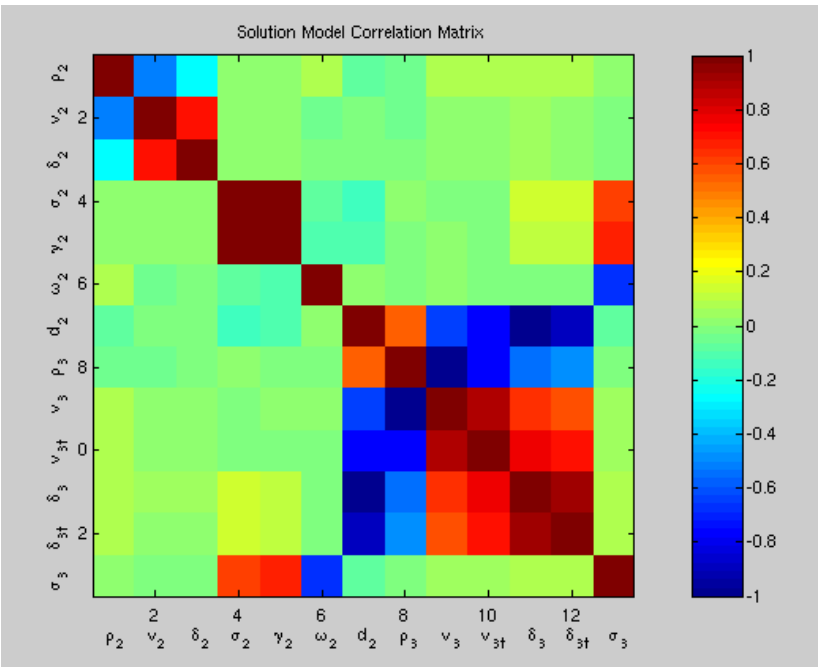
"Bottoms #1-33" – rock… gravel… sand… clay

Correlation Matrix R of Solution Parameters
(Local linear approximation)

$$\mathbf{R}(\mathbf{x}) = \begin{bmatrix} \mathbf{C}_{11}/\mathbf{C}_{11}^2 & \mathbf{C}_{12}/\mathbf{C}_{11}\mathbf{C}_{22} \\ \mathbf{C}_{21}/\mathbf{C}_{11}\mathbf{C}_{22} & \mathbf{C}_{22}/\mathbf{C}_{22}^2 \end{bmatrix}$$



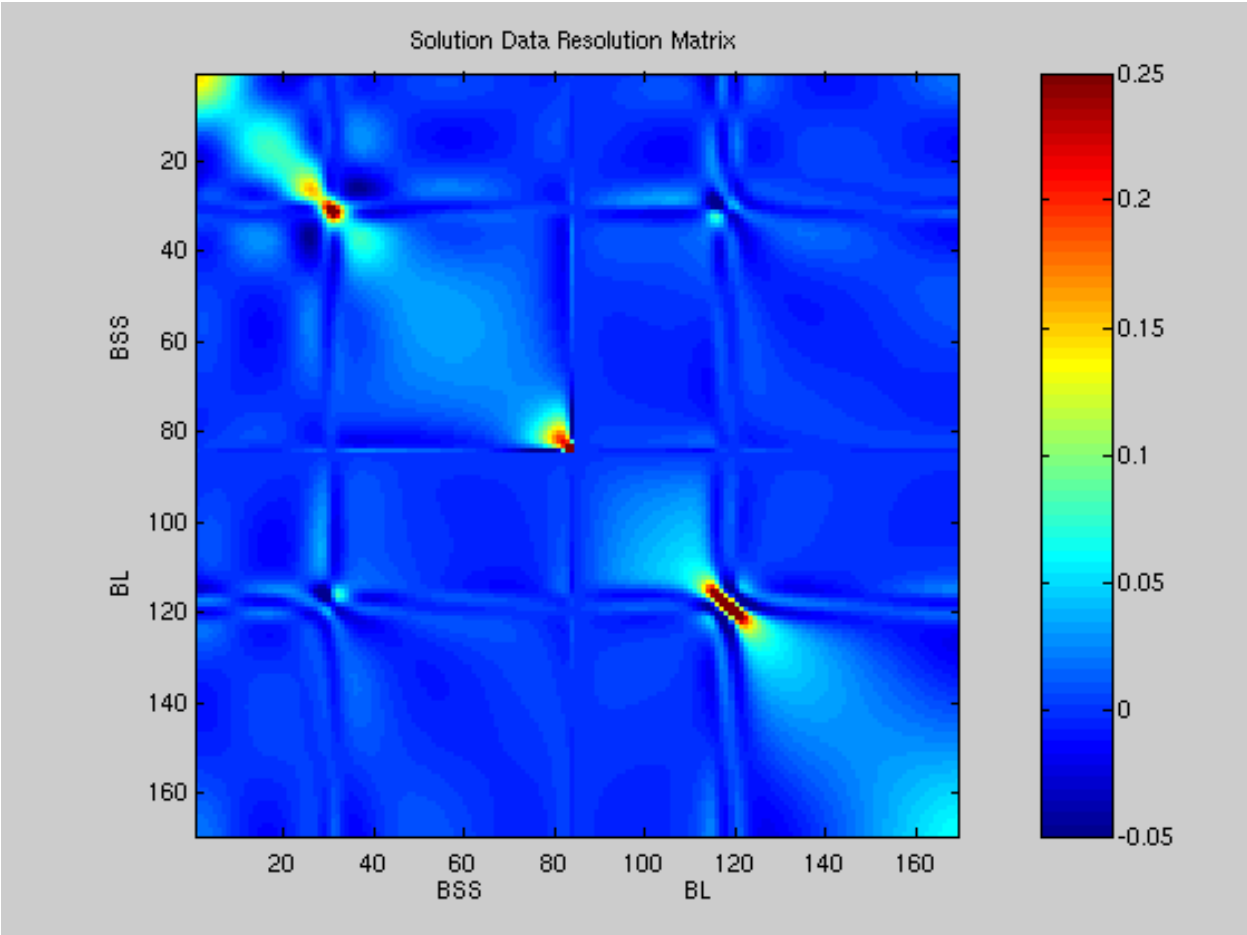
for the previous example inversion run



an easier-to-interpret run

Data Resolution Matrix at the Solution

$$\mathbf{N} = \mathbf{G}\mathbf{G}^{\dagger}$$



Caveats & Future Work

- This work inverts synthetic data – ultimately must invert real data at arbitrary angles.
- Extend the problem to invert reverberation rather than just bottom loss & backscattering.
- This work inverts only one frequency – ultimately must invert multiple frequencies at once.
- Investigate uniqueness problem further by using a global optimization technique such as genetic algorithms or simulated annealing.

3 Sources of BL/BSS Variability

Each involves a Gaussian distribution with mean and standard deviation.
Sources of variability can be “turned off” by using mean & setting std dev to zero.

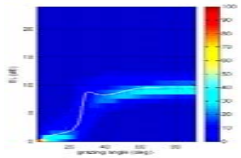
ASIAEX data
variations in measured grainsize

Global data
statistical uncertainty of grainsize regression

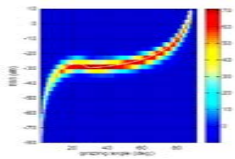
grainsize $M_z = \log_2(r / 1mm)$

ASIAEX data
variations in sediment thickness

measured grainsizes → grainsize to geoacous parameter regression → geoacous parameter distribs → 2 layer gabim-analytic model →



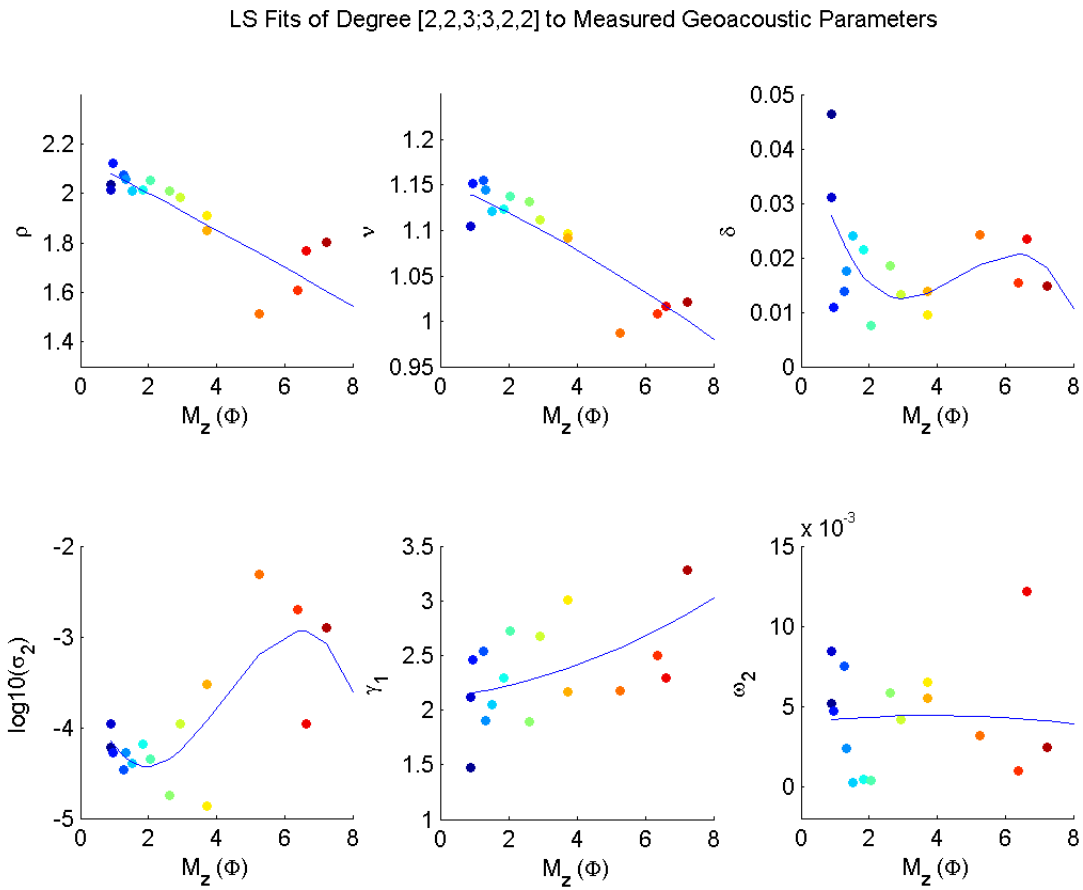
BL/BSS hists



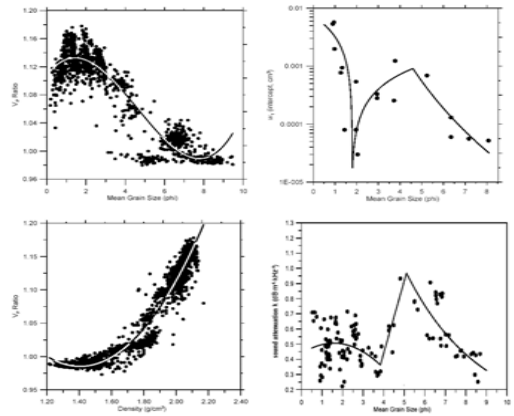
Monte Carlo runs sample those distribution 1000 times,
making 1000 pairs of BL & BSS curves which form the histograms.

Statistical Uncertainty of Grainsize Regression

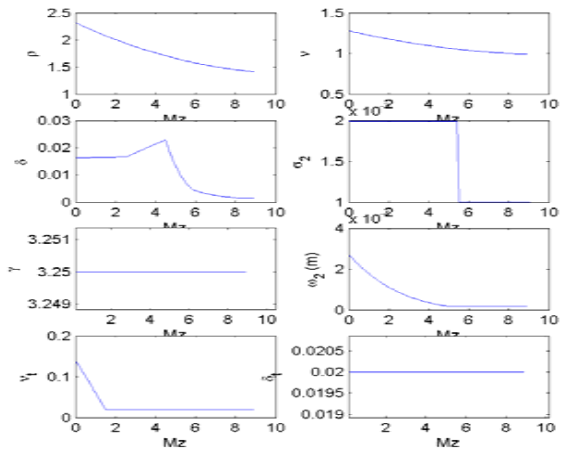
desire continuity and correlated errors about mean fits



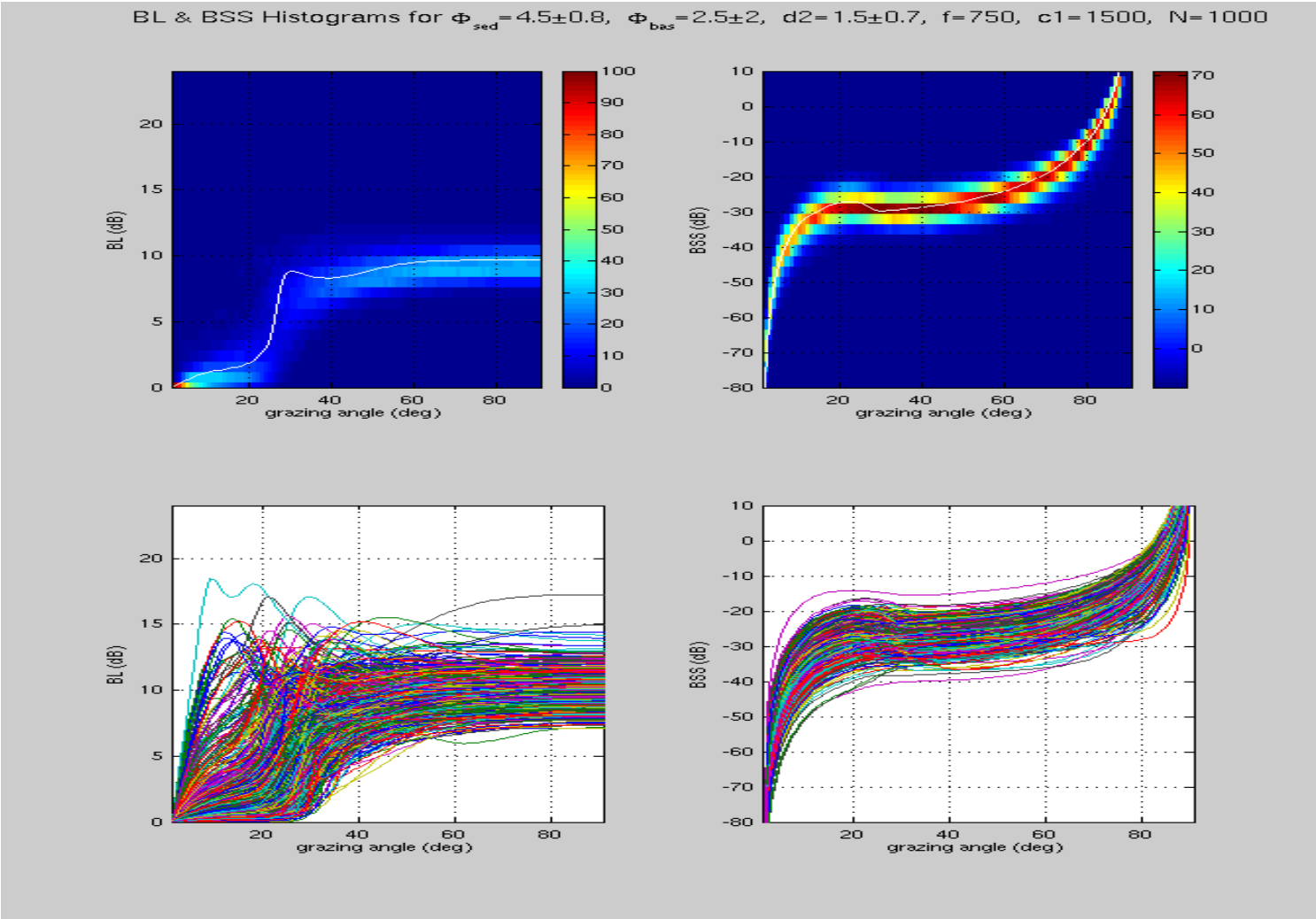
Grainsize Algorithm Update
(Briggs, Jackson, Moravan)



GABIM/Hamilton regressions

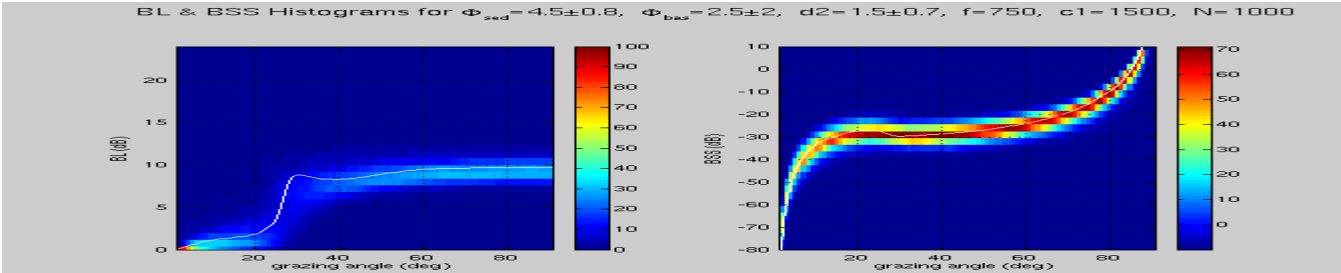


BL/BSS Curves and Their Histograms

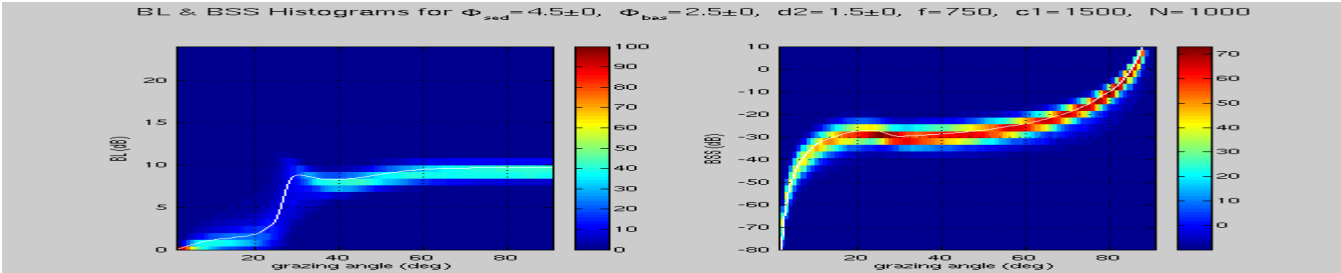


Results for Sources of Variability On/Off

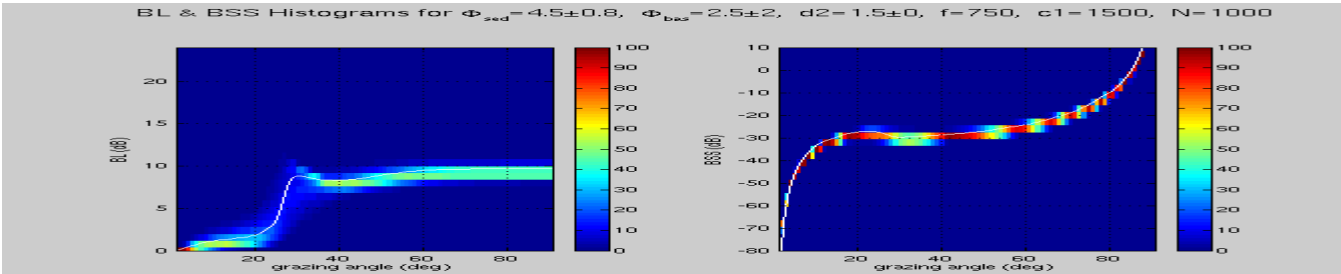
Grainsizes = **on**
Regression = **on**
Thickness = **on**



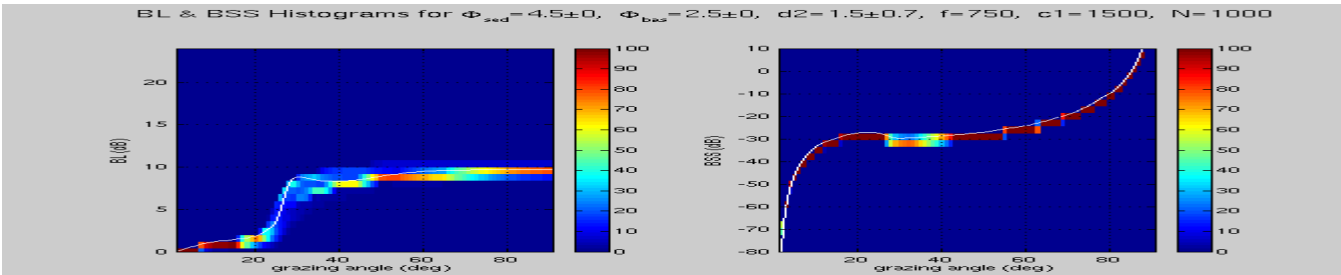
Grainsizes = off
Regression = **on**
Thickness = off



Grainsizes = **on**
Regression = off
Thickness = off



Grainsizes = off
Regression = off
Thickness = **on**



Conclusions - 1

- The Frechet derivatives are the sensitivity functions for the model and data.
  - Large derivatives $\Rightarrow$ data are sensitive to the model
  - Small derivatives $\Rightarrow$ data insensitive to the model
- Fast efficient implementation permits:
  - Rapid characterization of model/data sensitivities
  - Investigation of model parameter coupling and model uniqueness.
  - Experimental design (Resolution matrices are independent of actual data values.)
  - Quantitative characterization of nonlinear inverse solution by linearization about global minimum.

Conclusions - 2

- Application of Monte Carlo techniques to BSS/BL indicates grain size material parameter regression is large source of uncertainty.
- Comparison of BSS/BL from mean parameter values with the histograms of BSS/BL illustrates nonlinearity of the model.

Seabed Variability (C. Holland lead): June 2004 Review – 1

•Publications:

- Odom, R.I., “Frechet derivatives for shallow water ocean acoustic inverse problems,” (abstract), JASA, **113**, p2191, 2003.

- Odom, “Model and data resolution for ocean acoustic for ocean seabed inverse problems,” (in preparation for JASA/IEEE-JOE).

- Ganse, A., Title TBD, content is on Monte Carlo studies (IEEE-JOE)

• Contributions:

-Construction of model and data resolution matrices from analytical expressions for the Frechet derivatives.

· Large magnitude off-diagonal terms indicate model non-uniqueness.

Seabed Variability (C. Holland lead): June 2004 Review - 2

- Contributions continued:
  - · Important because the resolution matrices provide direct information about data resolution and model uniqueness
  - · Valuable for experimental design
  - - Monte Carlo simulations for bottom reflection loss and bottom backscattering (A. Ganse).
    - · Employs Frechet derivatives together with grain size grain size information to show variability of BSS and BRL with respect to grain size range.
- Seabed Team contributions:
  - - Beginning to quantify how depositional processes affect acoustics by their control of grain size.

Seabed Variability (C. Holland lead): June 2004 Review - 3

- Seabed Team contributions continued:
  - Brought together quite a disparate group of individuals with very different perspectives and cross-disciplinary perspectives.

- Database weaknesses
  - Bottom grain size data is extremely sparse (also noted by J. Fulford).
    - Yields large variances, and sometimes the data is so sparse that it is impossible to compute a real variance. The "engineering solution" to this problem is to use a variance computed from other regions.
    - Only real solution is to collect more data.