PHYSICAL-ACOUSTICAL DATA ASSIMILATION AND CHARACTERIZATION AND TRANSFER OF UNCERTAINTIES TO THE SONAR: ERROR SUBSPACE STATISTICAL ESTIMATION

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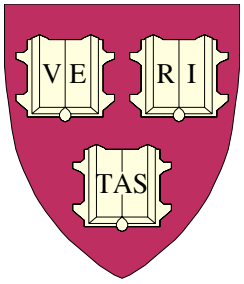
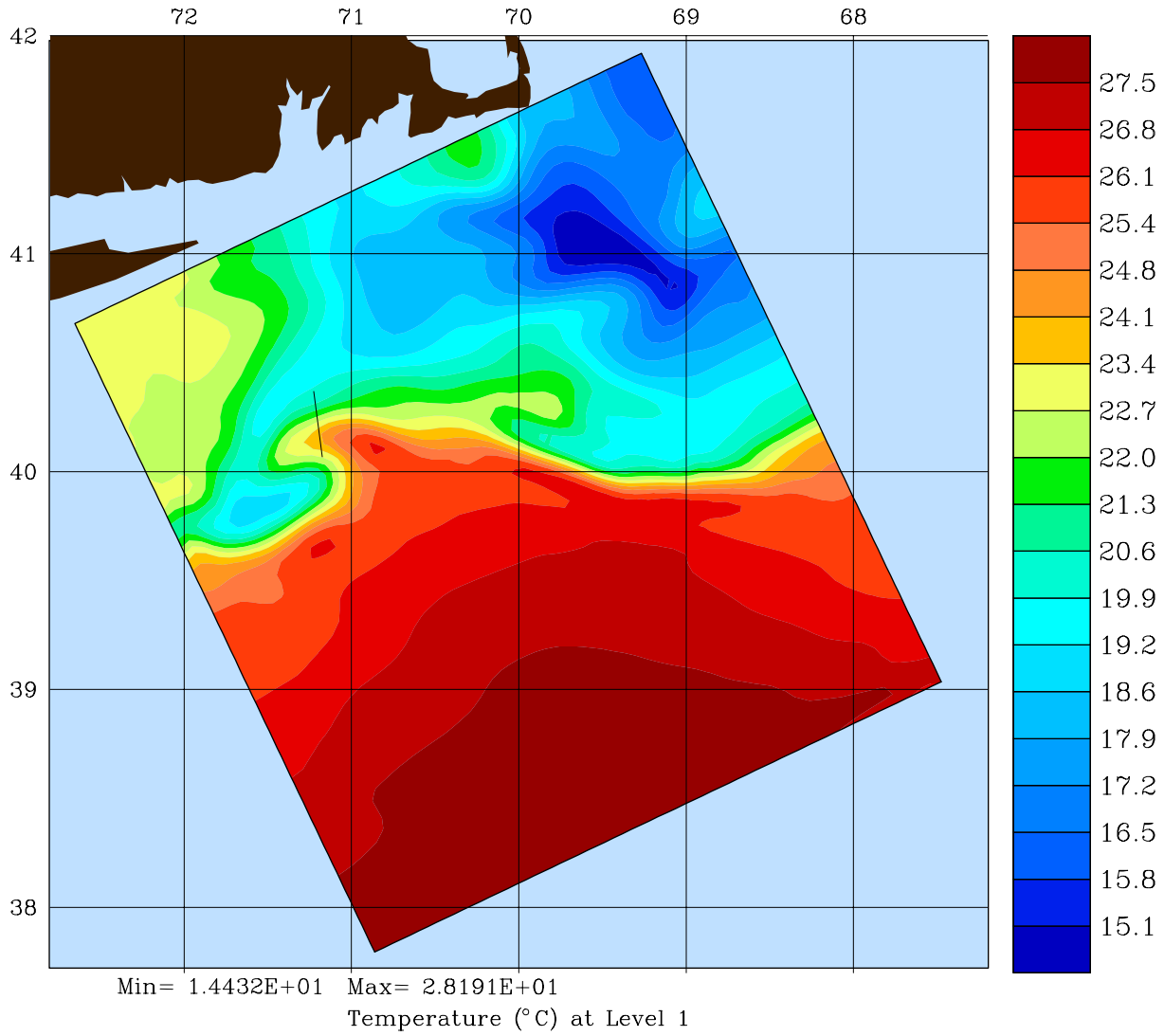



72
71
70
69
68
42
41
40
39
38
27.5
26.8
26.1
25.4
24.8
24.1
23.4
22.7
22.0
21.3
20.6
19.9
19.2
18.6
17.9
17.2
16.5
15.8
15.1
Min= 1.4432E+01 Max= 2.8191E+01
Temperature (°C) at Level 1

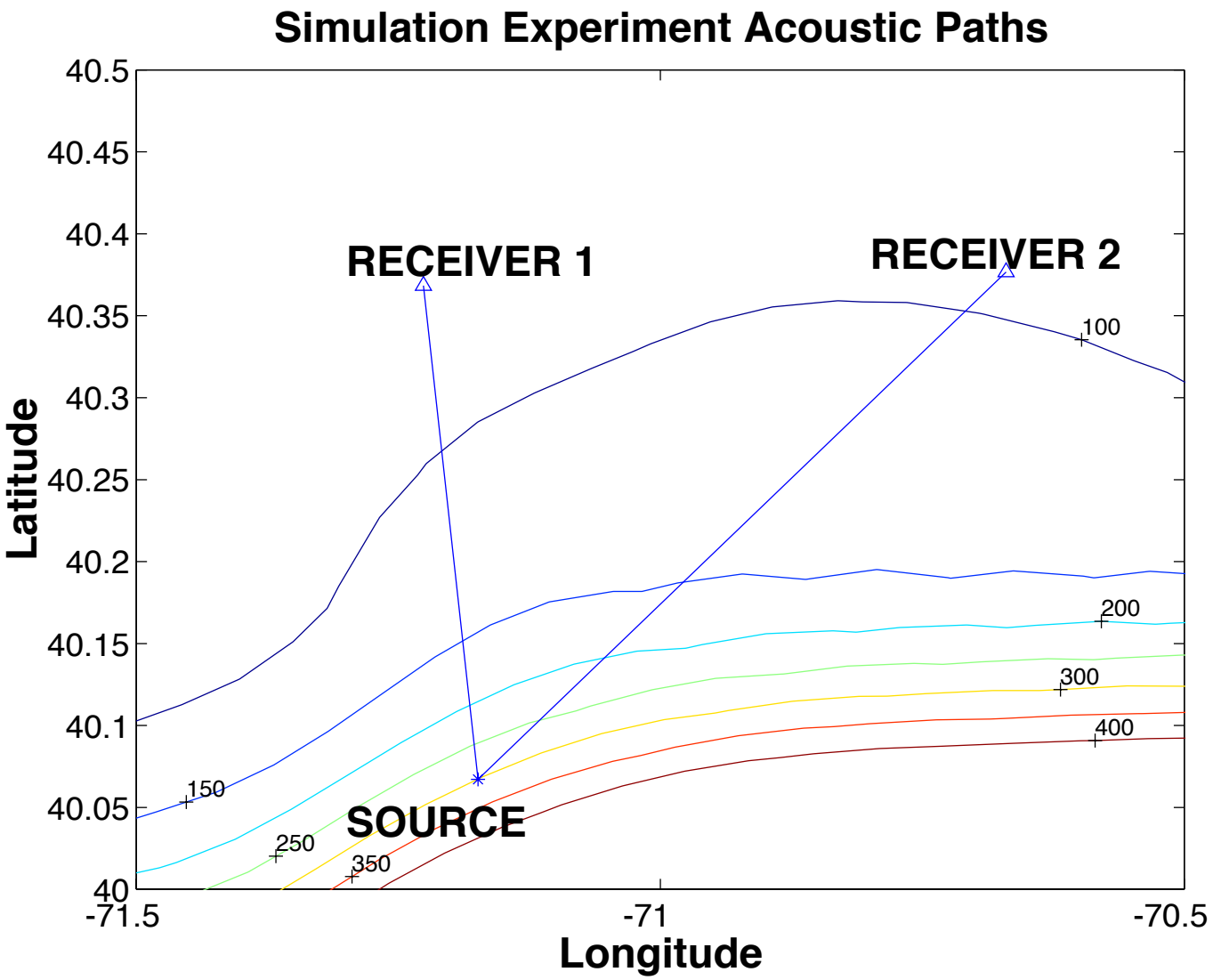
COUPLED PHYSICAL-ACOUSTICAL DYNAMICAL MODELS

- **Physical model: Primitive-Equation (PDE, x, y, z, t: HOPS)**

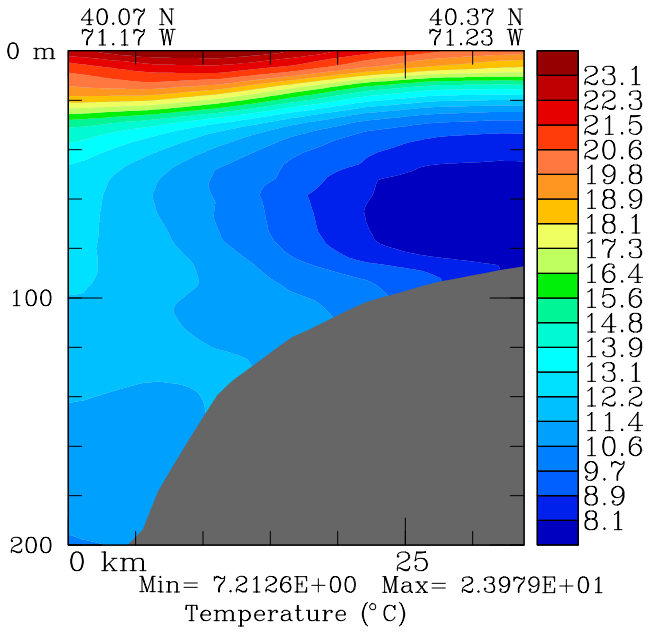
Horiz. Momen.	$\frac{D\mathbf{u_h}}{Dt} + f\,\mathbf{e}_3 \wedge \mathbf{u_h} = -\frac{1}{\rho_0}\nabla_h\, p_w + \nabla\cdot(A\nabla\mathbf{u}_h)$	(1-2)
Vert. Momen.	$\rho g + \frac{\partial p_w}{\partial z} = 0$	(3)
Thermal energy	$\frac{DT}{Dt} = \nabla\cdot(K\nabla T)$	(4)
Cons. of salt	$\frac{DS}{Dt} = \nabla\cdot(K^s\nabla S)$	(5)
Cons. of mass	$\nabla\cdot\mathbf{u} = 0$	(6)
Eqn. of state	$\rho(\mathbf{r}, z, t) = \rho(T, S, p_w)$	(7)
Sound speed eqn.	$c(\mathbf{r}, z, t) = C(T, S, p_w)$	(8)

- **Acoustical model: Coupled Normal-Mode model (PDE, f, r, z, t: NPS)**

Wave eqn.	$\rho\, c^2(\mathbf{r}, z, t)\,\nabla\cdot\left(\frac{1}{\rho}\nabla p_s(\mathbf{r}, z, t)\right) = \frac{\partial^2 p_s(\mathbf{r},z,t)}{\partial t^2}$	
p_s transfer fct.	$\nabla^2 P_s - \frac{1}{\rho}\nabla\rho\cdot\nabla P_s + k^2 P_s = -2\frac{r_0}{r}\,\delta_{(r-r_0)(z-z_0)}$	
	where $k \doteq 2\pi f/c(\mathbf{r}, z, t)$	(9)
Coupled	With $P_s(r, z; f) \doteq \sum_n \frac{r_0}{\sqrt{r}} P_n(r; f) Z_n(z; r, f)$	(10)
Normal-modes	$\left\{\frac{\partial^2}{\partial z^2} - \frac{1}{\rho(r,z)}\frac{\partial\rho(r,z)}{\partial z}\frac{\partial}{\partial z} + (k(r,z)^2 - k_n(r;f)^2)\right\}$ $\times Z_n(z; r, f) = 0$	(11)
Modal amplit.	$\left(\frac{d^2}{dr^2} + k_n^2\right) P_n = -\sum_m \left(\gamma_{mn}\frac{d}{dr} + C_{mn}\right) P_m$	(12)

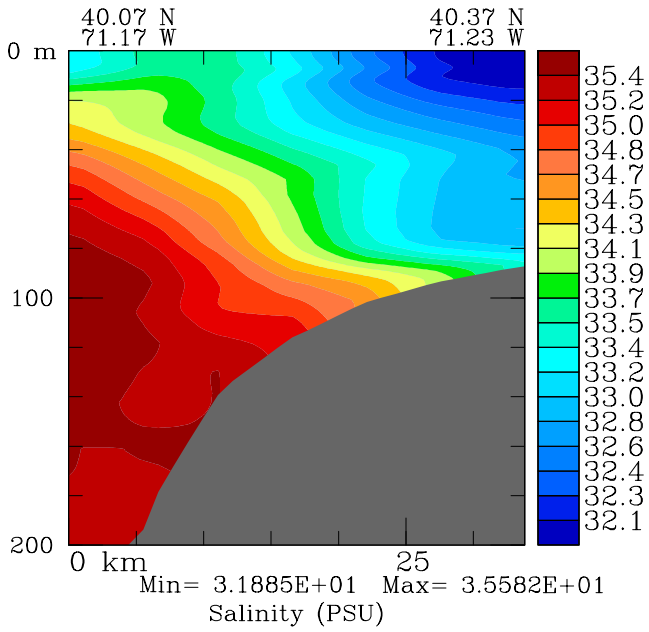


Acoustic paths considered (as in Shelfbreak-PRIMER), overlaid on bathymetry.



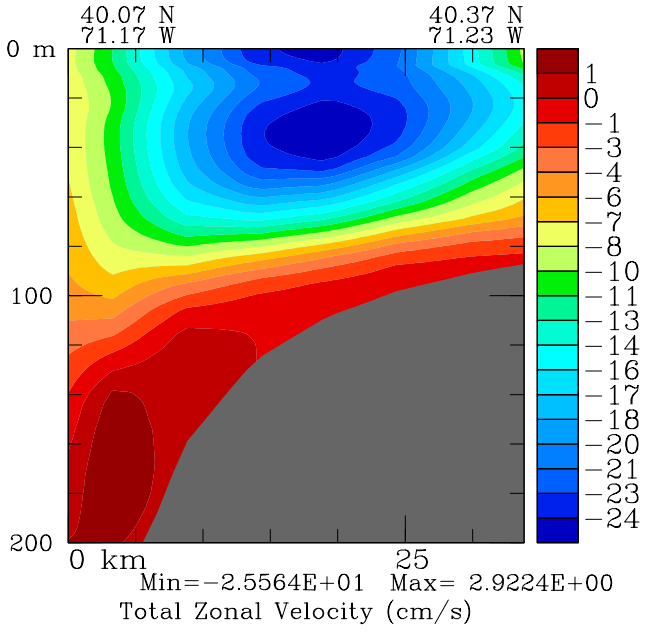
Min= 7.2126E+00 Max= 2.3979E+01

Temperature (°C)



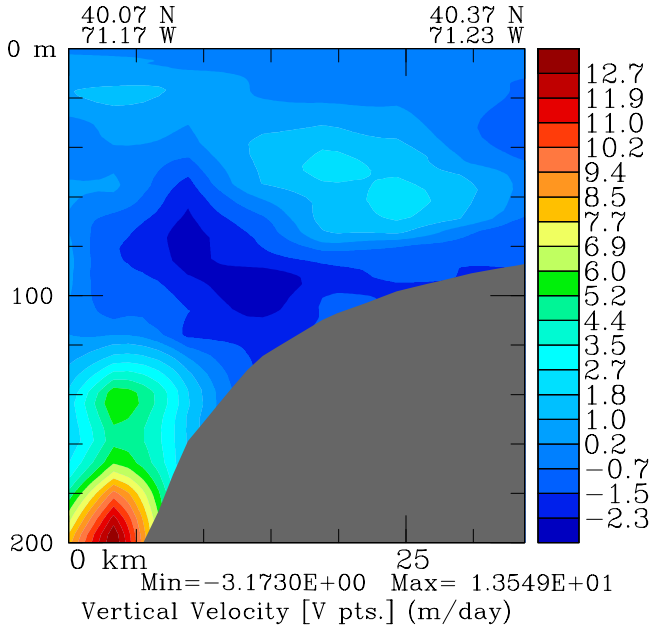
Min= 3.1885E+01 Max= 3.5582E+01

Salinity (PSU)



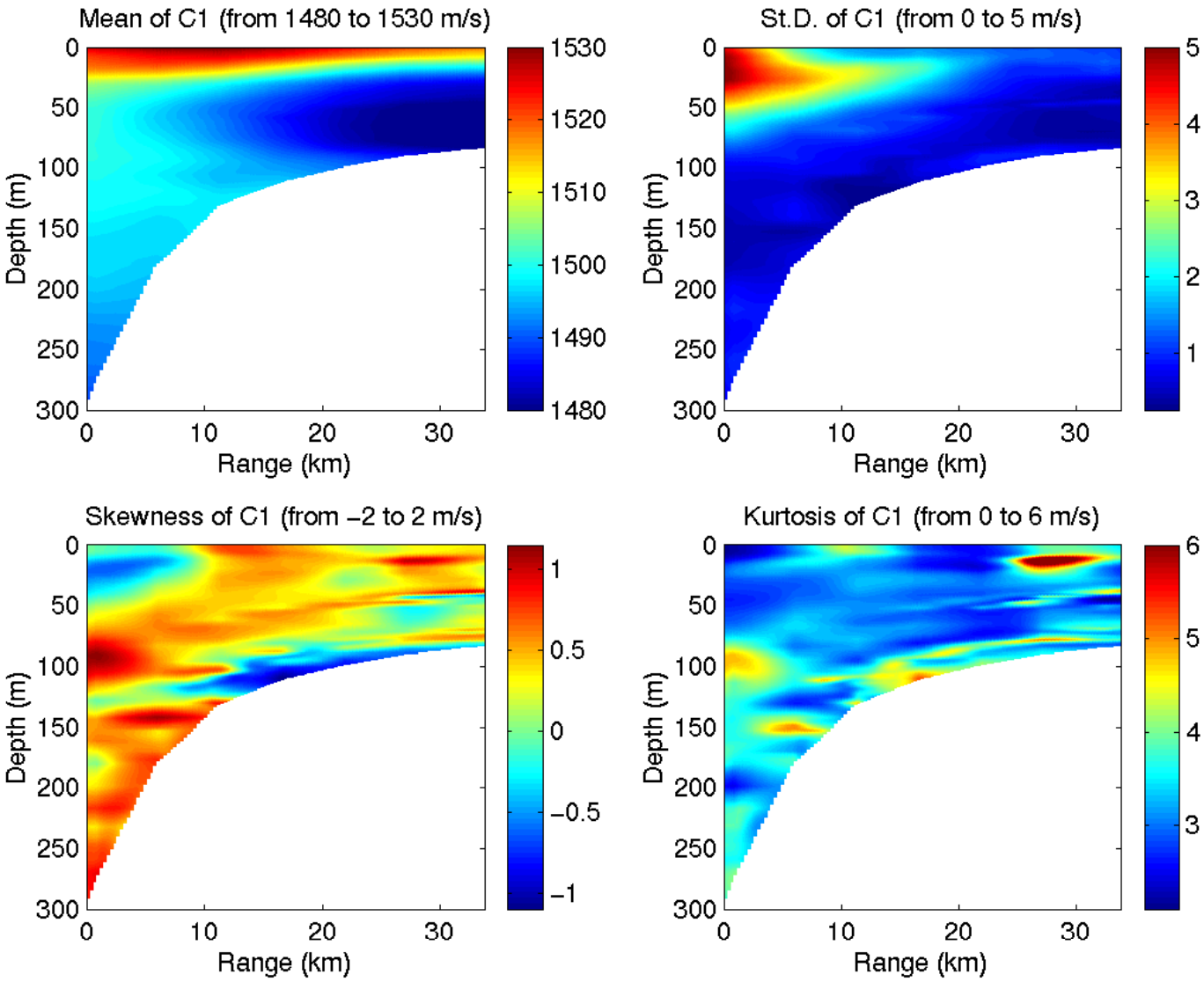
Min=−2.5564E+01 Max= 2.9224E+00

Total Zonal Velocity (cm/s)

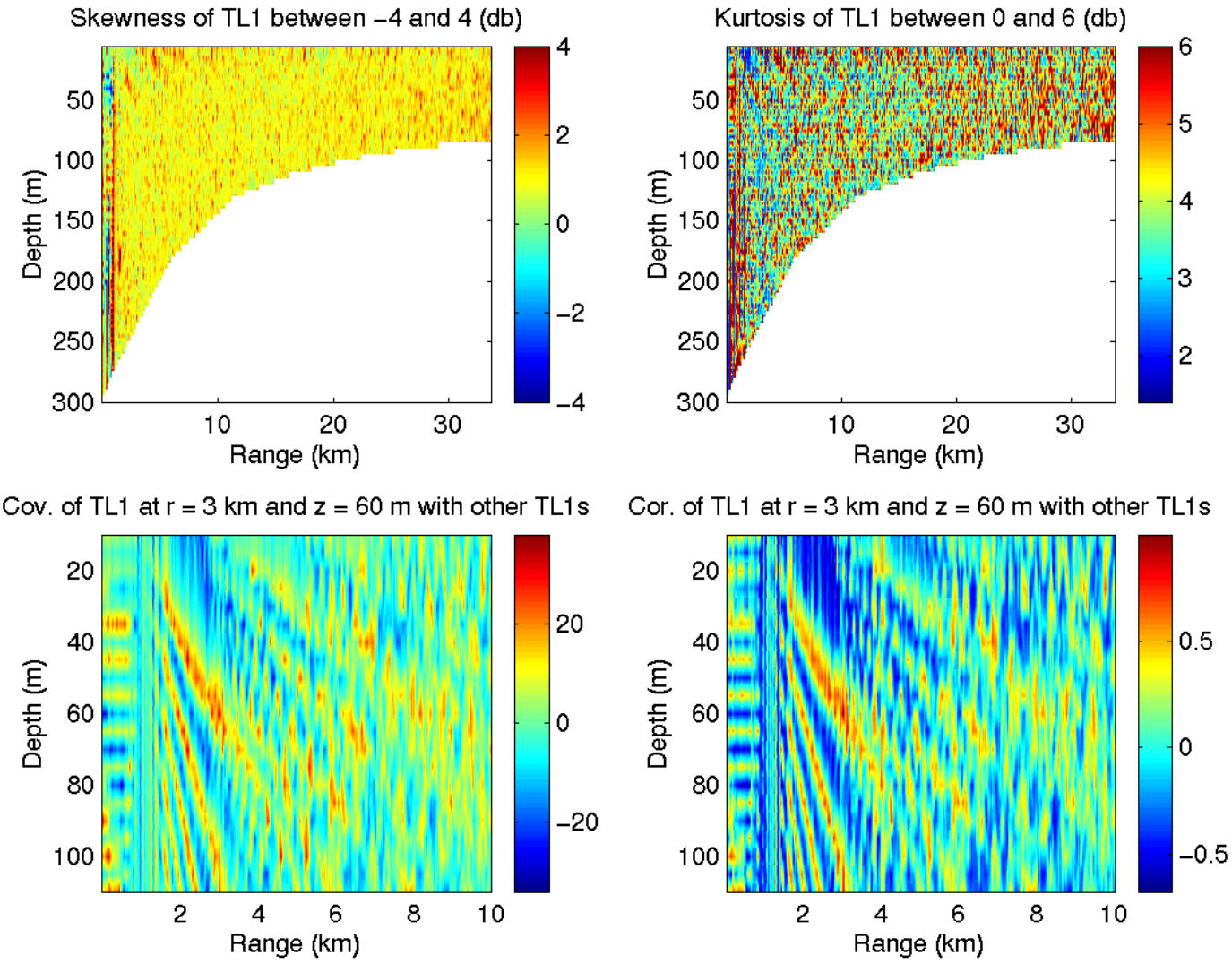


Min=−3.1730E+00 Max= 1.3549E+01

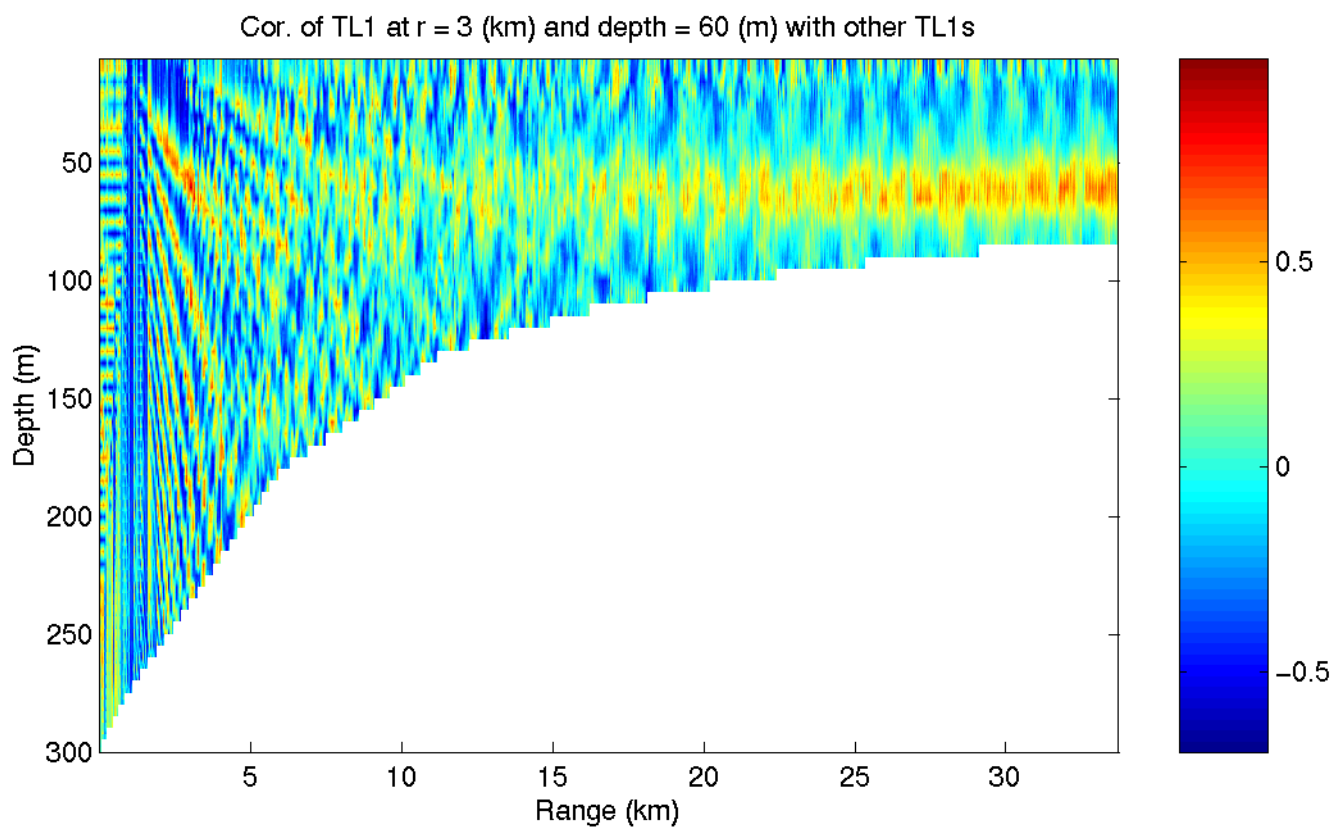
Vertical Velocity [V pts.] (m/day)

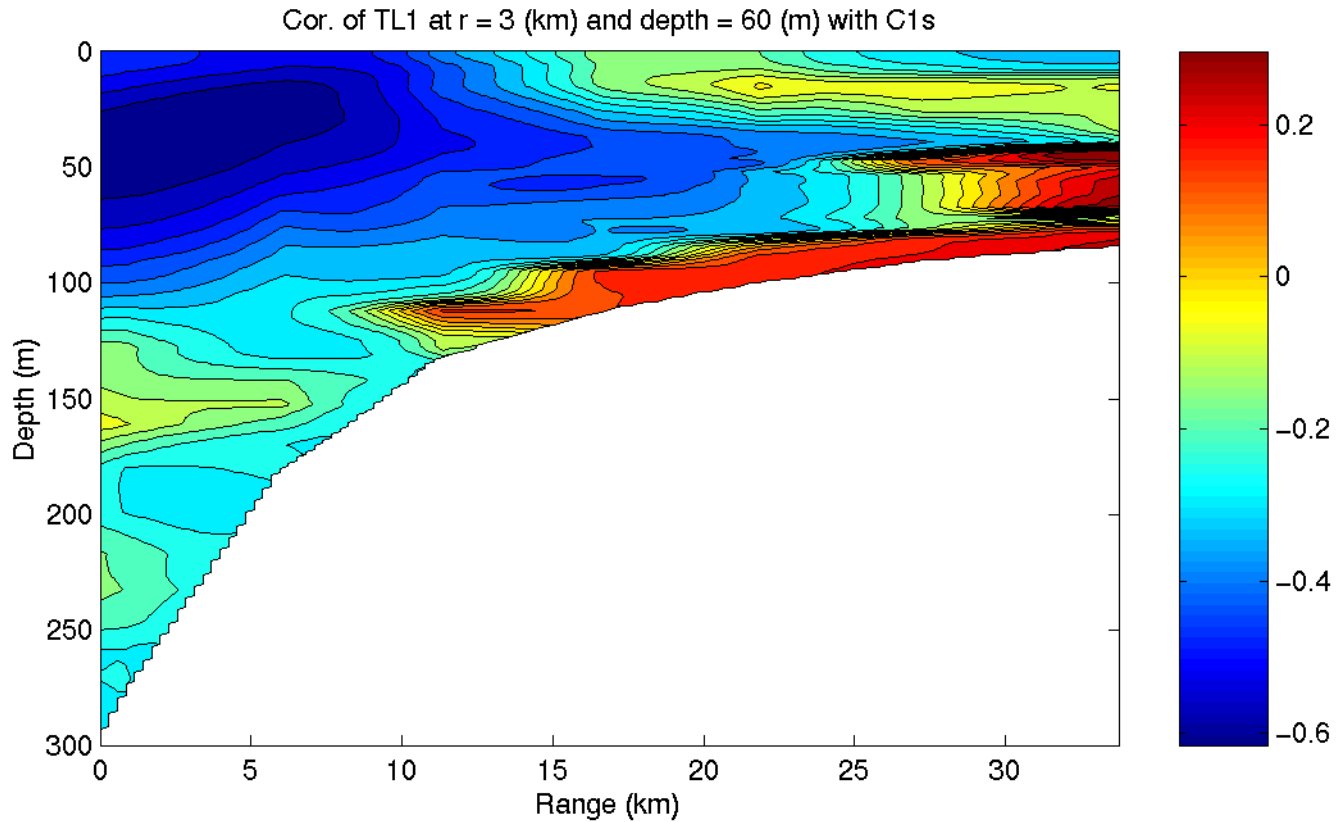


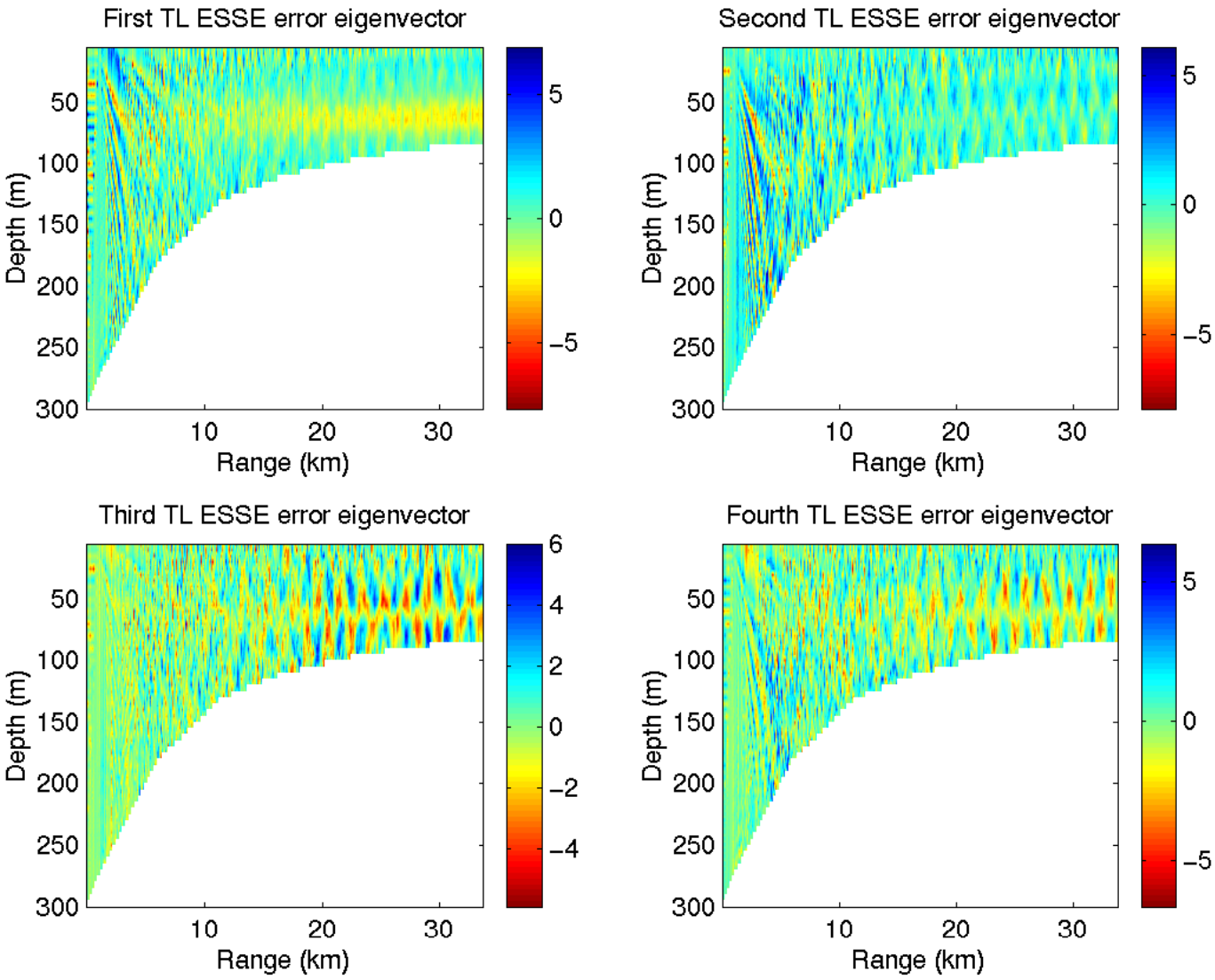
Mean of C1 and statistics of error estimate for C1



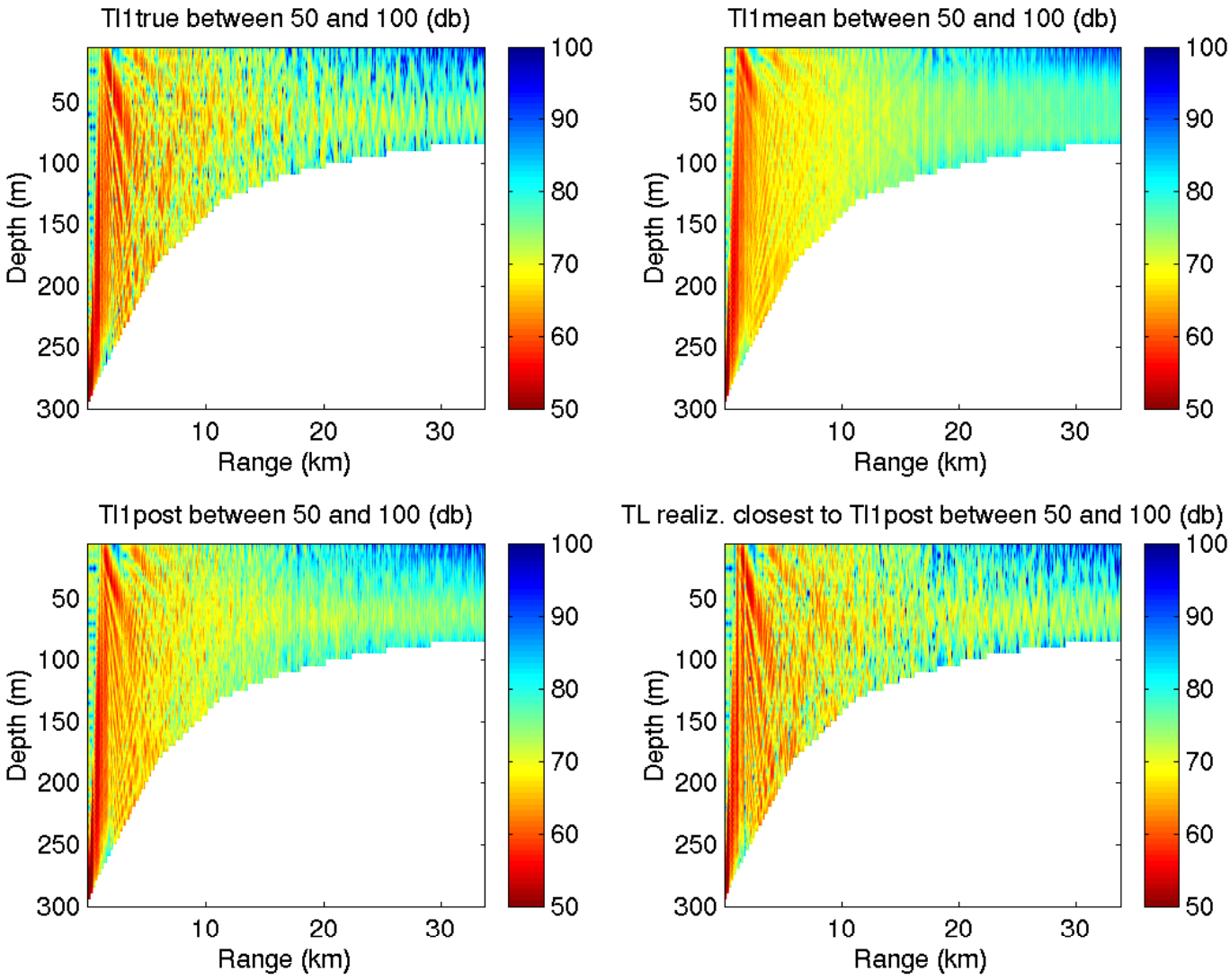
TL error statistics: sample skewness, kurtosis and zoom on a covariance/correlation field.


Cor. of TL1 at r = 3 (km) and depth = 60 (m) with other TL1s
Depth (m)
50
100
150
200
250
300
5
10
15
20
25
30
Range (km)
0.5
0
−0.5

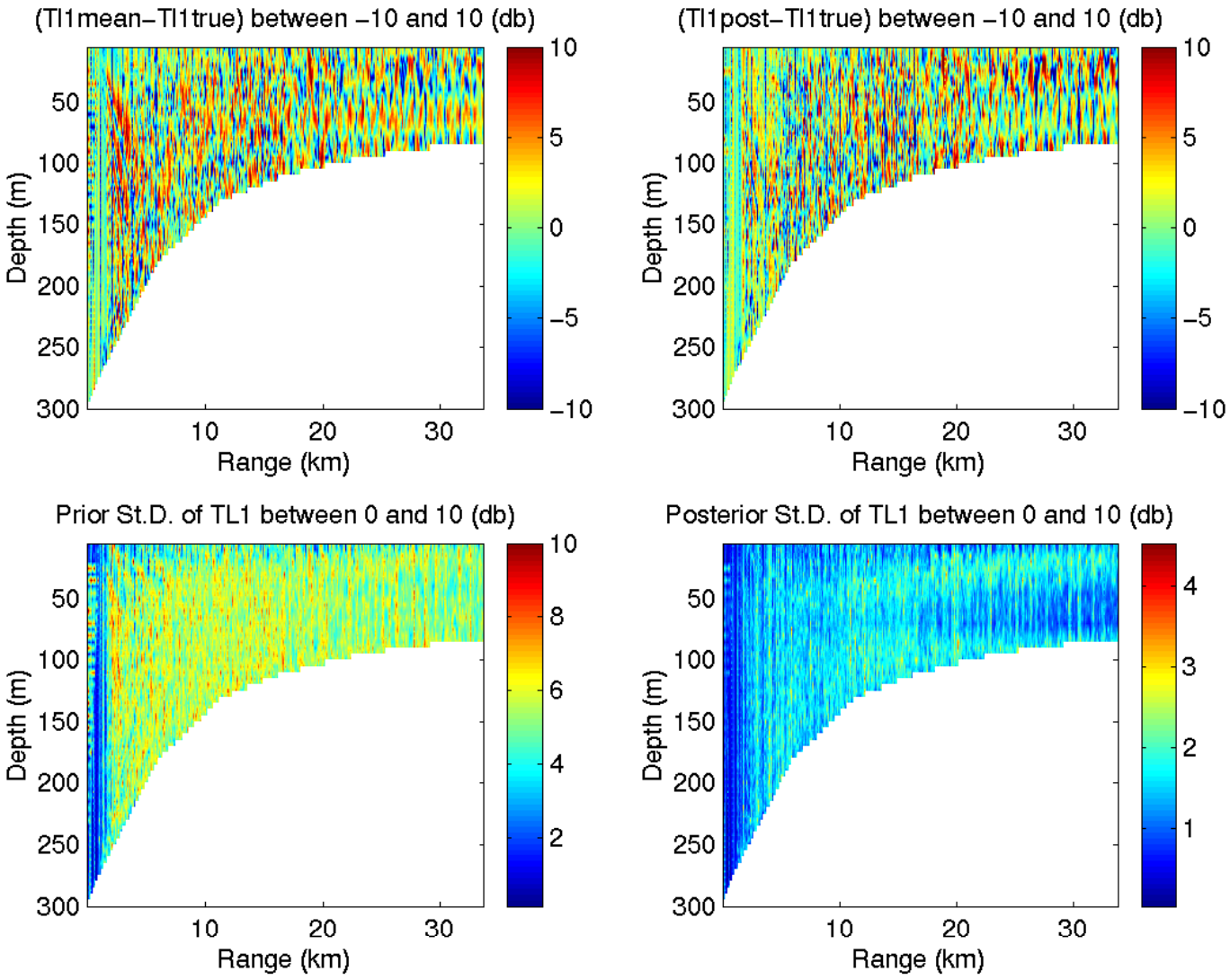

Cor. of TL1 at r = 3 (km) and depth = 60 (m) with C1s
Depth (m)
0
50
100
150
200
250
300
0
5
10
15
20
25
30
Range (km)
0.2
0
−0.2
−0.4
−0.6



First four ESSE error eigenvectors for TL along section 1: they explain 12.4%, 6%, 4.2% and 3.6% of the total TL error variance, respectively

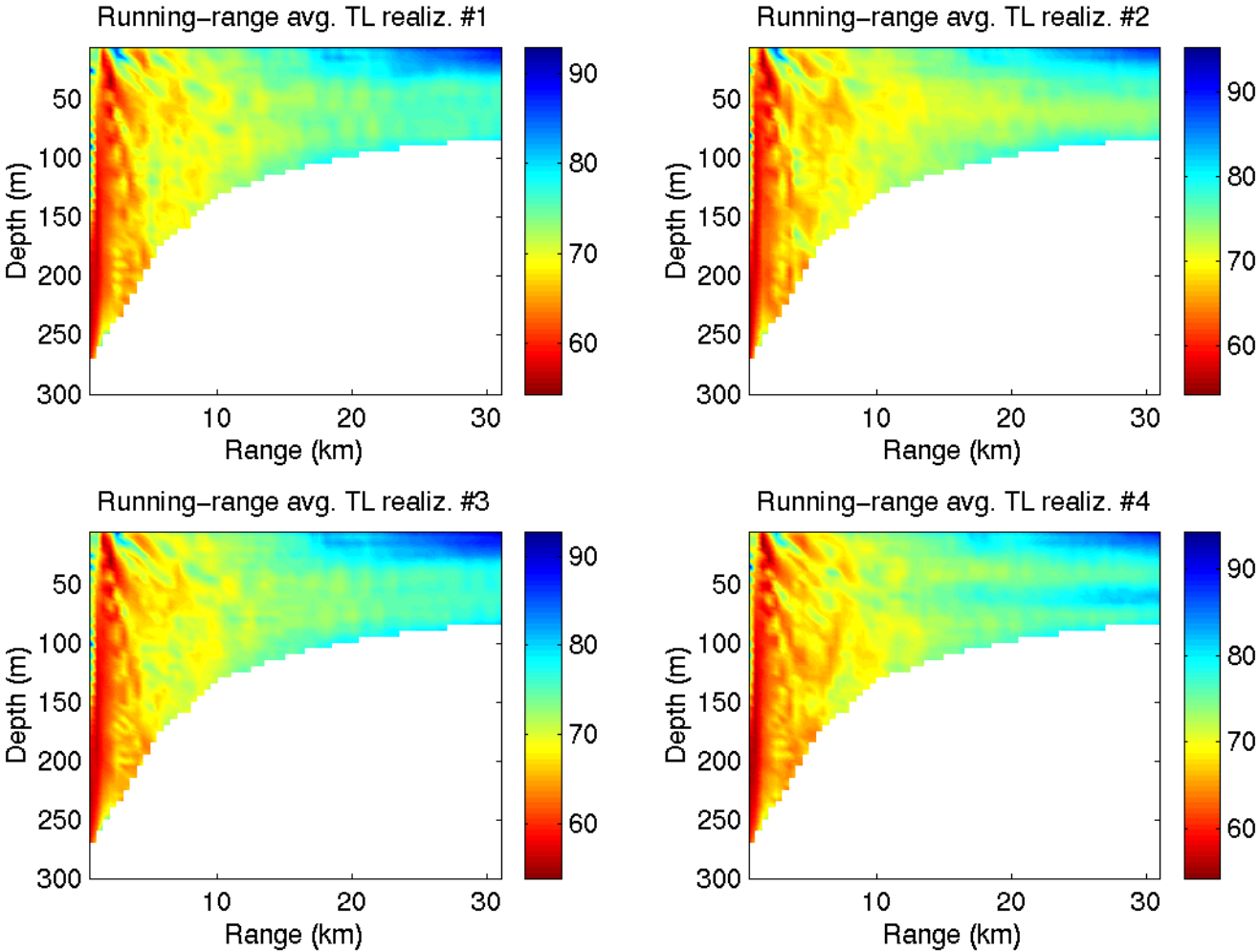


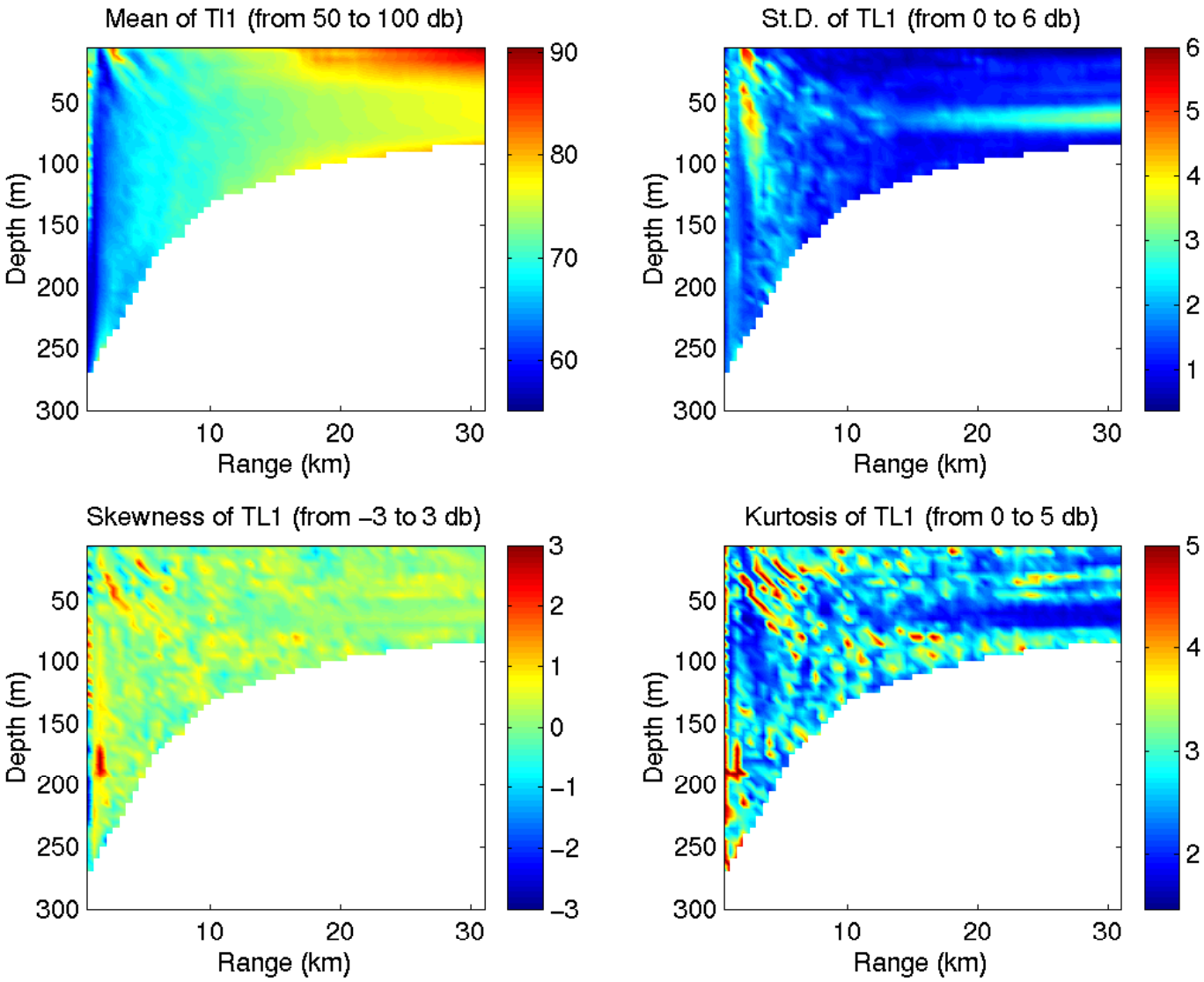
ESSE assimilation results (Twin Experiment):
"True" TL (from which towed-receiver data are sub-sampled),
a priori **TL (ensemble mean forecast),**
a posteriori **TL (after data assimilation) and**
TL realization closest to *a posteriori* **TL.**



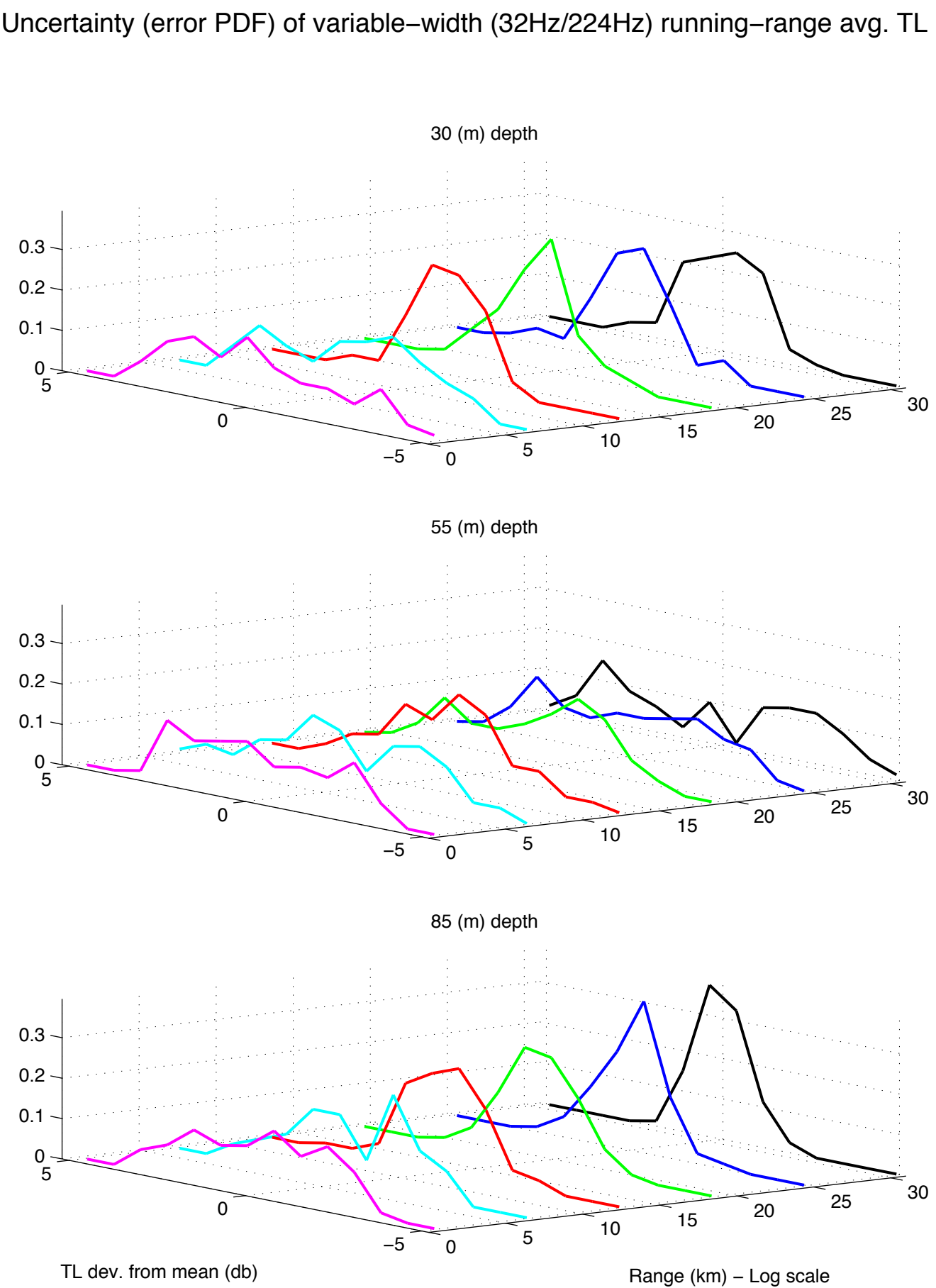
A priori **(before assi.) and** *a posteriori* **(after assi.) residuals, and** *a priori* **and** *a posteriori* **error Std. dev. for TL along section 1.**

Var.−width (32Hz/224Hz) running−range avg. TL realiz. #1−4 (from 50 to 100 db)





Mean and statistics of error estimate for the var.-width (32Hz/224Hz) running-range avg. TL1


Uncertainty (error PDF) of variable−width (32Hz/224Hz) running−range avg. TL
30 (m) depth
55 (m) depth
85 (m) depth
0.3
0.2
0.1
0
5
0
−5
0
5
10
15
20
25
30
TL dev. from mean (db)
Range (km) − Log scale