

# **ROMS/TOMS Tangent Linear and Adjoint Models: Testing and Applications**

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## **LONG-TERM GOALS**

Our long-term technical goal is to produce a tested tangent linear model (TLM) and adjoint model (ADM) for ROMS/TOMS (Regional Ocean Modeling System/Terrain-Following Ocean Modeling System) that is suitable for general use by the ROMS/TOMS community. To develop computational platforms based on the TLM and ADM for variational data assimilation, ensemble forecasting and sensitivity analysis. Our long-term scientific goal is to model and predict the mesoscale circulation and the ecosystem response to physical forcing in the various regions of the world ocean through state estimation.

## **OBJECTIVES**

We seek to develop an adjoint model for ROMS/TOMS, which is a state-of-the-art ocean model for high-resolution scientific and operational applications (Haidvogel et al., 2000; Shchepetkin and McWilliams, 2003). We also plan to develop algorithms for fitting the model to observations via adjoint techniques. The resulting codes will be suitable for exploring the predictability of the circulation in regional ocean models in a variety of dynamical regimes.

## **APPROACH**

This is fundamentally a collaborative effort involving University of Colorado (A. Moore), Rutgers University (H. Arango) and Scripps (B. Cornuelle, Post-Doc E. Di Lorenzo, A. Miller, and D. Neilson). Our approach is to write the tangent linear and adjoint models for ROMS/TOMS by hand. With each participant in the project contributing expertise in coding and model testing, the approach is feasible. As the development is accomplished, the assimilation scheme is tested in various scenarios involving observations. The Scripps contingent will test the adjoint for ROMS/TOMS in the California Current CalCOFI region where they are presently applying ROMS (under NASA funding) to a

physical-biological data synthesis and a model forecast scenario. Arango and Moore will test the adjoint in the Mid-Atlantic Bight (under NSF funding) for coupled atmosphere-ocean hindcast experiments using data collected at the observational network centered at the Long-Term Ecosystem Observatory (LEO-15).

## **WORK COMPLETED**

The ROMS/TOMS adjoint team met many times over the past three years in intensive tangent linear and adjoint model writing and testing sessions. A working 2D and 3D tangent linear and adjoint model is now running and being used in various idealized and realistic applications.

The computational platforms for computing eigenmodes, adjoint eigenmodes, singular vectors, forcing singular vectors, stochastic optimals, and pseudospectra of the tangent linear propagator have been completed and tested. The 4 Dimensional Variational data assimilation (4DVar) is being constructed and tested. The observational infrastructure, cost function, pre-conditioning and descent algorithms for 4Dvar have been built but still need testing.

Currently, we are developing a computational platform for ensemble prediction based on the TLM and ADM of ROMS/TOMS. The plan is to use Singular Vectors (SVs) to perturb the initial conditions of each ensemble member. Perturbing the system along the most unstable directions of the state space defined by the SVs yields information not only about the first moment (ensemble mean) of the Probability Density Function (PDF) but also about the tails of the PDF, specifically its second moment (the ensemble spread).

Some of the results are described by Moore et al. (2003) and Arango et al. (2003), including the tangent linear and adjoint components of ROMS, and examples of singular vectors and stochastic optimals for a time-evolving double gyre ocean model.

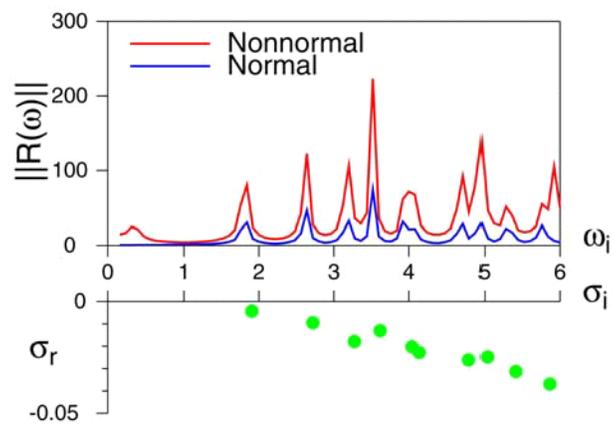
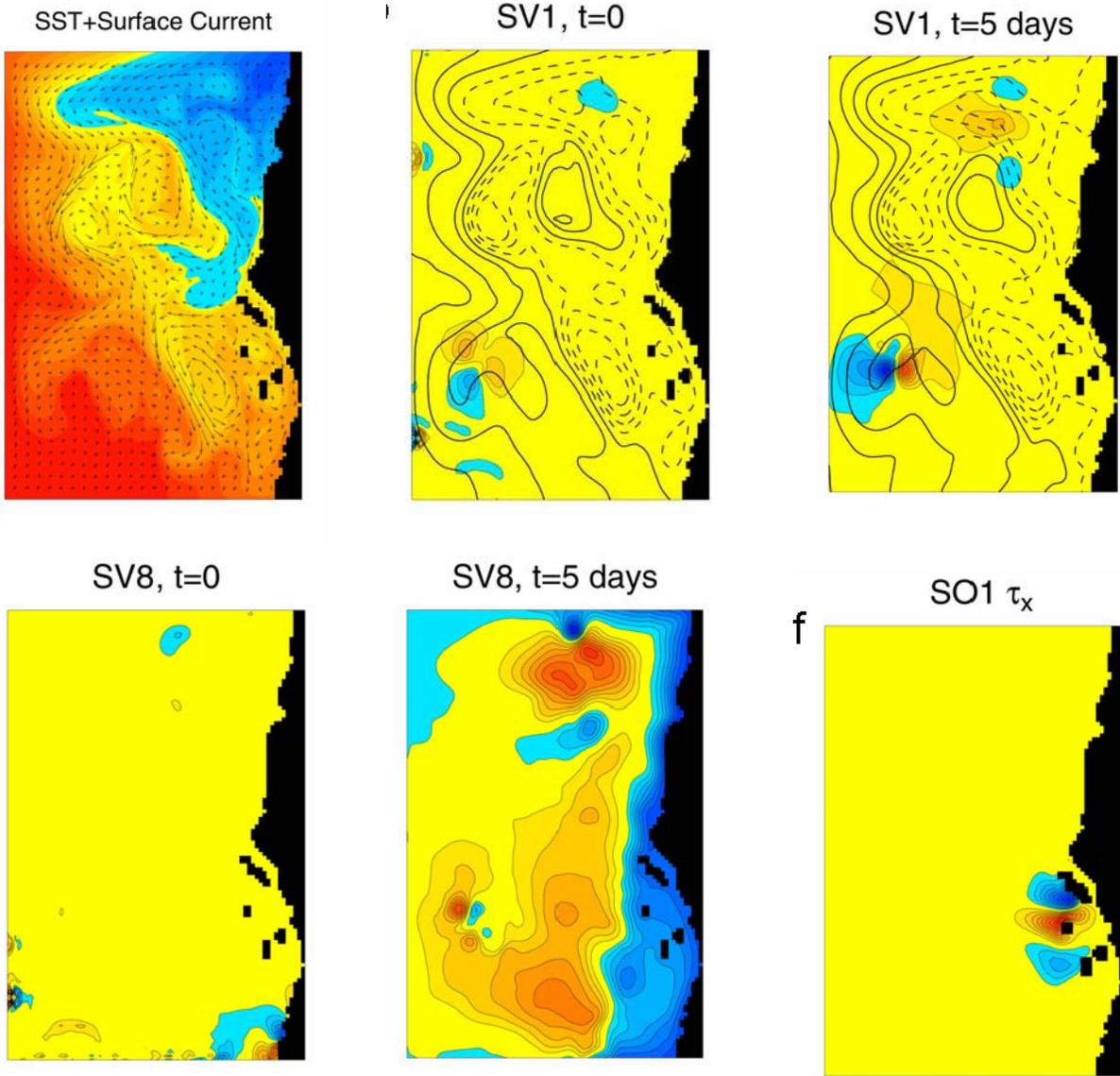
## **RESULTS**

The tangent linear and adjoint models can be used to study error growth and predictability. In the linear limit, the most rapidly growing of all possible perturbations, for a chosen norm, are the singular vectors of the tangent linear propagator  $\mathbf{R}$ . Rapid growth is often achieved by singling out the highly nonnormal eigenmodes of  $\mathbf{R}$ . Examples of SVs using ROMS in the Southern California Bight are shown in Figure 1. A snapshot of the nonlinear model solution about which  $\mathbf{R}$  is linearized is shown in Figure 1a in which the California Current System (CCS) and associated meanders can be seen. Figures 1bc (colored contours) show the initial and final structures of perturbation sea level for a SV that maximizes the growth of perturbation energy over a 5-day period. The solid contours show the corresponding nonlinear model sea level, which is a surrogate for surface stream function. The SV favors the major straining and shearing regions for growth. A second SV for the same flow is also shown which is initially confined to the southern boundary (Figure 1d) and grows to fill much of the domain during the next 5-days (Figure 1e). This latter SV provides information about the sensitivity of the interior flow to the open boundary conditions. Each of the SVs shown grow in amplitude by an order of magnitude.

The tangent linear and adjoint models also can be used to compute Forcing Singular Vectors (FSVs) which represent the patterns of forcing, constant in time, that are most disruptive to the system in that

they yield the largest growth of the chosen norm over the interval. FSVs can provide information about the effect of forcing biases on the ocean circulation, and are used in numerical weather prediction to quantify the effect of model bias on forecast errors. The Stochastic Optimals (SOs) represent the most disruptive patterns of stochastic forcing (Farrell and Ioannou, 1993; Kleeman and Moore, 1997). The SOs provide information about the influence of stochastic variations in ocean forcing due to say *weather* on the circulation. If the circulation itself is a result of stochastic forcing, then the SOs are associated with the component of the circulation that possesses the greatest predictability (Chang et al., 2003ab). Figure 1f shows an example of the zonal wind stress component of a SO for the Southern California Bight. This represents the pattern of stochastic forcing that maximizes the perturbation energy variance over a 5-day period. The gravest FSV for the same flow is very similar.

The response of the ocean circulation to any Fourier component of the forcing is also of interest, since typically the ocean forcing and its errors and uncertainties will possess energy over a wide range of frequencies. The usual ideas of resonance apply only to normal systems while for nonnormal systems the ideas of *pseudoresonance* apply (Trefethen et al., 1993). In this case, the response of the ocean to forcing at a particular frequency can be significantly enhanced by the nonnormal dynamics of the system compared to what one would expect from usual resonance arguments. In particular, the response of the circulation can be very large at frequencies that are far removed from the eigenmode frequencies. Pseudoresonance and nonnormality can be conveniently quantified by computing pseudospectra. A pseudospectrum for the Southern California Bight circulation of Figure 1a is shown in Figure 1g. The lower panel of Figure 1g shows a portion of the eigenspectrum of  $R$  where  $\sigma = \sigma_r + i\sigma_i$  denotes the eigenfrequency. The upper panel shows the response of the TLM to forcing of frequency  $\omega_i$  based (a) on traditional resonance arguments (blue curve), and (b) on pseudoresonance due to nonnormality of the flow (red curve). Figure 1g indicates that the response of the system at any forcing frequency is significantly elevated due to the shearing and straining components of the circulation.



**Figure 1. Southern California Bight tangent singular vectors.**

## IMPACT/APPLICATIONS

The tangent linear and adjoint model for ROMS/TOMS will provide a powerful tool for exploring data assimilation issues, such as: sensitivity to initial conditions, uncertainties in surface forcing, predictability, and ocean dynamics.

## TRANSITIONS

The work completed here will be part of the ROMS/TOMS utilities that will be freely available to all interested users.

## RELATED PROJECTS

Moore, Arango, Miller and Cornuelle have a project funded by NSF (lead PI: A. Bennett, OSU) entitled “Modular Ocean Data Assimilation”. The goal is to use the infrastructure of the Inverse Ocean Modeling (IOM) system of Chua and Bennett (2001) in conjunction with the ROMS/TOMS tangent linear and adjoint models for ocean data assimilation. IOM requires a somewhat different tangent linear model (but fortunately the same adjoint) that is based on the full fields rather than the perturbations fields. Both versions are now available for the 2D and 3D ROMS/TOMS kernel. Miller and Cornuelle are funded by NASA to explore Green’s Functions model fitting techniques (Miller and Cornuelle, 1999) with ROMS in the Southern California Bight of the California Current System <http://osep.ucsd.edu/index.cgi?rsadjoint>. Those results will prove useful in comparing with results from applying the adjoint to these same data.

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